## 13. Making \& Using Formulae

1. A rectangular clipboard has a triangular plastic pocket attached as shown in Figure 1.

The pocket is attached along edges TD and DB as shown in Figure 2.
B is $x$ centimeters from the corner C .


Figure 1


Figure 2
2. The number of diagonals, d , in a polygon with n sides is given by the formula:

$$
d=\frac{n(n-3)}{2}
$$

A polygon has 20 diagonals.
How many sides does it have?
3. Esther has a new mobile phone and considers the following daily rates.


## Green Call

40 pence per minute for the first 2 minutes

2 pence per minute after the first two minutes.
a) For Easy Call, find the cost of ten minutes in a day.
b) For Easy Call, find a formula for the cost of " $m$ " minutes in a day, $m>3$
c) For Green Call, find a formula for the cost of " $m$ " minutes in a day, $m>2$
d) Green Call claims that its system is cheaper.

Find algebraically, the least number of minutes (to the nearest minute) which must be used each day for this claim to be true.
4. The intensity of light, I, emerging after passing through a liquid with concentration, $c$, is given by the equation

$$
I=\frac{20}{2^{c}} \quad c \geq 0
$$

a) Find the intensity of light when the concentration is 3 .
b) Find the concentration of the liquid when the intensity is 10
c) What is the maximum possible intensity ?
5. A rectangular wall vent is 30 centimetres long and 20 centimetres wide.

It is to be enlarged by increasing both the length and the width by $x$ centimetres.
a) Write down the length of the new vent.
b) Show that the Area, $A$, square centimeters, of the new vent is given by

$$
A=x^{2}+50 x+600
$$

c) The area of the new vent must be at least $40 \%$ more than the original area.

Find the minimum dimensions to the nearest centimeters, of the new vent.
6. A glass vase, in the shape of a cuboid with a square base is 20 centimetres high.

It is packed in a cardboard cylinder with radius 6 centimetres and height 20 centimetres.

The corners of the vase touch the Inside of the cylinder as shown.

Show that the volume of the space between the vase and the cylinder is $720(\pi-2)$ cubic centimetres.

7. The cost of renting one of three apartments in Greece depends on the number of people sharing. If there are less than the standard number of people sharing an apartment, (known as under-occupancy), an extra fee is charged.
If there are more than the standard number, then a reduction is given to every person in the room, (based on each extra adult).
The table below shows how the cost is calculated.

| Style <br> of <br> Appartment | No. <br> Rooms | Cost <br> per person <br> per week | Based on <br> number <br> sharing | Under-occupancy <br> extrafee per <br> person (£) | Reduction <br> per extra <br> adult ( $£$ ) |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Mailia | 1 | 425 | 2 | 40 | 30 <br> (max. 2 extra) |
| Mavrikos | 2 | 310 | 4 | 45 | 25 <br> (max. 2 extra) |
| Tsilivi | 3 | 450 | 6 | 55 | 40 <br> (max. 4 extra) |

a) Find the total cost of 4 adults staying at Malia Appartments for 1 week.
b) Find a formula to calculate the total cost $£ C$, of $P$ people staying at Malia for 1 week, where $P$ is greater than 2 but less than 5 .
8. Use the information in the diagram to find a relationship connecting $a, b$ and $y$

9. Anna hired a mobile phone at a fixed charge of $£ 17.50$ per month.

She is also charged for her total call time each month.
15 minutes of this total call time are free. The rest of her call time is charged at 35 pence per minute.
a) What is the total cost for Anna's phone in a month when her total call time is 42 minutes.
b) Write down a formula for the total cost, $£ C$, for Anna's phone in a month when her total call time is $t$ minutes, where $t \geq 15$.
10. A gardener creates an L-shaped flower bed. He uses the house walls and concrete edging for the boundary as shown in figure 1.

He plans his flower bed as shown in figure 2.
a) He uses a total of $\mathbf{6}$ meters of edging.

$$
\begin{aligned}
& \mathrm{AB}=\mathrm{ED}=x \text { metres. } \\
& \mathrm{BC}=\mathrm{DC}
\end{aligned}
$$

Show that the length in metres, of BC , can be expressed as $\mathrm{BC}=3-x$.
b) Hence show that the area, $A$, in square
metres, of the flower bed can be expressed
b) Hence show that the area, $A$, in square
metres, of the flower bed can be expressed as

$$
A=6 x-3 x^{2}
$$


figure 1.

figure 2
c) Calculate algebraically the maximum area of the flower bed.
11. The cost of taking a school group to the theatre can be calculated from the information shown below.

* $\mathbf{1}$ adult goes free for every $\mathbf{1 0}$ pupils *

| Number of pupils | Cost per pupil | Cost per paying adult |
| :---: | :---: | :---: |
| Less than 10 | $£ 5.00$ | $£ 8.00$ |
| 10 to 19 | $£ 4.50$ | $£ 7.00$ |
| 20 to 29 | $£ 4.00$ | $£ 6.00$ |
| 30 to 39 | $£ 3.00$ | $£ 5.00$ |

a) Find the cost for a group of 12 pupils and 3 adults.

2 RE
b) Write down a formula to find the $\operatorname{cost}, £ C$, of taking a group of $p$ pupils and $d$ adults where $20 \leq p \leq 29$.
12. Traffic authorities are investigating the number of cars travelling along a busy stretch of road.

They assume that all cars are travelling at a speed of $v$ metres per second.
The number of cars, N , which pass a particular point on the road in one minute is given by the formula

$$
N=\frac{30 v}{2+v}
$$

In one minute, 26 cars pass a point on the road.
Find the speed of the cars in metres per second.
13. While on holiday, John's family decide to hire a car.

There are two different schemes for hiring the same type of car, Eurocar and Apex.

| EUROCAR HIRE |
| :---: |
| No deposit required |
| £15 per day |

> APEX HIRE
> $£ 50$ deposit required
> plus
> $£ 10$ per day
a) Write down a formula to find the cost, $£ C$, of hiring the car from Eurocar for $d$ days.
b) Write down a formula to find the cost, $£ C$, of hiring the car from Apex for $d$ days.
c) John's family have $£ 170$ to spend on car hire.

Which scheme should they use to have the car as long as possible?
Show clearly all your working.
4 RE
14. The area, A, of a quadrilateral drawn inside a circle can be found using the formula

$$
A=\sqrt{(s-a)(s-b)(s-c)(s-d)}
$$

where


$$
s=\frac{(a+b+c+d)}{2}
$$

Use this formula to find the area of the quadrilateral shown in the diagram.

## Give your answer correct to 2 significant figures.

15. The travelling expenses claimed by a salesperson depend on the engine capacity of the car and the number of miles travelled per week as shown in the table below.

| ENGINE CAPACITY | EXPENSES PER MILE |
| :---: | :---: |
| Less than or equal to 1 litre | $£ 0.25$ for each of the <br> first 250 miles travelled |
| greater than 1 litre <br> but less than or <br> equal to 1.2 litres | $£ 0.27$ for each of the <br> first 250 miles travelled |
| greater than 1.2 litres | $£ 0.29$ for each of the <br> first 250 miles travelled |

Where the number of miles traveled in a week is greater than 250 , $£ 0.15$ can be claimed for each additional mile.
a) Find the expenses claimed by a salesperson in a week when 550 miles are travelled and the engine capacity is 1.6 litres.

2 RE
b) Write down a formula to find the expenses $£ E$, claimed for $t$ miles travelled, where $t$ is greater than 250 , and the engine capacity is 1.6 litres

3 RE
16. The integral part of a positive real number is the part of the number which is an integer.

## EXAMPLES The integral part of 5.6 is 5

This can be written as $[\mathbf{5 . 6}]=\mathbf{5}$
The integral part of 6.2 is 6
This can be written as $[6.2]=6$
a) Find [16.7]
b) Identical boxes are packed on a board for storage. The boxes are all packed the same way round (two boxes are shown in the diagram).

i) The base of each box measures 150 millimetres by 110 millimetres.

The board measures
1.3 metres by 1 metre.

The number of boxes that can fit
 along the 1.3 metre length is given by

$$
\left[\frac{1300}{150}\right] \text { Find }\left[\frac{1300}{150}\right]
$$

ii) Write down an expression for the number of boxes which can be packed on the board shown on the right.

17. A rectangular sheet of plastic 18 cm by 100 cm is used to make a gutter for draining rain water.

The gutter is made by bending the sheet of plastic as shown below in diagram 1.

diagram 1
a) The depth of the gutter is $x$ centimetres as shown in diagram 2 below.

Write down an expression in $x$ for the width of the gutter.

b) Show that the volume, V cubic centimetres, of this gutter is given by

$$
V=1800 x-200 x^{2}
$$

c) Find the dimensions of the gutter which has the largest volume.

Show clearly all your working.
18. The cost of sending a parcel depends on the weight of the parcel and the time of delivery. The cost is calculated as shown below.

| TIME OF DELIVERY | COST |
| :--- | :--- |
| by 10 am <br> the next working day | $£ 18.20$ for 10 kg <br> and $£ 0.85$ for each extra kg. |
| by noon <br> the next working day | $£ 13.50$ for 10 kg <br> and $£ 0.75$ for each extra kg. |
| by $\mathbf{5}$ pm <br> the next working day | $£ 10.50$ for 10 kg <br> and $£ 0.50$ for each extra kg. |

a) Find the cost of sending a parcel, of weight 14 kg , for delivery by noon the next working day.
b) Write down a formula to find the cost, $£ C$, of sending a parcel, of weight $w \mathrm{~kg}$, where $w$ is greater than 10 .

The parcel has to be delivered by noon the next working day.
19. The opening on this box of tissues is in the shape of an ellipse.

The graphs of two ellipses and their equations are shown below.



Sketch the ellipse with equation $\frac{x^{2}}{36}+\frac{y^{2}}{16}=1$
20. Pipes with equal diameters are arranged in a stack.


To find the number of pipes, P , in the stack, the following formula can be used.

$$
P=\frac{(b+a)(b-a+1)}{2}
$$

where $b$ is the number of pipes on the bottom row and $a$ is the number of pipes on the top row.
a) Use this formula to find the number of pipes in a stack where $b=40$ and $a=15$.
b) In a particular stack, the number of pipes on the bottom row is twice the number on the top row.
Show that in this stack $P=\frac{3 a^{2}+3 a}{2}$ where $a$ is the number of pipes on the top row.
c) Would it be possible to arrange exactly 975 pipes in the kind of stack described in part b)
Justify your answer.
21. The diagram opposite shows two parallel lines meeting a third at $72^{\circ}$.
a) Find the value of $b$.
b) The diagram opposite shows the general case of two parallel lines meeting a third line.
Prove that in every case, the sum of the shaded angles is $180^{\circ}$.

22. An extract from a camping holiday brochure is shown below.

|  | For 14 nights |  |  |  |  | Over 14 <br> nights |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Season | Two <br> adults | Each <br> extra <br> adult | Each <br> young <br> adult <br> aged 14 <br> to17 | Each <br> child <br> aged <br> 10 to 13 | Each <br> child <br> aged <br> 0 to 9 |
| Low | $£ 399$ | $£ 74$ | $£ 40$ | Free | Each <br> additional <br> night |  |
| Mid | $£ 555$ | $£ 85$ | $£ 50$ | Free | Free | Free |
| High | $£ 699$ | $£ 95$ | $£ 60$ | $£ 46$ | Free | $£ 39$ |

a) Find the cost of a holiday for 2 adults and a child, aged 8 , for 17 nights during mid-season.
b) Write down a formula to find the cost, $£ \mathrm{C}$, of a holiday in mid-season for 2 adults and a child aged 8 lasting $t$ nights, where $t$ is greater than 14 .

3 RE
23. A square picture frame is shown.

The border of the frame (shaded in the diagram) has uniform width and an area of 48 square inches.
a) Show that $(x-y)(x+y)=48$
b) Given that $x$ and $y$ are whole numbers each greater than 10 , find suitable replacements for $x$ and $y$.

24. a) ABCD is a square of side 2 cms

Write down the ratio of the length AB to the length of AC.
b) Show that in every square, the ratio of the length of a side to the length of a diagonal is $1: \sqrt{ } 2$

25. The total time a walk takes in hillwalking depends on the horizontal distance covered ( $h$ kilometres) and the vertical height climbed $v$ metres.


For each kilometre of horizontal distance, 12 minutes should be allowed.
a) i) Write down the time which should be allowed for $h$ kilometres of horizontal distance.
ii) for each 100 metres of vertical height, 10 minutes should be allowed.

Write down the time which should be allowed for $v$ metres of vertical height.
iii) Show that the total time $T$ hours which should be allowed for the walk is given by the formula

$$
T=\frac{120 h+v}{600}
$$

b) For safety reasons, hillwalkers should be off the hills by 1900 hours.

Would it be safe to start the walk shown at 1300 hours?

27. Mr and Mrs Paton want to have their house valued before putting it up for sale.

The fee they have to pay for having this done depends on the value of their house. The fee is calculated as follows

| Value of house | Fee to be paid |
| :--- | :--- |
| First $£ 2000$ of value | $£ 5.00$ |
| Each additional $£ 500$ up to $£ 15000$ | $£ 1.00$ per $£ 500$ |
| Each additional $£ 1000$ over $£ 15000$ | $£ 1.00$ per $£ 1000$ |

a) The Paton's house is valued at $£ 33000$

What fee will they have to pay?
b) Write down a formula to find the total fee payable when a house is valued at $£ P$ thousand, where $P$ is a whole number greater than 15 .
26. The mass, $M$ grams, of a given radio-active isotope decreases with time according to the formula

$$
M=80(2)^{-t}
$$

where $t$ is the time in years.
a) The isotope weighs 80 grams at the start.

Show on the grid below,
how the mass of this isotope changes over the following 4 years.
b) Calculate how many years it takes for an isotope weighing 80 grams to decrease to a weight of $\frac{5}{8}$ of a gram.


