

## 2005 Solution

$$1a) f(x) = x^3 \tan 2x$$

$$\begin{aligned} f'(x) &= 3x^2 \tan 2x + x^3 \cdot 2 \sec^2 2x \\ &= 3x^2 \tan 2x + 2x^3 \sec^2 2x \end{aligned}$$

$$\begin{aligned} b) \quad y &= \frac{1+x^2}{1+x} & \frac{dy}{dx} &= \frac{2x(1+x) - (1+x^2)}{(1+x)^2} \\ & & &= \frac{2x + 2x^2 - 1 - x^2}{(1+x)^2} \\ & & &= \frac{x^2 + 2x - 1}{(1+x)^2} \end{aligned}$$

$$2. \quad 2y^2 - 2xy - 4y + x^2 = 0$$

Horizontal tangent so gradient is 0

$$\text{Find } \frac{dy}{dx} = 0$$

$$4y \frac{dy}{dx} - 2 \left[ 1 \cdot y + x \frac{dy}{dx} \right] - 4 \frac{dy}{dx} + 2x = 0$$

$$4y \frac{dy}{dx} - 2y - 2x \frac{dy}{dx} - 4 \frac{dy}{dx} + 2x = 0$$

$$4y \frac{dy}{dx} - 2x \frac{dy}{dx} - 4 \frac{dy}{dx} = 2y - 2x$$

$$\frac{dy}{dx} (4y - 2x - 4) = 2y - 2x$$

$$\frac{dy}{dx} = \frac{2y - 2x}{4y - 2x - 4} = \frac{2(y - x)}{2(2y - x - 2)}$$

$$= \frac{y - x}{2y - x - 2}$$

Since  $dy/dx = 0$

$$\frac{y - x}{2y - x - 2} = 0$$

$$y - x = 0 \Rightarrow x = y$$

To find  $x$  values sub into first equation

$$2y^2 - 2xy - 4y + x^2 = 0 \quad \text{sub in } x=y$$

$$2x^2 - 2x^2 - 4x + x^2 = 0$$

$$x^2 - 4x = 0$$

$$x(x - 4) = 0$$

$$\underline{x = 0} \quad \underline{x = 4}$$

$$3. \quad f(x) = e^x \quad f(0) = e^0 = 1$$

$$f'(x) = e^x \quad f'(0) = 1$$

$$f''(x) = e^x \quad f''(0) = 1$$

$$f'''(x) = e^x \quad f'''(0) = 1$$

$$f^{(4)}(x) = e^x \quad f^{(4)}(0) = 1$$

Maclaurin

$$f(x) = \frac{f(0)}{0!} + \frac{f'(0)x}{1!} + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!}$$

$$+ \frac{f^{(4)}(0)x^4}{4!}$$

$$f(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \dots$$

Expansion for  $e^{x^2}$  (Replace  $x$  by  $x^2$ )

$$= 1 + x^2 + \frac{(x^2)^2}{2} + \frac{(x^2)^3}{6} + \dots$$

$$= 1 + x^2 + \frac{x^4}{2} + \dots$$

⚡ (Not needed)

Expansion for  $e^{x+x^2}$  could be  $e^x (e^{x^2})$

or sub  $x+x^2$  into the first expansion for

$e^x$

$$\text{i.e. } e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \dots$$

Now sub in  $x+x^2$  for  $x$

$$e^{x+x^2} = 1 + (x+x^2) + \frac{(x+x^2)^2}{2} + \frac{(x+x^2)^3}{6} + \frac{(x+x^2)^4}{24}$$

Pascals  $\Delta$   
 1 3 3 1  $(x+x^2)^3$   
 $(x)^3 + 3(x)^2(x^2) + 3(x)(x^2)^2 + (x^2)^3$   
 $= x^3 + 3x^4 + 3x^5 + x^6$   
 ↑ ↑  
 Not needed

1 4 6 4 1  
 $(x+x^2)^4$   
 $1(x)^4 + 4(x)^3(x^2) + 6(x)^2(x^2)^2 + 4(x)(x^2)^3 + (x^2)^4$   
 $x^4 + 4x^5 + 6x^6 + \dots$   
 Not needed

$$e^{x+x^2} = 1 + (x+x^2) + \frac{x^2 + 2x^3 + x^4}{2} + \frac{x^3 + 3x^4}{6} + \dots$$

$$+ \frac{x^4}{24}$$

$$= (1) + (x+x^2) + \left(\frac{1}{2}x^2 + x^3 + \frac{x^4}{2}\right) + \left(\frac{x^3}{6} + \frac{1}{2}x^4\right) + \frac{x^4}{24} + \dots$$

$$= 1 + x + \frac{3}{2}x^2 + \frac{7}{6}x^3 + \frac{25}{24}x^4 + \dots$$

$\frac{x^4}{2} + \frac{1x^4}{6} + \frac{x^4}{24}$   
 $= \frac{12x^4 + 2x^4 + x^4}{24}$   
 $= \frac{15x^4}{24}$   
 $= \frac{5x^4}{8}$   
 4.

$$4. \quad S_n = 8n - n^2, \quad n \geq 1$$

$$S_1 = 8(1) - 1^2 = 8 - 1 = 7 \Rightarrow u_1 = 7$$

$$S_2 = 8(2) - 2^2 = 16 - 4 = 12 \Rightarrow u_2 = 5 \text{ (take away)}$$

$$S_3 = 8(3) - 3^2 = 24 - 9 = 15 \Rightarrow u_3 = 3$$

$\{7, 5, 3, \dots\}$  Sequence.

$\underbrace{\quad \quad}_{-2} \quad \underbrace{\quad \quad}_{-2}$

Arithmetic Sequence

$$u_n = a + (n-1)d$$

$$= 7 + (n-1)(-2)$$

$$= 7 - 2n + 2$$

$$= 9 - 2n$$

$$5. \quad \int_0^3 \frac{x}{\sqrt{1+x}} dx$$

$$u = 1+x$$

$$\frac{du}{dx} = 1$$

so  $du = dx$

$$= \int_0^3 \frac{x}{\sqrt{u}} du$$

need to change to u's

$$u = 1+x$$

$$\text{so } x = u-1$$

$$= \int_1^4 \frac{u-1}{\sqrt{u}} du$$

Change limits

$$x=0$$

$$u = 1+0 = 1$$

$$x=3$$

$$u = 1+3 = 4$$

5.

$$\begin{aligned}
&= \int_1^4 \frac{u}{\sqrt{u}} - \frac{1}{\sqrt{u}} du \\
&= \int_1^4 u^{1/2} - u^{-1/2} du = \left[ \frac{u^{3/2}}{3/2} - \frac{u^{1/2}}{1/2} \right]_1^4 \\
&= \left[ \frac{2}{3} \sqrt{u}^3 - 2\sqrt{u} \right]_1^4 \\
&= \left( \frac{2}{3} \sqrt{4}^3 - 2\sqrt{4} \right) - \left( \frac{2}{3} \sqrt{1}^3 - 2\sqrt{1} \right) \\
&= \left( \frac{2}{3}(8) - 4 \right) - \left( \frac{2}{3} - 2 \right) = \frac{16}{3} - 4 - \frac{2}{3} + 2 \\
&= -2 + \frac{14}{3} \\
&= 4^{2/3} - 2 = 2^{2/3}.
\end{aligned}$$

6.  $\left( \begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 2 & \lambda & 1 & 0 \\ 3 & 3 & 9 & 5 \end{array} \right) r_3 \rightarrow r_3 - 3r_1 \quad \left( \begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & \lambda-2 & -3 & -2 \\ 0 & 0 & 3 & 2 \end{array} \right)$

$$r_2 \rightarrow r_2 - 2r_1$$

$$r_2 \rightarrow \left( \begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & \lambda-2 & -3 & -2 \\ 0 & 0 & 3 & 2 \end{array} \right) \rightarrow \left( \begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & \lambda-2 & -3 & -2 \\ 0 & 0 & 3 & 2 \end{array} \right)$$

From  $r_3 \quad 3z = 2 \Rightarrow z = 2/3$

From  $r_2$

$$(\lambda-2)y - 3z = -2$$

$$\Rightarrow (\lambda-2)y = -2 + 2$$

$$\cdot (\lambda-2)y = 0 \quad \Rightarrow y = 0.$$

From  $r_1$   $x + y + 2z = 1$

$$x + 0 + 2\left(\frac{2}{3}\right) = 1$$

$$x = 1 - \frac{4}{3} = -\frac{1}{3}$$

so when  $\lambda \neq 2$   $x = -\frac{1}{3}, y = 0, z = \frac{2}{3}$   
 $\left\{-\frac{1}{3}, 0, \frac{2}{3}\right\}$ .

When  $\lambda = 2$

$$r_2 \rightarrow \begin{array}{ccc|c} 0 & 0 & -3 & -2 \end{array}$$

but  $r_3 \rightarrow \begin{array}{ccc|c} 0 & 0 & 3 & 2 \end{array}$  which are  
the same so there are many  
solutions as equation is redundant.

$$7. \quad A = \begin{pmatrix} 0 & 4 & 2 \\ 1 & 0 & 1 \\ -1 & -2 & -3 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 0 & 4 & 2 \\ 1 & 0 & 1 \\ -1 & -2 & -3 \end{pmatrix} \begin{pmatrix} 0 & 4 & 2 \\ 1 & 0 & 1 \\ -1 & -2 & -3 \end{pmatrix} = \begin{pmatrix} 2 & -4 & -2 \\ -1 & 2 & -1 \\ 1 & 2 & 5 \end{pmatrix}$$

$$\text{So } A^2 + A = \begin{pmatrix} 2 & -4 & -2 \\ -1 & 2 & -1 \\ 1 & 2 & 5 \end{pmatrix} + \begin{pmatrix} 0 & 4 & 2 \\ 1 & 0 & 1 \\ -1 & -2 & -3 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} \\ = 2I$$

$$\text{So } (A^2 + A = 2I)$$

$$A^2 + A = 2I \quad \text{multiply by } A^{-1}$$

$$A^{-1} \cdot A^2 + A^{-1} \cdot A = 2A^{-1}I$$

$$A + I = 2A^{-1} \quad \left( \begin{array}{l} \text{Remember} \\ A^{-1}A = I \end{array} \right)$$

$$\Rightarrow A^{-1} = \frac{1}{2}(A + I)$$

$$= \frac{1}{2}A + \frac{1}{2}I$$

$$\text{So } p = \frac{1}{2} \quad q = \frac{1}{2}$$

$$8. \quad \begin{aligned} x - 4y + 2z &= 1 \\ x - y - z &= -5 \end{aligned}$$

\* To find the direction vector of the line of intersection cross the vector planes.

$$\begin{array}{ccc|ccc} \mathbf{i} & 1 & 1 & \mathbf{i} & 1 & 1 \\ \mathbf{j} & -4 & -1 & \mathbf{j} & -4 & -1 \\ \mathbf{k} & 2 & -1 & \mathbf{k} & 2 & -1 \end{array}$$

$$\begin{aligned} & (4\mathbf{i} - \mathbf{k} + 2\mathbf{j}) - (-4\mathbf{k} - 2\mathbf{i} - \mathbf{j}) \\ &= 4\mathbf{i} - \mathbf{k} + 2\mathbf{j} + 4\mathbf{k} + 2\mathbf{i} + \mathbf{j} = \underline{6\mathbf{i}} + \underline{3\mathbf{j}} + \underline{3\mathbf{k}} \end{aligned}$$

$$\mathbf{v} = \begin{pmatrix} 6 \\ 3 \\ 3 \end{pmatrix}$$

To find a point let  $z=0$

$$\left. \begin{array}{l} x - 4y = 1 \\ x - y = -5 \end{array} \right\} \Rightarrow -3y = 6 \Rightarrow y = -2$$

sub to find  $x$

$$x = 1 + 4y = 1 - 8 = -7$$

So point is  $(-7, -2, 0)$

So equation of line is:

$$\frac{x+7}{6} = \frac{y+2}{3} = \frac{z}{3} \quad [ = \lambda ]$$

Show this line lies on the

plane  $x + 2y - 4z = -11$  sub line into plane!! 9.

$$x = 6\lambda - 7, \quad y = 3\lambda - 2, \quad z = 3\lambda.$$

$$x + 2y - 4z = -11$$

$$6\lambda - 7 + 2(3\lambda - 2) - 4(3\lambda)$$

$$6\lambda - 7 + 6\lambda - 4 - 12\lambda = -11 \Rightarrow \text{lies on plane.}$$

9.  $z + 2i\bar{z} = 8 + 7i$

$z = a + ib$  sub into expression

$$\underline{a} + i\underline{b} + 2i(\underline{a} - i\underline{b}) = 8 + 7i$$

$$\underline{a} + i\underline{b} + 2i\underline{a} - 2i^2\underline{b} = 8 + 7i$$

$$a + ib + 2ia + 2b = 8 + 7i$$

Equate real and imaginary parts.

$$a + 2b = 8$$

$$b + 2a = 7$$

$$\begin{array}{r} \xrightarrow{x-2} \\ \xrightarrow{x-2} \\ \hline 2a + b = 7 \\ -2a - 4b = -16 \\ \hline -3b = -9 \end{array}$$

$$\Rightarrow b = 3$$

sub to find a

$$a = 8 - 2b$$

$$= 8 - 2(3)$$

$$= \underline{\underline{2}}$$

$$\text{So } z = \underline{\underline{2 + 3i}}$$

$$10. \sum_{r=1}^n \frac{1}{r(r+1)(r+2)} = \frac{1}{4} - \frac{1}{2(n+1)(n+2)}$$

When  $n=1$

$$L.S = \frac{1}{1(2)(3)} = \frac{1}{6}$$

$$R.S = \frac{1}{4} - \frac{1}{2(2)(3)} = \frac{1}{4} - \frac{1}{12} = \frac{3}{12} - \frac{1}{12} = \frac{1}{6}$$

$\Rightarrow$  True when  $n=1$

Assume true when  $n=k$

$$\text{So } \sum_{r=1}^k \frac{1}{r(r+1)(r+2)} = \frac{1}{4} - \frac{1}{2(k+1)(k+2)}$$

Consider  $n=k+1$

Adding on

$$\frac{1}{(k+1)(k+1)(k+2)} = \frac{1}{(k+1)(k+2)(k+3)}$$

Looking to  
prove equal to

$$\frac{1}{4} - \frac{1}{2(k+1+1)(k+1+2)} = \frac{1}{4} - \frac{1}{2(k+2)(k+3)}$$

When  $n = k+1$

$$\sum_{r=1}^{k+1} = \sum_{r=1}^k \frac{1}{r(r+1)(r+2)} + \frac{1}{(k+1)(k+2)(k+3)}$$

$$\text{So } \frac{1}{4} - \frac{1}{2(k+1)(k+2)} + \frac{1}{(k+1)(k+2)(k+3)}$$

$$= \frac{1}{4} + \left[ \frac{1}{(k+1)(k+2)(k+3)} - \frac{1}{2(k+1)(k+2)} \right]$$

$$= \frac{1}{4} + \left[ \frac{2 - 1(k+3)}{2(k+1)(k+2)(k+3)} \right]$$

$$= \frac{1}{4} + \frac{2 - k - 3}{2(k+1)(k+2)(k+3)}$$

$$= \frac{1}{4} + \frac{(-k-1)}{2(k+1)(k+2)(k+3)}$$

$$= \frac{1}{4} - \frac{(k+1)}{2(k+1)(k+2)(k+3)}$$

$$= \frac{1}{4} - \frac{1}{2(k+2)(k+3)} = \frac{1}{4} - \frac{1}{2((k+1)+1)((k+1)+2)}$$

So true when  $n=1$ , assumed true when  $n=k$ ,  
proved true when  $n=k+1 \Rightarrow \text{True } \forall n \geq 1$

$$11. \quad y = \frac{x^3}{x-2}$$

a) Vertical Asymptote

Denominator is zero

$$\text{so } x-2=0 \Rightarrow \underline{\underline{x=2}}$$

b) Stationary points when  $dy/dx=0$

$$\frac{dy}{dx} = \frac{3x^2(x-2) - x^3(1)}{(x-2)^2}$$

$$= \frac{3x^3 - 6x^2 - x^3}{(x-2)^2} = \frac{2x^3 - 6x^2}{(x-2)^2}$$

$$\text{so } \frac{2x^3 - 6x^2}{(x-2)^2} = 0$$

$$2x^3 - 6x^2 = 0$$

$$2x^2(x-3) = 0$$

$$\Rightarrow \begin{array}{l} 2x^2 = 0 \\ x = 0 \end{array} \quad \rightsquigarrow \begin{array}{l} x-3 = 0 \\ x = 3 \end{array}$$

Stationary points (0,0) (3,27)

$$(c) \quad y = \left| \frac{x^3}{x-2} \right| + 1$$

$\nearrow$   
 change neg parts to positive parts  
 then move up 1 box.

$$\text{so } (0,0) \rightarrow (0,1)$$

$$(3,27) \rightarrow (3,28)$$

$$12. a) \quad z^4 = (\cos \theta + i \sin \theta)^4 \quad \text{Pascals } \Delta$$

$$1 \quad 4 \quad 6 \quad 4 \quad 1$$

$$z^4 = 1(\cos \theta)^4 + 4(\cos \theta)^3(i \sin \theta) + 6(\cos \theta)^2(i \sin \theta)^2 + 4(\cos \theta)(i \sin \theta)^3 + (i \sin \theta)^4$$

$$= \cos^4 \theta + 4i \sin \theta \cos^3 \theta - 6 \sin^2 \theta \cos^2 \theta - 4i \sin^3 \theta \cos \theta + \sin^4 \theta$$

$$= (\cos^4 \theta - 6 \sin^2 \theta \cos^2 \theta + \sin^4 \theta) + i(4 \sin \theta \cos^3 \theta - 4 \sin^3 \theta \cos \theta)$$

$$b) \quad \text{De Moivre's} \quad z^4 = \cos 4\theta + i \sin 4\theta$$

$$c) \quad \left( \frac{\cos 4\theta}{\cos^2 \theta} = p \cos^2 \theta + q \sec^2 \theta + r \right) \quad \text{show this}$$

compare real parts (a) & (b)

$$\cos 4\theta = \cos^4 \theta - 6 \sin^2 \theta \cos^2 \theta + \sin^4 \theta$$

Firstly change sin's to cos's

we  $\{\sin^2 x + \cos^2 x = 1\}$  so  $\sin^2 \theta = 1 - \cos^2 \theta$

$$\cos 4\theta = \cos^4 \theta - 6(1 - \cos^2 \theta) \cos^2 \theta + (1 - \cos^2 \theta)^2$$

$$= \cos^4 \theta - 6 \cos^2 \theta + 6 \cos^4 \theta + 1 - 2 \cos^2 \theta + \cos^4 \theta$$

$$\cos 4\theta = 8 \cos^4 \theta - 8 \cos^2 \theta + 1$$

divide by  $\cos^2 \theta$

$$\frac{\cos 4\theta}{\cos^2 \theta} = \frac{8 \cos^4 \theta}{\cos^2 \theta} - \frac{8 \cos^2 \theta}{\cos^2 \theta} + \frac{1}{\cos^2 \theta}$$

$$= 8 \cos^2 \theta - 8 + \sec^2 \theta$$

$$= 8 \cos^2 \theta + \sec^2 \theta - 8$$

so  $p = 8$ ,  $q = 1$  and  $r = -8$

$$13. \quad \frac{1}{x^3+x} = \frac{1}{x(x^2+1)}$$

$$\text{so } \frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1} \leftarrow \begin{array}{c} \text{Irreducible} \\ \text{quadratic} \end{array}$$

$$\text{so } 1 = A(x^2+1) + (Bx+C)x$$

$$\text{let } x=0$$

$$1 = A + 0 \quad \Rightarrow \quad A=1$$

$$\text{let } x=1$$

$$1 = A(1+1) + (B+C)1$$

$$1 = 2A + B + C$$

$$1 = 2 + B + C$$

$$\Rightarrow B + C = -1 \quad \text{--- (1)}$$

$$\text{let } x=-1$$

$$1 = 2A + (-B+C) - 1$$

$$1 = 2A + B - C$$

$$1 = 2 + B - C$$

$$\Rightarrow B - C = -1 \quad \text{--- (2)}$$

Solve

$$\begin{array}{r} B+C=-1 \\ B-C=-1 \\ \hline 2B=-2 \end{array} \Rightarrow B=-1, C=0$$

$$\text{So } \frac{1}{x^3+x} = \frac{1}{x} + \frac{-x}{x^2+1}$$

$$= \frac{1}{x} - \frac{x}{x^2+1}$$

$$I(k) = \int_1^k \frac{1}{x^3+x} dx$$

$$= \int_1^k \left( \frac{1}{x} - \frac{x}{x^2+1} \right) dx$$

$$= \left[ \ln x - \frac{1}{2} \ln(x^2+1) \right]_1^k$$

$$= \left( \ln k - \frac{1}{2} \ln(k^2+1) \right) - \left( \ln 1 - \frac{1}{2} \ln 2 \right)$$

$$= \ln k - \frac{1}{2} \ln(k^2+1) - \ln 1 + \frac{1}{2} \ln 2$$

$$= \ln k - \ln(k^2+1)^{1/2} - 0 + \frac{1}{2} \ln 2 = \ln k - \ln(k^2+1)^{1/2} + \ln 2^{1/2}$$

$$= \ln \left[ \frac{k \times 2^{1/2}}{(k^2+1)^{1/2}} \right] = \ln \left[ \frac{\sqrt{2} k}{\sqrt{k^2+1}} \right]$$

$$e^{I(k)} = e^{\ln \left[ \frac{\sqrt{2} k}{\sqrt{k^2+1}} \right]} = \frac{\sqrt{2} k}{\sqrt{k^2+1}}$$

$$\int \frac{x}{x^2+1} dx$$

let  $u = x^2+1$   
 $\frac{du}{dx} = 2x$

$$\int \frac{x}{u} \cdot \frac{du}{2x}$$

$$= \int \frac{1}{2} u du$$

$$\text{Value of } \lim_{k \rightarrow \infty} \frac{\sqrt{2} k}{\sqrt{k^2+1}}$$

as  $k$  gets very very large look at values of above.

Try to simplify expression where  $k$  is in one part of formula to simplify

$$\frac{\sqrt{2} k}{\sqrt{k^2+1}} \xrightarrow[\text{top/bottom by } k]{\text{divide}} \frac{\sqrt{2}}{\frac{\sqrt{k^2+1}}{k}}$$

$$= \frac{\sqrt{2}}{\sqrt{\frac{k^2}{k^2} + \frac{1}{k^2}}}$$

$$= \frac{\sqrt{2}}{\sqrt{1 + \frac{1}{k^2}}} \leftarrow$$

as this becomes very large  $\rightarrow 0$

$$\text{so } \frac{\sqrt{2}}{\sqrt{1+0}} \rightarrow \sqrt{2}$$

$$14. \quad \frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 20 \sin x$$

Auxilliary Equation

$$\alpha^2 - 3\alpha + 2 = 0$$

$$(\alpha - 1)(\alpha - 2) = 0$$

$$\underline{\underline{\alpha = 1}} \quad \text{or} \quad \underline{\underline{\alpha = 2}}$$

C.F.  $y = Ae^x + Be^{2x}$

P.I.  $y = C \sin x + D \cos x$

$$\frac{dy}{dx} = C \cos x - D \sin x$$

$$\frac{d^2y}{dx^2} = -C \sin x - D \cos x$$

Now sub in to first equation

$$\begin{aligned} (-C \sin x - D \cos x) - 3(C \cos x - D \sin x) + 2(C \sin x + D \cos x) \\ = 20 \sin x \end{aligned}$$

Now equate !!

$$-C \sin x - D \cos x - 3C \cos x + 3D \sin x + 2C \sin x + 2D \cos x$$

$$= 20 \sin x$$

sin x's

$$-C + 3D + 2C = 20$$

$$\Rightarrow C + 3D = 20 \quad \text{--- (1)}$$

cos x's

$$-D - 3C + 2D = 0$$

$$D - 3C = 0 \quad \text{--- (2)}$$

$$C + 3D = 20 \quad \text{---} \times 3 \rightarrow 3C + 9D = 60$$

$$-3C + D = 0$$


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$$10D = 60$$

$$\underline{D = 6}$$

sub to find C

$$C + 3D = 20$$

$$C + 18 = 20 \quad \Rightarrow \underline{C = 2}$$

Overall solution

$$y = C.F + P.I$$

$$y = Ae^{2x} + Be^{2x} + 2 \sin x + 6 \cos x$$

Need to find particular solution i.e. find

A and B.

$$y = A e^x + B e^{2x} + 2 \sin x + 6 \cos x$$

Given  $y=0$   
 $\frac{dy}{dx}=0$   
 $x=0$

sub in  $y=0, x=0$

$$0 = A e^0 + B e^0 + 2 \sin 0 + 6 \cos 0$$

$$0 = A + B + 0 + 6 \Rightarrow A + B = -6$$

Find  $\frac{dy}{dx}$

$$\frac{dy}{dx} = A e^x + 2B e^{2x} + 2 \cos x - 6 \sin x$$

so  $0 = A e^0 + 2B e^0 + 2 \cos 0 - 6 \sin 0$

$$0 = A + 2B + 2 - 0$$

$$\Rightarrow A + 2B = -2$$

$$\begin{array}{r}
 A + B = -6 \quad \times (-1) \rightarrow -A - B = 6 \\
 A + 2B = -2 \\
 \hline
 B = 4
 \end{array}$$

$$\begin{array}{l}
 \text{So } A + B = -6 \\
 A + 4 = -6 \quad \Rightarrow A = -10
 \end{array}$$

Solution is

$$\underline{y = -10e^x + 4e^{2x} + 2\sin x + 6\cos x}$$

$$15. \quad f(x) = \sqrt{\sin x} = (\sin x)^{1/2}$$

$$f'(x) = \frac{1}{2} (\sin x)^{-1/2} \times \cos x$$

$$= \frac{\cos x}{2\sqrt{\sin x}}$$

$$15b) f(x) = g(x)^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2} (g(x))^{-1/2} \times g'(x)$$

$$= \frac{g'(x)}{2\sqrt{g(x)}}$$

So k=2.

$$\text{So } * \int \frac{x}{\sqrt{1-x^2}} dx \leftarrow \frac{g'(x)}{2\sqrt{g(x)}}$$

$$\int \frac{2x}{2\sqrt{1-x^2}}$$

so  $g(x)$  must be  $1-x^2$

$$g'(x) = -2x$$

$$= - \int \frac{-2x}{2\sqrt{1-x^2}} dx$$

still the same as \*

$$= -\sqrt{1-x^2} + C$$

$$15c) \int_0^{\frac{1}{2}} \sin^{-1} x \, dx$$

	Diff	Int
u	$\sin^{-1} x$	1
$\frac{dv}{dx}$	$\frac{1}{\sqrt{1-x^2}}$	$x$

$$\int_0^{\frac{1}{2}} \sin^{-1} x \, dx = x \sin^{-1} x - \int \frac{x}{\sqrt{1-x^2}} \, dx$$

$$= \left( x \sin^{-1} x + \sqrt{1-x^2} \right) \Big|_0^{\frac{1}{2}}$$

$$= \left( \frac{1}{2} \sin^{-1} \left( \frac{1}{2} \right) + \sqrt{1 - \frac{1}{4}} \right) - (0 + \sqrt{1})$$

$$= \left( \frac{1}{2} \times \frac{\pi}{6} + \sqrt{\frac{3}{4}} \right) - 1$$

$$= \frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1$$