

# Past Paper Solution 2008.

$$1. \quad a = 2 \quad S_n = \frac{n}{2} \{2a + (n-1)d\}$$

$$U_{20} = 97$$

$$U_{20} = a + (n-1)d$$

$$97 = 2 + 19d$$

$$\Rightarrow 19d = 95$$

$$\boxed{d = 5}$$

$$S_{50} = \frac{50}{2} \{4 + 49(5)\} = \underline{\underline{6225}}$$

$$2. \text{ a) } f(x) = \cos^{-1}(3x)$$

$$f'(x) = -\frac{1}{\sqrt{1-(3x)^2}} \times 3 = -\frac{3}{\sqrt{1-9x^2}}$$

$$\text{b) } x = 2 \sec \theta$$

$$y = 3 \sin \theta$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

$$= \frac{3 \cos \theta}{2 \sec \theta \cdot \tan \theta}$$

$$= 3 \cos \theta \cdot \frac{\cos \theta}{2} \cdot \frac{\cos \theta}{\sin \theta}$$

$$= \frac{3}{2} \frac{\cos^3 \theta}{\sin \theta}$$

2.

3.  $y = f(x)$

$$y = -f(x+1)$$

Move graph 1 place left

Reflect in  $x$ -axis

Asymptotes

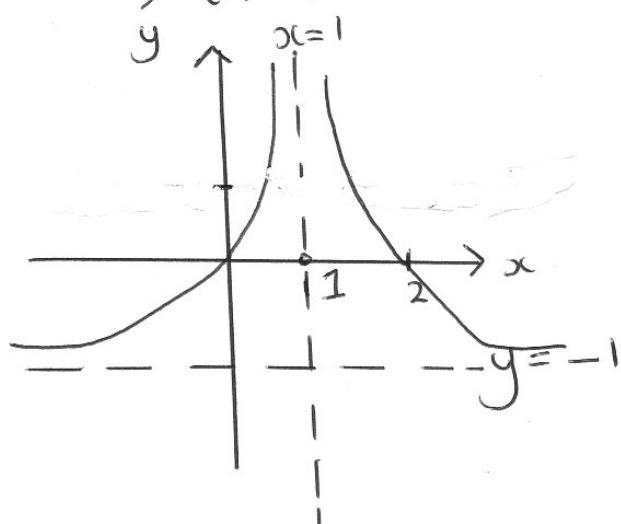
$$x=2 \xrightarrow{\text{move 1 place}} x=1 \xrightarrow{\text{reflect}} \text{cloud} \text{ same!}$$

$$y=1 \xrightarrow{\text{move 1 place}} \text{same} \xrightarrow{\text{reflect}} \text{cloud} \text{ } y=-1$$

So new asymptotes are  $\text{cloud } x=1 \text{ and } y=-1$

Point  $(1, 0)$   $\xrightarrow{\text{move 1}}$   $(0, 0)$   $\xrightarrow{\text{reflect}}$   $(0, 0)$

$(3, 0)$   $\rightarrow (2, 0) \rightarrow (2, 0)$



$$Q4. \frac{12x^2 + 20}{x(x^2 + 5)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 5}$$

$$\Rightarrow 12x^2 + 20 = A(x^2 + 5) + (Bx + C)x$$

Let  $x = 0$

$$20 = 5A \Rightarrow A = 4$$

Let  $x = 1$

$$12 + 20 = 6A + B + C$$

$$8 = B + C$$

Let  $x = -1$

$$12 + 20 = 6A + B - C$$

$$8 = B - C$$

$$B + C = 8$$

$$B - C = 8$$

$$\frac{B + C}{2B} = \frac{8}{16} \Rightarrow B = 8, C = 0$$

$$\text{so } \frac{12x^2 + 20}{x(x^2 + 5)} = \frac{4}{x} + \frac{8x}{x^2 + 5}$$

$$\begin{aligned}
 \int \frac{12x^2 + 20}{x(x^2 + 5)} dx &= \int_1^2 \frac{4}{x} + \frac{8x}{x^2 + 5} dx \\
 &= \int_1^2 \frac{4}{x} dx + \int_1^2 \frac{8x}{x^2 + 5} dx \\
 &= [4 \ln x]_1^2 + \int \frac{8x}{u} \cdot \frac{du}{2x} \quad \text{let } u = x^2 + 5 \\
 &\quad \frac{du}{dx} = 2x \\
 &\quad dx = \frac{du}{2x} \\
 &= [4 \ln x]_1^2 + \int \frac{4}{u} du \\
 &= [4 \ln x]_1^2 + [4 \ln u]_1^2 \\
 &= [4 \ln x]_1^2 + [4 \ln(x^2 + 5)]_1^2 \\
 &= [4 \ln 2 - 4 \ln 1] + [4 \ln 9 - 4 \ln 6] \\
 &= 4 \ln \frac{2 \times 9}{1 \times 6} = 4 \ln \frac{18}{6} = 4 \ln 3
 \end{aligned}$$

$$Q5. \quad xy^2 + 3x^2y = 4 \quad 5$$

$$1. \quad y^2 + x \cdot 2y \frac{dy}{dx} + 6xy + 3x^2 \frac{dy}{dx} = 0$$

$$y^2 + 2xy \frac{dy}{dx} + 6xy + 3x^2 \frac{dy}{dx} = 0$$

$$2xy \frac{dy}{dx} + 3x^2 \frac{dy}{dx} = -6xy - y^2$$

$$\frac{dy}{dx}(2xy + 3x^2) = -6xy - y^2$$

$$\frac{dy}{dx} = \frac{-6xy - y^2}{2xy + 3x^2}$$

Equation of tangent when  $x=1$

sub to find  $y$

$$xy^2 + 3x^2y = 4$$

$$y^2 + 3y - 4 = 0$$

$$(y-1)(y+4) = 0$$

$$y=1 \quad \text{or} \quad y=-4$$

$$\text{so } y=1$$

$y > 0$  start  
of question

$$x=1 \\ y=1$$

$$\frac{dy}{dx} = \frac{-6xy - 4^2}{2xy + 3x^2}$$

6.

$$= \frac{-6-1}{2+3} = -\frac{7}{5}$$

Equation of tangent

$$\frac{dy}{dx} = -7/5$$

$$x=1 \\ y=1$$

$$y-1 = -\frac{7}{5}(x-1)$$

$$5y - 5 = -7x + 7$$

$$\underline{5y + 7x = 12}$$

Q6.

a) Matrix is singular if  $ad-bc=0$

$$\begin{pmatrix} 1 & x \\ x & 4 \end{pmatrix}$$

$$4-x^2=0 \\ (2+x)(2-x)=0$$

$$\Rightarrow x = \pm 2$$

b) When  $x=2$

$$A^2 = A \cdot A = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & 10 \\ 10 & 20 \end{pmatrix}$$

$$PA = \begin{pmatrix} 5 & 10 \\ 10 & 20 \end{pmatrix}$$

7.

$$\Rightarrow A^2 = 5 \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix} \Rightarrow P=5$$

$$\begin{aligned} A^4 &= A^2 \cdot A^2 = 5A \cdot 5A \\ &= 25 A^2 \\ &= 25 \cdot (5A) \\ &= 125A \end{aligned}$$

$$q = 125$$

7.  $\int 8x^2 \sin 4x \, dx$

	Diff	Int
u	$8x^2$	$\sin 4x$
$\frac{du}{dx}$	$16x$	$-\frac{1}{4} \cos 4x$

$$uv - \int v \frac{du}{dx} \, dx = 8x^2 \left( -\frac{1}{4} \cos 4x \right) - \int * -4x \cos 4x \, dx$$

\*

	Diff	Int
$x$	$\cos 4x$	$-\frac{1}{4} \sin 4x$
1		

$$= -2x^2 \cos 4x + 4 \int * x \cos 4x \, dx$$

DO AGAIN

8.

$$\begin{aligned} \text{So } \int x \cos 4x \, dx &= \frac{1}{4} x \sin 4x - \int \frac{1}{4} \sin 4x \, dx \\ &= \frac{1}{4} x \sin 4x + \frac{1}{16} \cos 4x + C \end{aligned}$$

Now sub back

$$\begin{aligned} \int 8x^2 \sin 4x \, dx &= -2x^2 \cos 4x + 4 \left[ \frac{1}{4} x \sin 4x + \frac{1}{16} \cos 4x \right] + C \\ &= -2x^2 \cos 4x + x \sin 4x + \frac{1}{4} \cos 4x + C \end{aligned}$$

$$\begin{aligned} Q8. \quad &\left(x^2 + \frac{1}{x}\right)^{10} \\ &\binom{10}{r} (x^2)^{10-r} \left(\frac{1}{x}\right)^r \end{aligned}$$

Look at  $x$  terms.

$$\begin{aligned} &(x^2)^{10-r} \cdot \left(\frac{1}{x}\right)^r \\ &= \frac{x^{20-2r}}{x^r} = x^{20-3r} \end{aligned}$$

$x^{14}$  term  $\Rightarrow 20-3r = 14$

$$\begin{aligned} 3r &= 6 \\ r &= 2. \end{aligned}$$

$$\therefore \binom{10}{2} (x^2)^8 \left(\frac{1}{x}\right)^2$$

9.

$$\begin{aligned}
 &= \frac{10!}{8!2!} \cdot \frac{x^{16}}{x^2} = \frac{10 \times 9}{2 \times 1} x^{14} \\
 &= \underline{\underline{45 x^{14}}}
 \end{aligned}$$

$$Q9. \quad \frac{d}{dx} \tan x = \sec^2 x$$

$$1 + \tan^2 x = \sec^2 x$$

$$\begin{aligned}
 L.S. &= 1 + \tan^2 x \\
 &= 1 + \frac{\sin^2 x}{\cos^2 x} \\
 &= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} \\
 &= \frac{1}{\cos^2 x} = \underline{\underline{\sec^2 x}} = R.S.
 \end{aligned}$$

$$\begin{aligned}
 \int \tan^2 x \, dx &= \int \sec^2 x - 1 \, dx \\
 &= \tan x - x + C.
 \end{aligned}$$

Q10.  $v = t^3 - 12t^2 + 32t$  10.

$$a = \frac{dv}{dt} = 3t^2 - 24t + 32$$

at  $(t=0)$   $a = 32 \text{ ms}^{-2}$

$$\begin{aligned} (b) \quad s &= \int v \, dt = \int t^3 - 12t^2 + 32t \, dt \\ &= \frac{t^4}{4} - \frac{12t^3}{3} + \frac{32t^2}{2} + C \\ &= \frac{1}{4}t^4 - 4t^3 + 16t^2 + C \end{aligned}$$

when  $\begin{cases} t=0 \\ s=0 \end{cases}$

so  $s=0, t=0$   
 $\Rightarrow (C=0)$

$$so \quad 0 = \frac{1}{4}t^4 - 4t^3 + 16t^2$$

Returns to origin when  $s=0$

$$\Rightarrow 0 = \frac{1}{4}t^4 - 4t^3 + 16t^2$$

$$\begin{aligned} (\times 4) \quad 0 &= t^4 - 16t^3 + 64t^2 \\ 0 &= t^2(t^2 - 16t + 64) \end{aligned}$$

11.

$$O = t^2(t - 8)(t + 8)$$

$t = 0$  or  $t = 8$  seconds.

Q11. A. If  $m^2$  is divisible by 4 then  
m is divisible by 4

If  $m = 2$ ,  $m^2 = 4$   
 $\Rightarrow$  Not true.

B. cube of odd p + square of even q  
is always odd

$$\begin{aligned}
 \text{let } p &= 2k+1 & \Rightarrow (2k+1)^3 + (2k)^2 \\
 q &= 2k & = (8k^3 + 12k^2 + 6k + 1) + 4k^2 \\
 & & = 8k^3 + 16k^2 + 6k + 1 \\
 & & = 2(4k^3 + 8k^2 + 3k) + 1 \\
 & \Rightarrow \text{Always odd.} \\
 & \Rightarrow \text{True.}
 \end{aligned}$$

$$Q12 \quad f(x) = x \ln(2+x)$$

12.

$$f(x) = \frac{f(0)}{0!} + \frac{f'(0)x}{1!} + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \dots$$

$$f(x) = x \ln(2+x)$$

$$f'(x) = 1 \cdot \ln(2+x) + \frac{x}{2+x}$$

Quotient Rule

$$f''(x) = \frac{1}{2+x} + \frac{1(2+x) - x(1)}{(2+x)^2}$$

$$= \frac{1}{2+x} + \frac{2}{(2+x)^2}$$

$$\begin{cases} f(0) = 0 \\ f'(0) = \ln 2 \end{cases}$$

$$\begin{cases} f''(0) = \frac{1}{2} + \frac{1}{2} \\ = 1 \end{cases}$$

$$f'''(x) = -(2+x)^{-2} - 4(2+x)^{-3}$$

$$= -\frac{1}{(2+x)^2} - \frac{4}{(2+x)^3}$$

$$\begin{cases} f'''(0) = -\frac{1}{4} - \frac{4}{8} \\ = -\frac{3}{4} \end{cases}$$

$$so \quad x \ln(2+x)$$

$$= x \ln 2 + \frac{x^2}{2} - \frac{x^3}{8} + \dots$$

Composite series

13.

$$x \ln(2+x) = x \ln 2 + \frac{x^2}{2} - \frac{x^3}{8}$$

$x \ln(2-x)$  ← sub  $(-x)$  when you see  $x$

$$\text{So } x \ln(2-x) = - \left[ -x \ln 2 + \frac{(-x)^2}{2} - \frac{(-x)^3}{8} \right]$$

This is not  
-ve so  
put - at start

$$= x \ln 2 - \frac{x^2}{2} - \frac{1}{8} x^3$$

Hence Fund  $x \ln(4-x^2)$

$$= x \left[ \ln(2+x)(2-x) \right]$$

$$= x \left[ \ln(2+x) + \ln(2-x) \right]$$

$$= x \ln(2+x) + x \ln(2-x)$$

so add above

$$\left( x \ln 2 + \frac{x^2}{2} - \frac{x^3}{8} \right) + \left( x \ln 2 - \frac{x^2}{2} - \frac{1}{8} x^3 \right)$$

$$= 2x \ln 2 - \frac{1}{4} x^3$$

Q13.  $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2x^2$

14

Auxilliary Equation

$$\lambda^2 - 3\lambda + 2 = 0$$

$$(\lambda - 1)(\lambda - 2) = 0$$

$$\lambda = 1 \text{ or } \lambda = 2.$$

So. C.F  $y = A e^{\lambda x} + B e^{2\lambda x}$

P.I Try  $\left\{ \begin{array}{l} y = Cx^2 + Dx + E \\ \frac{dy}{dx} = 2Cx + D \\ \frac{d^2y}{dx^2} = 2C \end{array} \right.$

$$\frac{d^2y}{dx^2} = 2C$$

Sub into above

$$2C - 3(2Cx + D) + 2(Cx^2 + Dx + E) = 2x^2$$

$$2C - 6Cx - 3D + 2Cx^2 + 2Dx + 2E = 2x^2$$

Equate  $x^2$ 's

$$2Cx^2 = 2x^2$$

$$2C = 2 \Rightarrow C = 1$$

15.

Equate  $\alpha$ 's

$$-6c\alpha + 2D\alpha = 0\alpha$$

$$\Rightarrow -6c + 2D = 0$$

$$-3c + D = 0$$

$$-3(1) + D = 0 \Rightarrow D = \underline{\underline{3}}$$

Equate numbers

$$2C - 3D + 2E = 0$$

$$2 - 3D + 2E = 0$$

$$\Rightarrow 2 - 9 + 2E = 0$$

$$2E = 7$$

$$E = \frac{7}{2}$$

$$\Rightarrow P.I \quad y = x^2 + 3x + \frac{7}{2}$$

Solution

$$y = Ae^x + Be^{2x} + x^2 + 3x + \frac{7}{2}$$

16.

Given  $y = \frac{1}{2}$

$$\frac{dy}{dx} = 1$$

$$\partial L = 0$$

$$y = Ae^x + Be^{2x} + x^2 + 3x + \frac{7}{2}$$

$$\frac{1}{2} = Ae^0 + Be^0 + 0 + 0 + \frac{7}{2}$$

$$\frac{1}{2} = A + B + \frac{7}{2}$$

$$\Rightarrow \left\{ \begin{array}{l} A+B = -3 \\ \end{array} \right. \quad \text{--- (1)}$$

$$\frac{dy}{dx} = Ae^x + 2Be^{2x} + 2x + 3$$

$$1 = Ae^0 + 2Be^0 + 0 + 3$$

$$1 = A + 2B + 3$$

$$\left\{ \begin{array}{l} A+2B = -2 \\ \end{array} \right. \quad \text{--- (2)}$$

$$\begin{array}{rcl} A+B = -3 & \rightarrow & A+B = -3 \\ A+2B = -2 & & \begin{array}{l} -A-2B = 2 \\ \hline -B = -1 \end{array} \\ & & \Rightarrow B = 1 \end{array}$$

Solution  $y = -4e^x + e^{2x} + x^2 + 3x + \frac{7}{2}$   $A = -4$

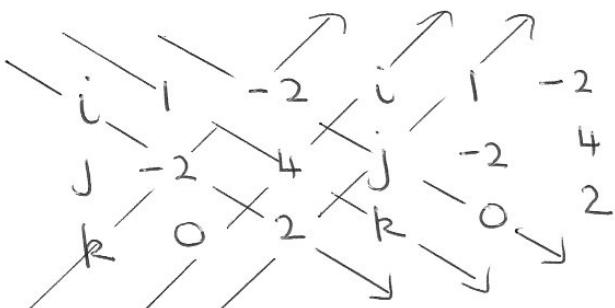
Q14. Equation of the plane need normal and a point. 17.

Normal is  $\vec{AB} \times \vec{BC}$ .

$$\vec{AB} = \underline{b} - \underline{a} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}$$

$$\vec{BC} = \underline{c} - \underline{b} = \begin{pmatrix} 0 \\ 3 \\ 3 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \\ 2 \end{pmatrix}$$

$$\Rightarrow \vec{AB} \times \vec{BC} = \begin{vmatrix} i & 1 & -2 \\ j & -2 & 4 \\ k & 0 & 2 \end{vmatrix}$$



$$(-4\underline{i} + 4\underline{k}) - (4\underline{k} + 2\underline{j}) \\ = -4\underline{i} - 2\underline{j} = \begin{pmatrix} -4 \\ -2 \\ 0 \end{pmatrix}$$

Point A(1,1,1)

So Equation of plane

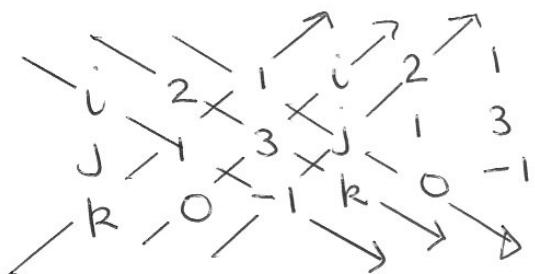
$$-4x - 2y = -4(1) - 2(1) \\ -4x - 2y = -6 \quad \text{or} \quad \underline{\underline{2x + y = 3}}$$

$$(b) \quad \begin{array}{l} \Pi_1 \\ \Pi_2 \end{array} \quad \left. \begin{array}{l} 2x+y=3 \\ x+3y-z=2 \end{array} \right\} \quad 18.$$

$$\begin{array}{l} (0, a, b) \\ \uparrow \uparrow \uparrow \\ x \ y \ z \end{array} \quad \begin{array}{l} \text{sub into } \Pi_1 \\ \text{sub into } \Pi_2 \end{array} \quad \begin{array}{l} 0+a=3 \Rightarrow \underline{\underline{a=3}} \\ 0+3a-b=2 \\ \Rightarrow 9-b=2 \\ \Rightarrow \underline{\underline{b=7}} \end{array}$$

$$\text{so } (0, 3, 7).$$

Direction of line of intersection from  $\underline{a} \times \underline{b}$



$$(-\underline{i} + 6\underline{k}) - (\underline{k} + \underline{o} - 2\underline{j})$$

$$= -\underline{i} + 2\underline{j} + 5\underline{k}$$

$\Rightarrow$  Equation of line: Direction Vector  $\begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix}$   
Pt  $(0, 3, 7)$

Equation is  $x = -t, y = 2t + 3, z = 5t + 7$

19.

c) Acute angle between 2 planes is  
the angle between the normals.

$$\pi_1 = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \quad \pi_2 = \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}$$

$$\cos \theta = \frac{\pi_1 \cdot \pi_2}{\|\pi_1\| \|\pi_2\|} = \frac{2+3+0}{\sqrt{5} \sqrt{11}} = 0.674$$

$$\theta = 47.6^\circ$$

Q15.  $f(x) = \frac{x}{\ln x}$

$$(a) f'(x) = \frac{1(\ln x) - x\left(\frac{1}{x}\right)}{(\ln x)^2} = \frac{\ln x - 1}{(\ln x)^2}$$

$$f''(x) = \frac{\frac{1}{x}(\ln x)^2 - (\ln x - 1)\left(2\ln x \cdot \left(\frac{1}{x}\right)\right)}{(\ln x)^4}$$

$$= \frac{\frac{1}{x}(\ln x)^2 - \frac{2}{x}\ln x(\ln x - 1)}{(\ln x)^4}$$

$$= \frac{\frac{1}{x}(\ln x)^2 - \frac{2}{x}(\ln x)^2 + \frac{2}{x}\ln x}{(\ln x)^4}$$

20.

$$= \frac{\ln x - 2 \ln x + 2}{x (\ln x)^3}$$

$$= -\frac{\ln x + 2}{x (\ln x)^3}$$

b) Stationary points when  $f'(x) = 0$

$$\frac{\ln x - 1}{(\ln x)^2} = 0$$

$$\ln x = 1$$

$$x = e^1 = e. \quad y = \frac{x}{\ln x} = \frac{e}{\ln e} \\ = e$$

$\Rightarrow$  Point  $(e, e)$

Nature  $\frac{d^2y}{dx^2} = -\frac{\ln x + 2}{x (\ln x)^3} = -\frac{\ln e + 2}{e (\ln e)^3}$

$$= -\frac{1+2}{e(1)^3} = \frac{1}{e} \\ = \underline{\underline{e^{-1}}}$$

$$\Rightarrow \frac{d^2y}{dx^2} > 0$$

$\Rightarrow$  Min T.P.

21.

(c) Point of inflection when

$$\frac{d^2y}{dx^2} = 0$$

$$\frac{-\ln x + 2}{x(\ln x)^3} = 0$$

$$\Rightarrow -\ln x + 2 = 0$$

$$\underline{\underline{\ln x = 2}}$$

$$\Rightarrow x = e^2, \quad y = \frac{x}{\ln x} = \frac{e^2}{\ln e^2} = \frac{e^2}{2\ln e} = \frac{1}{2}e^2$$

$\Rightarrow$  Point of inflection at  $\underline{\underline{(e^2, \frac{1}{2}e^2)}}$

$$Q16. \quad z = \cos\theta + i\sin\theta$$

22.

$$z^k = \cos k\theta + i\sin k\theta$$

$$\frac{1}{z^k} = \frac{1}{\cos k\theta - i\sin k\theta}$$

$$= \frac{1}{(\cos k\theta + i\sin k\theta)} \times \frac{\cos k\theta - i\sin k\theta}{\cos k\theta - i\sin k\theta}$$

$$= \frac{\cos k\theta - i\sin k\theta}{\cos^2 k\theta - i^2 \sin^2 k\theta}$$

$$= \frac{\cos k\theta - i\sin k\theta}{\cos^2 k\theta + \sin^2 k\theta} = \underline{\underline{\cos k\theta - i\sin k\theta}}$$

$$\begin{array}{ccc} z^k = \cos k\theta + i\sin k\theta & & z^{-k} = \cos k\theta - i\sin k\theta \end{array}$$

To get  $\cos k\theta$  add  $z^k + z^{-k}$  (sin terms cancel)

$$z^k + z^{-k} = 2\cos k\theta$$

$$\Rightarrow \cos k\theta = \frac{1}{2}(z^k + z^{-k})$$

To get  $\sin k\theta$  subtract  $z^k + z^{-k}$

$$z^k - z^{-k} = -i\sin k\theta + i\sin k\theta$$

$$= 2i\sin k\theta$$

$$\sin k\theta = \frac{1}{2i}(z^k - z^{-k})$$

Q16 (continued)

23.

$$\cos^2 \theta \cdot \sin^2 \theta \\ = (\cos k\theta)^2 \cdot (\sin k\theta)^2 \quad \text{where } k = 1$$

We know

$$\cos k\theta = \frac{1}{2} (z^k + z^{-k}) \Rightarrow \cos \theta = \frac{1}{2} (z + z^{-1})$$

$$\sin k\theta = \frac{1}{2i} (z^k - z^{-k}) \Rightarrow \sin \theta = \frac{1}{2i} (z - z^{-1})$$

square

$$\cos^2 \theta = \left[ \frac{1}{2} (z + z^{-1}) \right]^2 \\ = \frac{1}{4} (z^2 + 2 + z^{-2})$$

square

$$\sin^2 \theta = \left[ \frac{1}{2i} (z - z^{-1}) \right]^2 \\ = \frac{1}{4i^2} (z^2 - 2 + z^{-2}) \\ = -\frac{1}{4} (z^2 - 2 + z^{-2})$$

$$\begin{aligned} \text{so } \cos^2 \theta \cdot \sin^2 \theta \\ &= \frac{1}{4} (z^2 + 2 + z^{-2}) \times -\frac{1}{4} (z^2 - 2 + z^{-2}) \\ &= -\frac{1}{16} (z^2 + 2 + z^{-2}) (z^2 - 2 + z^{-2}) \\ &= -\frac{1}{16} \left[ (z^4 - 2z^2 + 1) + (2z^2 - 4 + 2z^{-2}) \right. \\ &\quad \left. + (1 - 2z^{-2} + z^{-4}) \right] \end{aligned}$$

$$= -\frac{1}{16} \left[ z^4 - 2 + z^{-4} \right] = -\frac{1}{16} \left( z^4 - 2 + \frac{1}{z^4} \right)^*$$

$$= -\frac{1}{16} \left[ z^2 - \frac{1}{z^2} \right]^2$$

Hence show  $\cos^2 \theta \cdot \sin^2 \theta = a + b \cos 4\theta$

\* Go back to

$$\cos^2 \theta \cdot \sin^2 \theta = -\frac{1}{16} \left( z^4 - 2 + \frac{1}{z^4} \right)$$

$z = \cos \theta + i \sin \theta$  so  $z^4 = \cos 4\theta + i \sin 4\theta$

$$\begin{aligned} & \cos^2 \theta \cdot \sin^2 \theta \\ &= -\frac{1}{16} \left( \cos 4\theta + i \sin 4\theta - 2 + \cos(-4\theta) + i \sin(-4\theta) \right) \end{aligned}$$

~~s/a  
t/c~~

$$* \cos(-4\theta) = \cos 4\theta$$

$$* \sin(-4\theta) = -\sin 4\theta$$

$$= -\frac{1}{16} \left( \cos 4\theta + i \sin 4\theta - 2 + \cos(4\theta) - i \sin 4\theta \right)$$

$$= -\frac{1}{16} (2 \cos 4\theta - 2)$$

$$= -\frac{1}{8} \cos 4\theta + \frac{1}{8} \quad \Rightarrow \quad a = \frac{1}{8}$$

$$b = -\frac{1}{8}$$