2500/202

SCOTTISH CERTIFICATE OF EDUCATION 1996 WEDNESDAY, 8 MAY 1.30 PM - 4.00 PM MATHEMATICS HIGHER GRADE Paper II

Read Carefully

- 1 Full credit will be given only where the solution contains appropriate working.
- 2 Calculators may be used.
- 3 Answers obtained by readings from scale drawings will not receive any credit.

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FORMULAE LIST

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre (-g, -f) and radius $\sqrt{(g^2 + f^2 - c)}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $a.b = |a| |b| \cos \theta$, where θ is the angle between a and b

or

$$\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$$
 where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

Trigonometric formulae:

$$\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$
$$\sin 2A = 2\sin A \cos A$$

Table of standard derivatives:

$$f(x) f'(x)$$

in ax a cos ax
os ax - a sin ax

S

Table of standard integrals:

$$f(x) \qquad \int f(x) \, dx$$
$$\sin ax \qquad \frac{-1}{a} \cos ax + C$$
$$\cos ax \qquad \frac{1}{a} \sin ax + C$$

1.	A curve has equation $y = x^4 - 4x^3 + 3$.			
	(a) Find algebraically the coordinates of the stationary points.	(6)		
	(b) Determine the nature of the stationary points.	(2)		

All questions should be attempted

2. A triangle ABC has vertices A(-3, -3), B(-1, 1) and C(7, -3).



- (a) Show that the triangle ABC is right-angled at B.
- (b) The medians AD and BE intersect at M.



(i)	Find the equations of AD and BE.		
ii)	Hence find the coordinates of M.		

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(5) (3)

(3)

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3. The first four levels of a stepped pyramid with a square base are shown in the diagram.



Each level is a square-based cuboid with a height of 3 m. The shaded parts indicate the steps which have a "width" of 1 m.

The height and "width" of a step at a corner are shown in this enlargement.



With coordinate axes as shown and 1 unit representing 1 metre, the coordinates of P and A are (12, 0, 0) and (24, 0, 0).

(a) Find the coordinates of Q and R.
(b) Find the size of angle QPR.
(7)

(a)	f(x) =	$= 2x + 1$, $g(x) = x^2 + k$, where k is a constant.	- 1 km
	(i)	Find $g(f(x))$.	(2)
	(ii)	Find $f(g(x))$.	(2)
(<i>b</i>)	(i)	Show that the equation $g(f(x)) - f(g(x)) = 0$ simplifies to $2x^2 + 4x - k = 0$.	(2)
	(ii)	Determine the nature of the roots of this equation when $k = 6$.	(2)
	(iii)	Find the value of k for which $2x^2 + 4x - k = 0$ has equal roots.	(3)

5. An artist has designed a "bow" shape which he finds can be modelled by the shaded area below. Calculate the area of this shape.



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(6)

4.

(3)

6. Diagram 1 shows:

- the point A(1, 2),
- the straight line *l* passing through the origin O and the point A,
- the parabola **p** with a minimum turning point at O and passing through A,
- and the circle *c*, centre O, passing through A.



(a) Write down the equations of the line, the parabola and the circle.

6. (continued)

The following transformations are carried out:

• the line is given a translation of 4 units down (ie -4 units in the direction of the y-axis),

Diagram 2 shows the line l', the image of line l, after this translation.

- the parabola is reflected in the x-axis,
- the circle is given a translation of 2 units to the right (ie +2 units in the direction of the x-axis).



- (b) Write down the equations of l', p'(the image of the parabola p) and c' (the image of the circle c).
 (4)
- (c) (i) Show that the line l' passes through the centre of the circle c'. (1)
 - (ii) Find the coordinates of the points where the line *l'* intersects the parabola *p'*.
 (3)

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(3)

(3)

- 7. $f(x) = 2 \cos x^{\circ} + 3 \sin x^{\circ}$.
 - (a) Express f(x) in the form $k\cos(x-\alpha)^\circ$ where k > 0 and $0 \le \alpha < 360$. (4)
 - (b) Hence solve algebraically f(x) = 0.5 for $0 \le x < 360$.

8. In the diagram below, a winding river has been modelled by the curve $y = x^3 - x^2 - 6x - 2$ and a road has been modelled by the straight line AB. The road is a tangent to the river at the point A(1, -8).

- (a) Find the equation of the tangent at A and hence find the coordinates of B. (8)
- (b) Find the area of the shaded part which represents the land bounded by the river and the road.



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9. Six spherical sponges were dipped in water and weighed to see how much water each could absorb. The diameter (x millimetres) and the gain in weight (y grams) were measured and recorded for each sponge. It is thought that x and y are connected by a relationship of the form $y = ax^b$.

By taking logarithms of the values of x and y, the table below was constructed.

$X (= \log_e x)$	2.10	2.31	2.40	2.65	2.90	3.10
$Y (= \log_e y)$	7.00	7.60	7.92	8.70	9.38	10.00

A graph was drawn and is shown below.



(a) Find the equation of the line in the form Y = mX + c. (3)

(b) Hence find the values of the constants a and b in the relationship $y = ax^{b}$. (4)

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10. Two curves, y = f(x) and y = g(x), are called orthogonal if, at each point of intersection, their tangents are at right angles to each other.

(a) Diagram 1 shows the parabola

 $x^{2} + (y - 5)^{2} = 13.$

(3, 7) and (-3, 7).

orthogonal.

with equation $y = 6 + \frac{1}{9}x^2$ and the circle M with equation

These two curves intersect at

Prove that these curves are





- (b) Diagram 2 shows the circle M, from (a) above, which is orthogonal to the circle N. The circles intersect at (3, 7) and (-3, 7).
 - (i) Write down the equation of the tangent to circle M at the point (-3, 7).
 - (ii) Hence find the equation of circle N.





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11. A window in the shape of a rectangle surmounted by a semicircle is being designed to let in the maximum amount of light.

The glass to be used for the semicircular part is stained glass which lets in one unit of light per square metre; the rectangular part uses clear glass which lets in 2 units of light per square metre.



The rectangle measures 2x metres by h metres.



(<i>a</i>)	(i)	If the perimeter of the whole window is 10 metres, express h in terms of x .	(2)
	(ii)	Hence show that the amount of light, L, let in by the window is given by $L = 20x - 4x^2 - \frac{3}{2}\pi x^2$.	(2)
(<i>b</i>)	Find the m	the values of x and h that must be used to allow this design to let in aximum amount of light.	(5)

[END OF QUESTION PAPER]