18. Sequences

1. Using the sequence

   1, 3, 5, 7, 9, …..

   a) Find S_3, the sum of the first 3 numbers. 1 RE

   b) Find S_n, the sum of the first n numbers. 2 RE

   c) Hence or otherwise, find the $(n + 1)^{\text{th}}$ term of the sequence 2 RE

2. The median of seven consecutive even integers is $2p+2$.

   (a) Write down, in terms of p, expressions for the seven integers. 2 RE

   (b) Show that the mean can be expressed as $2(p+1)$. 2 RE

3. a) Solve the equation

   $$2^n = 32$$ 1 KU

   b) A sequence of numbers can be grouped and added together as shown.

   The sum of 2 numbers: $(1 + 2) = 4 - 1$

   The sum of 3 numbers: $(1 + 2 + 4) = 8 - 1$

   The sum of 4 numbers: $(1 + 2 + 4 + 8) = 16 - 1$

   Find a similar expression for the sum of 5 numbers. 1 RE

   c) Find a formula for the sum of the first n numbers of this sequence. 2 RE

4. Study the pattern of numbers given below:

1^{st} pattern	$2 \times (1) - 1$	=	1
2^{nd} pattern	$2 \times (1 + 2) - 2$	=	4
3^{rd} pattern	$2 \times (1 + 2 + 3) - 3$	=	9
4^{th} pattern	$2 \times (1 + 2 + 3 + 4) - 4$	=	16

   a) Write down a similar expression for the 5^{th} pattern. 1 KU

   b) Write down the general formula for the n^{th} pattern. 2 RE

   c) If $2 \times (1 + 2 + 3 + \ldots.. + t) - t = 289$, find the value of t. 3 RE

5. A number pattern is shown below:

   $$1^3 + 1 = (1+1)(1^2 - 1 + 1)$$
   $$2^3 + 1 = (2+1)(2^2 - 2 + 1)$$
   $$3^3 + 1 = (3+1)(3^2 - 3 + 1)$$

   a) Write down a similar expression for $7^3 + 1$ 1 RE

   b) Hence write down an expression for $n^3 + 1$ 1 RE

   c) Hence find an expression for $8p^3 + 1$ 2 RE

6. 1, 3, 5, 7, ….

The **first** odd number can be expressed as $1 = 1^2 - 0^2$

The **second** odd number can be expressed as $3 = 2^2 - 1^2$

The **third** odd number can be expressed as $5 = 3^2 - 2^2$

a) Express the **fourth** odd number in this form. 1 RE

b) Express the number 19 in this form 1 RE

c) Write down a formula for the n^{th} odd number and simplify this expression. 2 RE

d) **Prove** that the product of two consecutive odd numbers is always odd 3 RE

7. A pattern of numbers is found as follows:

$3 + 2 - 1$	1st term
$6 + 3 - 3$	2nd term
$9 + 4 - 5$	3rd term
………..	
………..	

a) Write down the next 2 terms in this pattern 2 RE

b) Write an expression for the $\boldsymbol{n}^{\mathbf{th}}$ term in this pattern and express it in its simplest form. 3 RE

8. The difference between squares of any two consecutive whole numbers can be found using the following pattern.

$$2^2 - 1^2 = 3 = 2 + 1$$

$$3^2 - 2^2 = 5 = 3 + 2$$

$$4^2 - 3^2 = 7 = 4 + 3$$

a) Use this to find the difference between 24^2 and 23^2 2 RE

b) Write down an expression for the difference between the **squares of any two consecutive numbers**, and simplify it as much as possible. 4 RE

[Hint: let one of the consecutive numbers be n.]

9.

A Fibonacci sequence is a sequence of numbers.

After the first two terms, each term is the sum of the previous two terms.

$13 = 5 + 8$

e.g. **2, 3, 5, 8, 13, …..**

$5 = 2 + 3$

a) Write down the next three terms of this Fibonacci sequence.

5, -1, 4, ____, ____, ____, 1 RE

b) For the Fibonacci sequence

4, -3, 1, -2, -1, -3, -4, …….

Show that the sum of the first six terms is equal to four times the fifth term. 2 RE

c) If p and q are the first two terms of a Fibonacci sequence, **prove** that the sum of the first six terms is equal to four times the fifth term. 3 RE

10. A sequence of terms, starting with 1, is

1, 5, 9, 13, 17, …..

Consecutive terms in this sequence are formed by adding 4 to the previous term.

The total of consecutive terms of this sequence can be found using the following pattern.

Total of the first 2 terms:	$1 + 5$	$= 2 \times 3$
Total of the first 3 terms:	$1 + 5 + 9$	$= 3 \times 5$
Total of the first 4 terms:	$1 + 5 + 9 + 13$	$= 4 \times 7$
Total of the first 5 terms:	$1 + 5 + 9 + 13 + 17$	$= 5 \times 9$

a) Express the total of the first 9 terms of this sequence in the same way. 2 RE

b) The first n terms of this sequence are added.
Write down an expression, in n, for the total. 3 RE

11. A 3×3 square has been identified on the calendar shown opposite.

The numbers in the diagonally opposite corners of the square are multiplied.

These products are then subtracted in the order shown below.

(23 × 11) - (25 × 9) = 28

M	T	W	T	F	S	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

a) Repeat the process for a different 3×3 square.

Show clearly all your working. 1 RE

b) Prove that in every 3×3 square on the calendar above, the process gives the answer 28. 3 RE

12. Consecutive cubic numbers can be added using the following pattern.

$$1^3 + 2^3 = \frac{2^2 \times 3^2}{4}$$

$$1^3 + 2^3 + 3^3 = \frac{3^2 \times 4^2}{4}$$

$$1^3 + 2^3 + 3^3 + 4^3 = \frac{4^2 \times 5^2}{4}$$

a) Express $1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3$ 2 RE

b) Write down an expression for the sum of the first n consecutive cubic numbers. 3 RE

c) Write down an expression for $8^3 + 9^3 + 10^3 + \ldots\ldots + n^3$ 2 RE

13. The sequence of odd numbers starting with 3 is 3, 5, 7, 9, 11, …..

Consecutive numbers from this sequence can be added using the following pattern.

$3 + 5 + 7 + 9 = \mathbf{4 \times 6}$

$3 + 5 + 7 + 9 + 11 = \mathbf{5 \times 7}$

$3 + 5 + 7 + 9 + 11 + 13 = \mathbf{6 \times 8}$

a) Express 3 + 5 + ….. + 25 in the same way. 2 RE

b) The first n numbers in this sequence are added. Find a formula for the total. 3 RE

14. A sequence of numbers is **1, 5, 12, 22, ……..**

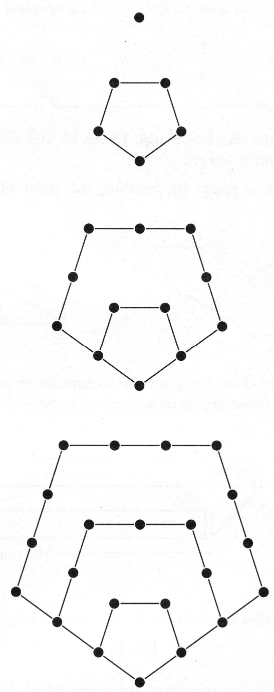
Numbers from this sequence can be illustrated in the following way using dots.

First Number (N = 1)

Second Number (N = 2)

Third Number (N = 3)

Fourth Number (N = 4)



a) What is the fifth number in this sequence?

Illustrate this in a sketch. 2 RE

b) The number of dots, D, needed to illustrate the Nth number in this sequence is given by the formula

$$D = aN^2 - bN$$

Find the values of a and b. 4 RE

15. Brackets can be multiplied out in the following way.

$$(y+1)(y+2)(y+3) = y^3 + (1+2+3)y^2 + (1\times2+1\times3+2\times3)y + 1\times2\times3$$
$$(y+2)(y+3)(y+4) = y^3 + (2+3+4)y^2 + (2\times3+2\times4+3\times4)y + 2\times3\times4$$
$$(y+3)(y+4)(y+5) = y^3 + (3+4+5)y^2 + (3\times4+3\times5+4\times5)y + 3\times4\times5$$

a) In the same way, multiply out $(y+4)(y+5)(y+6)$ 2 RE

b) In the same way, multiply out $(y+a)(y+b)(y+c)$ 2 RE

16. The following number pattern can be used to sum consecutive square whole numbers.

$$1^2 + 2^2 = \frac{2\times3\times5}{6}$$
$$1^2 + 2^2 + 3^2 = \frac{3\times4\times7}{6}$$
$$1^2 + 2^2 + 3^2 + 4^2 = \frac{4\times5\times9}{6}$$

a) Express $1^2 + 2^2 + 3^2 + \text{........} + 10^2$ in the same way. 1 RE

b) Express $1^2 + 2^2 + 3^2 + \text{........} + 10^2$ in the same way. 2 RE