2012 – Higher Answers

Paper 1 (Section A - Multiple Choice)

1) C	2) D	3) B	4) B	5) A	6) C	7) A	8) C	9) A	10) B
11) D	12) B	13) D	14) A	15) D	16) C	17) D	18) B	19) B	20) A

Paper 1 (Section B – Written Response)

21a)(i) Synthetic Division (with 4 outside), Remainder = 0 $\therefore$ (x – 4) is a factor (ii) (x – 4) (x – 2) (x + 1)
(iii) x = -1, 2, 4 b) Area = $\frac{32}{3}$ units^2

22a) k = 2, a = $\frac{\pi}{3}$ b) cuts X axis at $\frac{\pi}{6}, \frac{7\pi}{6}$ cuts Y axis at 1

23a) $y - 3 = \frac{1}{3}(x - 1)$ b) $y - (-2) = -3(x - 1)$ c) $\left(-\frac{1}{2}, \frac{5}{2}\right)$ d) $\sqrt{\frac{5}{2}}$

Paper 2

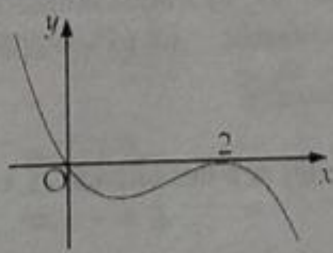
1a) (i) $(x + 4)^2 + 3$ (ii) $x^2 + 7$ b) $\Delta = -144 < 0$ $\therefore$ no real roots

2a) P (- 3 , - 1), Q (1 , 7)
b) Centre $C_{original}$ (3 , 1) Radius $C_{original} = \sqrt{40}$ Equation of Circle $(x + 5)^2 + (y - 5)^2 = 40$

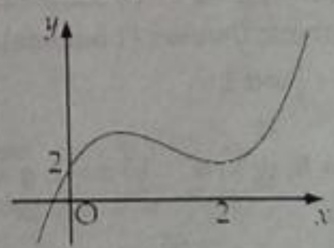
3) SPs $x = -\frac{2}{3}, 2$ f(2) = -2, as $f(-\frac{2}{3})$ is outside the domain evaluate endpoints
f(0) = 6 and f(3) = 3……so max 6 and min -2.

4)

Graph for (a)



Graph for (b)



5a)(i) $\overrightarrow{BA} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ $\overrightarrow{BC} = \begin{pmatrix} 2 \\ k+3 \\ -1 \end{pmatrix}$ (ii) proof b) k = -2 or – 4

6a) $-1 < \sin x < 1$ for $0 < x < -\frac{\pi}{2}$, $0 < \sin x < 1$ b) show that $\sin x = \frac{2}{3}$ or -1 [disregard -1] for x = 0.730

7a) Proof b) $y \approx 4^{0.8842} \approx 3.4$

2011 – Higher Answers

Paper 1 (Section A - Multiple Choice)

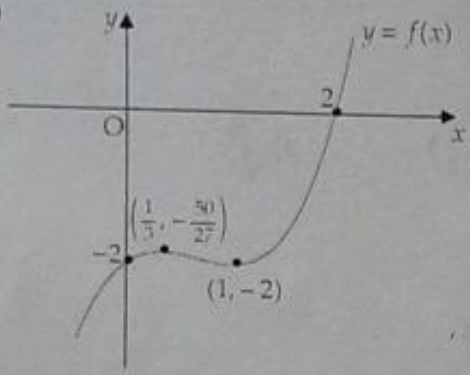
1) C	2) B	3) D	4) D	5) A	6) C	7) D	8) A	9) B	10) D
11) D	12) C	13) C	14) B	15) B	16) A	17) A	18) C	19) C	20) D

Paper 1 (Section B – Written Response)

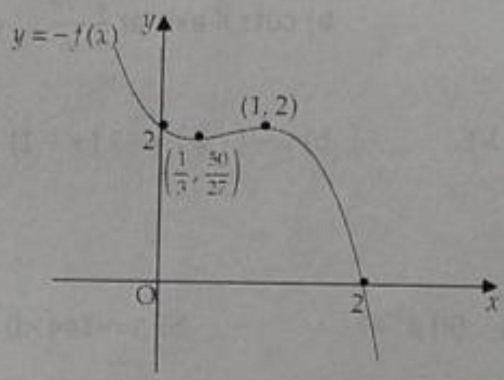
21a) $y - 12 = 3(x - 7)$ b) $E(5, 6)$ c)(i) $y - 10 = -2(x - 3)$ (ii) sub $x = 5$ to give $y = 6$

22a)(i) X axis $(2, 0)$ (ii) Y axis $(0, -2)$ b) Max $\left(\frac{1}{3}, -\frac{50}{27}\right)$, Min $(1, -2)$ must inc Nature Table or $f''(x)$

(c) (i)



(ii)



23a) 0, 60 and 300 (not 360!)   b) 0, 30, 150, 180, 210 and 330

Paper 2

1a) $B(4, 4, 0)$ b) $\overrightarrow{DB} = \begin{pmatrix} 2 \\ 2 \\ -6 \end{pmatrix}$, $\overrightarrow{DM} = \begin{pmatrix} 0 \\ -2 \\ -6 \end{pmatrix}$ c) angle BDM = 40.3°

2a) $3(x^3 - 1) + 1$ b) Substitute $g(f(x))$, multiply the terms of $h(x)$ by x and complete
c) Synthetic Division (1 outside). Remainder = 0 $\therefore (x - 1)$ is a factor   (ii) $(x - 1)(3x + 1)(x + 2)$
d) $-2, -\frac{1}{3}$ and 1

3a) $u_1 = 8, u_2 = -4$ b) $p = 2, q = -3$ c)(i) Limit = 0   (ii) as p lies outside $-1 < p < 1$

4) [Area on left = $\frac{63}{4}$] + [Area on right = $\frac{16}{3}$] = Total Area $\frac{253}{12}$ units^2

5) $k = 32, n = \frac{1}{2}$

6a) $R = \sqrt{34}$, $a = 5.253$ b) $t = 0.6$

7) Centre $C_1(-1, 1)$ Radius $C_1 = 11$, Centre $C_2(2, -3)$ Radius $C_2 = \sqrt{13 - p}$, Distance between centres = 5
Equating Radius to give $\sqrt{13 - p}, < 6$, $p > -23$

2010 HIGHER ANSWERS

APER I

A 2. C 3. D 4. A 5. B 6. D 7. C 8. B 9. C 10. B
D 12. A 13. B 14. C 15. C 16. A 17. B 18. B 19. C 20. A

1.(a) $2x + 5y = 72$
(b) Verify that $(6, 12)$ satisfies the equation $2x + 5y = 72$
(c) $BT : TQ = 2 : 1$

22.(a)(i) [Verify that $f(1) = 0$]
Since $f(1) = 0$, $(x - 1)$ is a factor of $f(x)$
(ii) $f(x) = (x-1)(x-1)(2x+5)$
(b) $x = 1, -\frac{5}{2}$ (c) $G(1, -1)$ [NB: CLEAR REASON AT END]
(d) $H\left(-2\frac{1}{2}, -8\right)$

23.(a)(i) "PROOF" (ii) $\sin a = \frac{3}{\sqrt{13}}$ (b) $\sin b = \frac{3}{5}$ $\cos b = \frac{4}{5}$
(c)(i) $\sin(a - b) = \frac{6}{5\sqrt{13}}$ (ii) $\sin(b - a) = -\frac{6}{5\sqrt{13}}$

PAPER II

1.(a) $M(0, 1, 0)$ $N(4, 2, 2)$ (b) $\vec{VM} = \begin{pmatrix} 0 \\ -1 \\ -3 \end{pmatrix}$ $\vec{VN} = \begin{pmatrix} 4 \\ 0 \\ -1 \end{pmatrix}$
(c) 76.7°

2.(a) $[13\cos(x + 22.6^\circ)]$ $k = 13$, $a = 22.6^\circ$
(b) MAXIMUM = 13 – OCCURS WHEN $x = 337.4^\circ$
MINIMUM = −13. – OCCURS WHEN $x = 157.4^\circ$

3.(a) (i) AND (ii) Since line meets the circle at 1 point only, $P(-1, 4)$, the line is a tangent to the circle.
(b) $(x - 1)^2 + (y - 6)^2 = 8$

4. 2.419, 3.864

5.(a)(i) $PQ = 6 - x^2$ (ii) "PROOF"
(b) [MAX. AREA OCCURS WHEN $x = \sqrt{2}$]. MAX. AREA $= 8\sqrt{2}$ UNITS^2

6.(a) "PROOF" (b) $A\left(4\frac{1}{2}, 0\right)$ (c) $4\frac{1}{2}$ UNITS^2

7.(a) "PROOF" (b) $x = 6561$

SQA HIGHER ANSWERS 2009

PAPER I

1. A	2. B	3. D	4. C	5. B	6. A	7. A	8. D	9. A	10. B
11. B	12. C	13. B	14. C	15. A	16. B	17. A	18. D	19. C	20. C

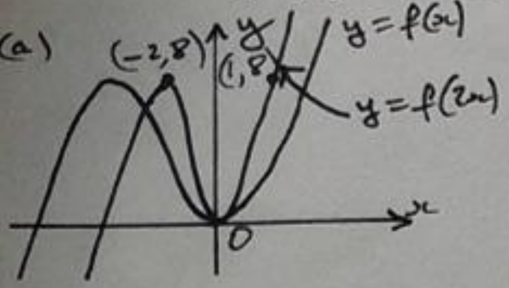
21.(a) $P(-3,0)$ (b) $2y = x + 3$ (c) $T(5,4)$

22.(a)(i) PROVE THAT $\vec{DE} = 3\vec{EF}$ (ii) $3:1$

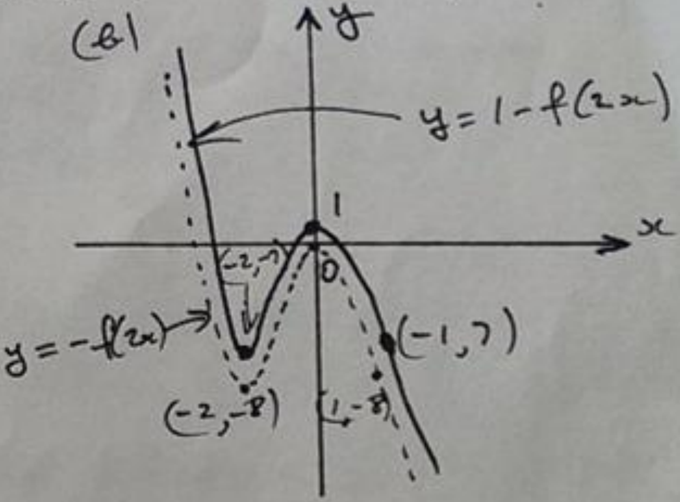
$\therefore$ DE, EF are parallel

$\therefore$ D, E, F are collinear ($\because$ of common point E) (b) $k = 7$

23.(a)



(b)



24.(a) $\frac{\sqrt{3}+1}{2\sqrt{2}}$ (b) PROOF

(c)(i) $\frac{\pi}{3} - \frac{\pi}{4}$

(ii) $\frac{\sqrt{3}}{\sqrt{2}}$ $\left(\text{or } \frac{\sqrt{6}}{2}\right)$

PAPER II

1. $(-1, 17)$ is a MAX. S.P. ; $(3, -15)$ is a MIN. S.P.

2. (a)(i) $3x^2 - 5$ (ii) $(3x+1)^2 - 2$ (b) $x = -\frac{1}{2}$.

3. (a)(i) PROVE that $f(1) = 0$. (ii) $(x-1)(x+4)(x+5)$

(b) $x = 1$ [NB EXPLAIN CLEARLY DELETION OF $-4, -5$]

4. (a) PROVE that $P(5,10)$ "works" in $(x+1)^2 + (y-2)^2 = 100$

(b) $3x + 4y = -45$ (c) $C_2 : (x-5)^2 + (y-10)^2 = 400$

$C_3 : (x+19)^2 + (y+22)^2 = 400$

5. (a) $m = 3$, $n = 2$

(b) $(0.564, 1.286)$, $(2.578, 1.286)$ (c) 12.365 units^2

6. (a) 76.3 MILLION (b) 161.2 YEARS

7. (a) $6\sqrt{3}$; $\frac{9}{4}$

(b) $\frac{3}{2}\sqrt{3}$; $\sqrt{25 - 12\sqrt{3}}$

(or equivalent approximations).

2008 SQA HIGHER ANSWERS

PER I

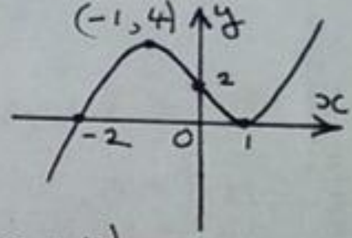
. C 2. D 3. C 4. B 5. A 6. B 7. C 8. D 9. B 10. A

1. B 12. C 13. A 14. B 15. C 16. A 17. C 18. C 19. B 20. D

21 (a) $(-1,4)$ is a MAX. S.P.
$(1,0)$ is a MIN. S.P.

(b) Since $f(1)=0$, $(x-1)$ is a factor
$(x-1)(x-1)(x+2)$

(c) AXES:- $(-2,0)$
$(1,0)$
$(0,2)$



22 (a) $(1,3)$, $(3,-3)$

(b) $A(1,3)$

23 (a) $h[f(x)] = \log_2(x^2 - x + 10)$
$h[g(x)] = \log_2(5-x)$

(b) $x = -10, 3$.

PAPER II

1. (a) "PROOF" (b) $3x + y = 10$ (c) $(3,1)$

2 (a) $P(8,0,4)$, $Q(0,4,3)$ (b) $\overrightarrow{PQ} = \begin{pmatrix}-8\\4\\-1\end{pmatrix}$, $\overrightarrow{PA} = \begin{pmatrix}0\\0\\-4\end{pmatrix}$ (c) $83\cdot62^\circ$

3. (a) (i) $p = \sqrt{7}$ (ii) $q = -3$ (b) $f(x) + g(x) = 4\cos(x + 0\cdot848)$
(c) $f'(x) + g'(x) = -4\sin(x + 0\cdot848)$

4. (a) $C_1(-4,-2)$ $r_1 = \sqrt{58}$
(b) Obtain $C_2(4,6)$ and therefore $C_1C_2 = 8\sqrt{2}$
(c) Subst. $y = 4 - x$ into the equation of either circle, BUT EASIER TO USE CIRCLE IN (a), to obtain $(3,1)$ AND $(-1,5)$

5. $x = 90^\circ, 199\cdot5^\circ, 340\cdot5^\circ$

6. (a) Observe that QR is the y-coordinate of Q, so you firstly need to find the equation of the sloping straight line.

(b) [Obtain $A_{RECTANGLE} = 6t - 2t^2$
Then MAX. AREA OCCURS WHEN $t = 1\frac{1}{2}$ ← TABLE OF VALUES REQUIRED]

MAX. AREA OCCURS WHEN Q IS $(1\frac{1}{2}, 3)$

7. [There are various ways of doing this either as a subtraction of areas or an addition of areas, but for either method, it is easier to find the area in the 1ST quadrant and doubling the answer. You will need to obtain $x = 2$ and $x = 3$ as points of intersection of 2 lines with parabola.]

[ADDITION METHOD IS "LIKELY" TO YIELD $2\left(20 + 5\frac{1}{3}\right)$
SUBTRACTION METHOD IS "LIKELY" TO YIELD $2\left(36 - 10\frac{2}{3}\right)$]

AREA $= 50\frac{2}{3}$ UNITS^2

2007 SQA HIGHER – ANSWERS

PAPER I

1. $y = 3x + 7$ 2. $C(7, 7, 8)$ 3. (a) $-1-2x^2$ (b) $4x-1$

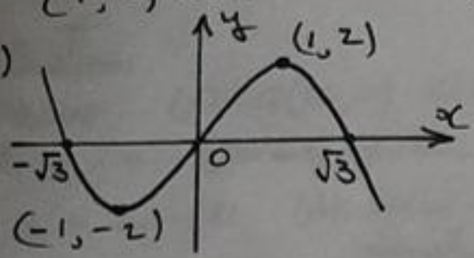
4. $k < -\frac{1}{4}$

5. $(x-15)^2 + (y-8)^2 = 4$ 6. $90°, 270°$

7 (a) $u_1 = 16$ $u_2 = 20$ $u_3 = 21$
(b) Limit exists $\because \frac{1}{4}$ lies between -1 and 1 (c) $k = \frac{64}{3}$

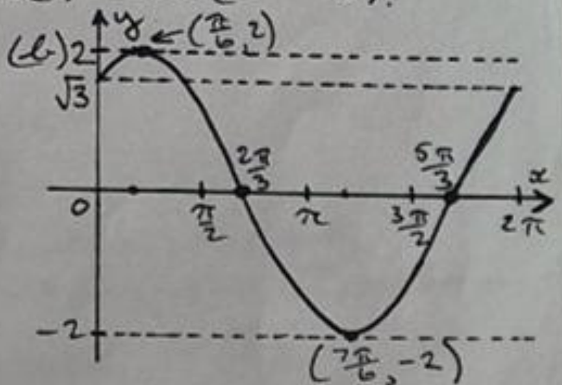
8. (a) Verify that $f(3)=0$ $\therefore$ Curve crosses x-axis at $x=3$ [$(3,0)$]
(b) $A(2,0)$ [NB: You must obtain $x=-1, 2, 3$ and explain why the solution MUST be $x=2$]
(c) $7\frac{1}{3}$ units^2

9 (a) $(0,0)$, $(\sqrt{3},0)$, $(-\sqrt{3},0)$
(b) $(-1,-2)$ is a MIN. S.P. } TABLE OF VALUES REQUIRED
$(1,2)$ is a MAX. S.P.
(c)



10. $\frac{3x}{(3x^2+2)^{1/2}}$

11. (a) $2\cos\left(x - \frac{\pi}{6}\right)$
(b)



PAPER II

1. (a) $G(0,2,2)$ (b) $\underset{\sim}{p} = \begin{pmatrix}0\\1\\1\end{pmatrix}$ $\underset{\sim}{q} = \begin{pmatrix}1\\2\\1\end{pmatrix}$ (c) $30°$

2. (a) $\frac{1}{\sqrt{2}}$ (b)(i) $\frac{4}{5}$ (ii) $\frac{4}{5}$ $\therefore \cos 2d = \sin 2c = \frac{4}{5}$

3. WORKING LEADING TO THIS CONCLUSION :– Since the line meets the circle at one point only, $(1,4)$, the line is a tangent to the circle.

4. (a) $a=2$ $b=3$ $c=-1$ $(y = 2\sin 3x - 1)$ (b) $x_p = 50°$ [EXPLAIN DELETION OF $10°$]

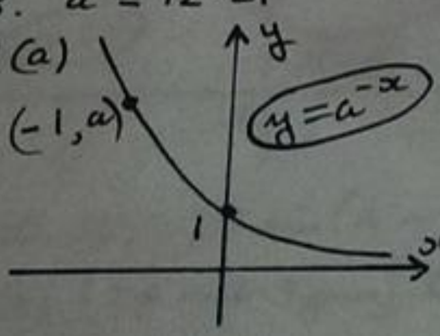
5. (a) $Q(12,10)$ (b) $P(4,10)$ (c) $C(8,11)$

6. (a)(i) $10\sqrt{2}$ m (ii) "PROOF" (b) [TABLE OF VALUES GIVING $x = 5\sqrt{2}$]
$\therefore$ Dimensions are $\frac{5\sqrt{2}}{2}$ m $\times$ $5\sqrt{2}$ m

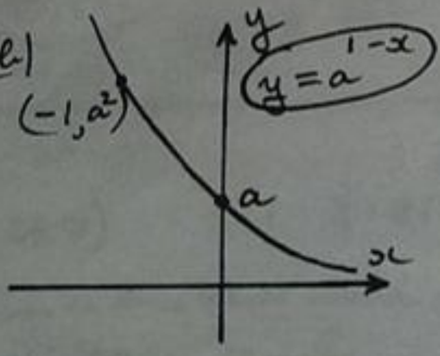
7. 0.3629

8. $a = 12.21$

9 (a)



(b)



10. (a)(i) $a = 2$ $b = 4$ (or vice-versa)
(ii) $k = \frac{3}{4}$
(b) $f(x) = \frac{1}{4}x^3 - \frac{9}{4}x^2 + 6x + 6$

11 (a) $a = \frac{1}{2}$ (b) $b = \frac{3}{2}$
(c) $\log_{10} y = 0.602x + 0.477$
– ie gradient $= 0.602$

2006 HIGHER ANSWERS

PER I

(a) $y = 2x - 1$ (b) $3x + y = 9$ (c) $(2, 3)$

. (a) $(x+2)^2 + (y-3)^2 = 18$ (b) $x + y = -5$

3. (a) (i) $4x - 3$ (ii) $4x + 3$ (b) -9

4. (a) LIMIT EXISTS $\because$ 0.8 LIES BETWEEN -1 AND 1 (b) 60

5. $(\frac{1}{2}, 0)$ IS A POINT OF INFLECTION Table of values required

6. (a) $1\frac{1}{4}$ units^2 (b) $(1\frac{1}{4} + 3\frac{1}{4}) = 4\frac{1}{2}$ units^2.

7. $0°, 60°, 180°, 300°, 360°$ 8. (a) $2(x+1)^2 - 5$ (b) $(-1, -5)$

9. (a) PROOF (b) VERIFY THAT $f(-3) = 0$ WITH CLEAR CONCLUSION
$(k+3)(k-1)(k+1)$

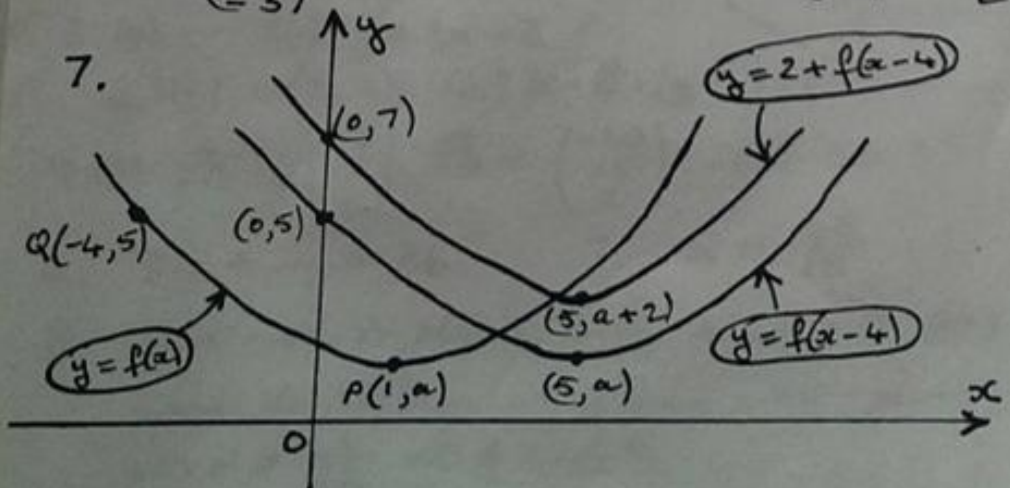
(c) $k = 1$ [Clearly explain deletion of -3 and -1] (d) $\frac{1}{11}$

10. $a = 2$

PAPER II

1. (a) PROOF (b) $Q(22, 0)$ $R(24, 6)$
2. $k = 24$ [Clearly explain deletion of 0]
3. (a) $y = 2x - 11$ (b) Since line meets circle at one point only, namely $Q(4, -3)$, line is a tangent to 2$^{\text{ND}}$ parabola.
4. [Firstly prove that $k = 6$]. Radius of larger circle $= \sqrt{37}$.
5. $y = 2x^2 - 2x^3 + 5$
6. (a) $\begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix}$ (b) 5 units (c) $\frac{1}{5}\begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix}$ $\left[\text{or } -\frac{1}{5}\begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix}\right]$

7.



8. (a) (i) $\frac{1}{\sqrt{5}}$ (ii) $\frac{4}{5}$
(b) $\frac{11}{5\sqrt{5}}$ $\left(\text{or } \frac{11\sqrt{5}}{25}\right)$

9. $-\frac{3}{x^4} + 2\sin 2x$

10. (a) $25\sin(x - 1.287)$ (b) 2.818 radians

11. [$t = 1030.9$ years] Claim is true that wood is over 1000 years old.

12. (a) (i) $PS = 6 - x$ $RS = 12 - \frac{8}{x}$ (b) PROOF
(b) MAX. AREA = 32 units^2 occurring when $x = 2$
MIN. AREA = 20 units^2 occurring when $x = 1$ or 4 Table of values required

2005 SQA HIGHER ANSWERS

PAPER I

1. $y = \sqrt{3}(x+2)$ 2.(a) [Firstly find A and B] $P(0,2)$ (b) 10 units.

3.(a) $F(10,5,3)$ (b) $\vec{AF} = \begin{pmatrix}-2\\5\\3\end{pmatrix}$ 4.(a) $(3x-1)^2+7$
(b)(i) $(\frac{1}{3}, 7)$ (ii) $y \geq 7$

5. $\frac{dy}{dx} = 8\cos x(1+2\sin x)^3$

6(a) $k = -\frac{1}{4}$ (b)(i) $u_1 = 3m+5$, $u_2 = 3m^2+5m+5$
(ii) $m = -2$ [CLEARLY EXPLAINING DELETION OF $m = \frac{1}{3}$]

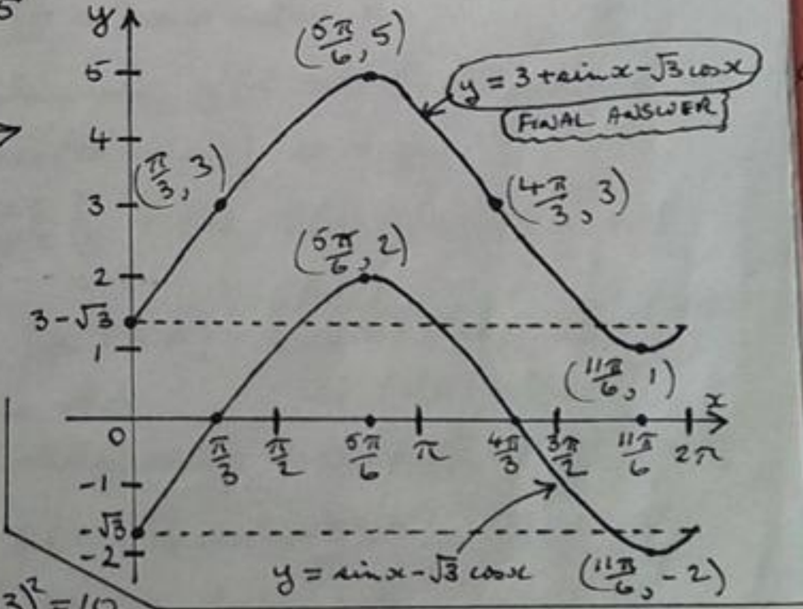
7.(a) $a = 4$ $b = 5$ (b) $x > 4$

8.(a) Prove that $f(3) = 0$ $\therefore$ $(x-3)$ is a factor of $f(x)$
$f(x) = (x-3)(x+1)(2x-3)$
(b) CROSSES x-AXIS AT : $(3,0)$, $(-1,0)$, $(1\frac{1}{2}, 0)$ y-AXIS AT $(0,9)$
(c) MAX $= 9$ MIN $= -35$

9. $\cos x = \frac{4}{5}$, $\sin x = \frac{3}{5}$

10(a) $2\sin(x - \frac{\pi}{3})$ (b) →

11.(a) $(x-t)^2 + y^2 = 4$
(b) $t = \sqrt{5}$



PAPER II

1. $2x^2 + \frac{1}{x} + c$

2.(a) — (b)(i) $-\frac{13}{85}$ (ii) $-\frac{84}{13}$

3.(a) — (b) $y = 3x-3$
(c)(i) $C(2,3)$ (ii) $(x-2)^2+(y-3)^2=10$

4.(a) $\vec{TA} = \begin{pmatrix}-5\\15\\1\end{pmatrix}$ $\vec{TB} = \begin{pmatrix}-40\\15\\2\end{pmatrix}$ (b) 50.9° 5. 36 units^2

6. $3x+2y=36$ 7. $x = \frac{43}{15}$

8. You need to obtain both $\sin x = 0$ AND $\cos x = \frac{1}{2k}$. Then you need to explain what $\sin x = 0$ gives, leaving therefore $\cos x = \frac{1}{2k}$ at A and C.

9.(a) £252 MILLION (b) 40 years – CARE REQUIRED TO OBTAIN FIRSTLY 39.995 ----

10. $13\frac{1}{2}$ 11.(a) Verify that $f(-1) = 0$ $\therefore$ $x=-1$ is a solution of the equation.
(b) $p \leq -1$ or $p \geq 3$ [QUADRATIC GRAPH REQUIRED]

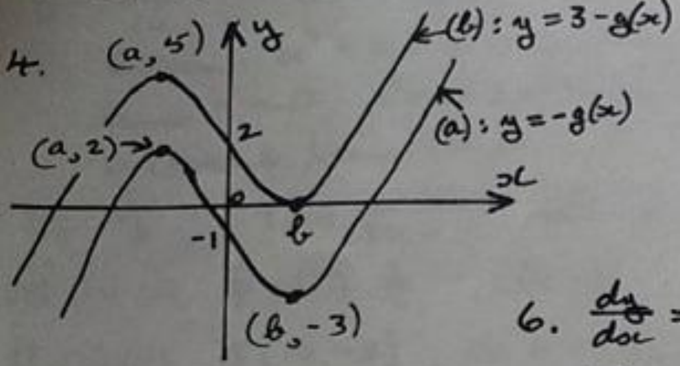
SQA – 2004 HIGHER ANSWERS

PAPER I

1. (a) $[B(5,-2)]$ $\therefore m_{AB} = 3$ (b) You need, using gradients, to verify 2 things clearly: that AB is perpendicular to $x+3y+1=0$ AND ALSO that AB is NOT perpendicular to $2x+5y=0$

2. (a)(i) Verify that $f(-1)=0$ (ii) $(x+1)(x+1)(x-3)$ (b) $(-1,0)$

3. $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ [REMEMBER THAT A SQUARE ROOT CAN BE POSITIVE OR NEGATIVE]

4.


5. (a) eg: $\vec{AB} = \begin{pmatrix}2\\4\\-4\end{pmatrix}$ $\vec{BC} = \begin{pmatrix}1\\2\\-2\end{pmatrix}$

$\vec{AB} = 2\vec{BC}$

$\therefore$ AB is parallel to BC

$\therefore$ A, B, C are collinear ($\because$ of common point B)

(b) $D(5, 20, -9)$

6. $\frac{dy}{dx} = 3\cos x - 2\sin 2x$ 7. $\frac{13}{3}$

8. (a) $(x-5)^2+2$ (b) clearly explain why $g'(x) > 0$ for all values of x

9. $x = 71$ 10. [Simplify $\cos(90° + 2x)$] $= -\frac{3}{5}$.

11. (a) $y = 6x(x-2)$ – ie $\left\{\begin{matrix}a=6\\b=2\end{matrix}\right\}$ (b) $f(x) = 2x^3 - 6x^2 + 8$

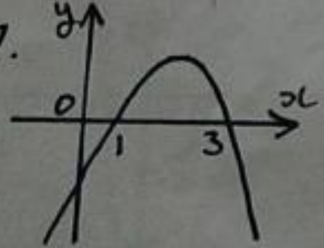
PAPER II

1. (a) $a = 26\cdot565°$ (b) $1\cdot5$ [to 1 D.P.] 2. (a) $\vec{QP} = \begin{pmatrix}-1\\3\\-2\end{pmatrix}$ $\vec{QR} = \begin{pmatrix}-5\\1\\1\end{pmatrix}$

(b) $P\hat{Q}R = 72\cdot0°$

3. $[\Delta = p^2 + 24]$ + clear conclusion that $\Delta > 0$ for all p $\therefore$ Roots are always real 4. (a) $-1 < k < 1$ (b) $k = \frac{2}{5}$

5. (a) $x = 2$ (b) $y = 12x - 8$ 6 (a) $\sqrt{34}\cos(x - 59\cdot0°)$ (b) $12\cdot3°$

7.


8 (a) PROOF (b) Solve pair of linear/quadratic equations to obtain 1 solution $(-3,3)$ + CLEAR CONCLUSION (c) $[Q(-3,3)] \to PQ = 4\sqrt{5}$ UNITS

9. (a) PROOF (b) $x = \sqrt{2}$ (TABLE OF VALUES REQUIRED)

10. (a) $A_0 = 4433\cdot4$ micrograms (b) $346\cdot6$ years

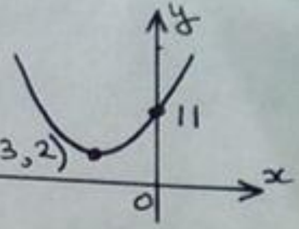
11. [Firstly clearly obtain area $= \int_1^3 \left(2x - \frac{1}{2}x^2 - \frac{3}{2}\right)dx$]

area $= \frac{2}{3}\,m^2$.

2003 HIGHER ANSWERS

PAPER I

1. $4y = x + 13$ 2.(a) $(x+3)^2 + 2$ (b) $(-3, 2)$



3. Prove that $\underset{\sim}{u} \cdot \underset{\sim}{v} = 0$ $\therefore$ $\underset{\sim}{u}$ is perpendicular to $\underset{\sim}{v}$.

4(a) Obtain $\left\{\begin{array}{l} 12p + q = 15 \\ 15p + q = 16 \end{array}\right\}$ $\therefore p = \frac{1}{3}$, $q = 11$ (b) $u_n \to 16\frac{1}{2}$ as $n \to \infty$

5. $\frac{3}{16}$ 6. $D(8, 3, -1)$

7. Obtain $x^2 + x + 3 = 0$ $\therefore \Delta = -11$

Since $\Delta < 0$, $x^2 + x + 3 = 0$ has no real solution $\therefore$ Line does not intersect parabola.

8. $\frac{2}{3}$ 9.(a) $\frac{1}{2x-1}$ (b) $\underset{\sim}{h}$ is not defined when $x = \frac{1}{2}$

10.(a) (i) $\frac{4}{5}$ (ii) $\frac{3}{5}$ (b) $\frac{4}{3}$

11.(a) (i) $A(12, -5)$; $OA = 13$ units (ii) $(x - 24)^2 + y^2 = 64$

(b) $p = \frac{5}{144}$, $q = -24$

12. $1 + \log_e 8 - \log_e 9$

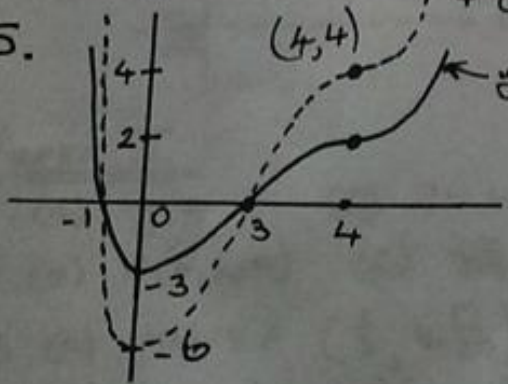
PAPER II

1.(a) Prove that $f(2) = 0$ (b) $(x-2)(3x-1)(2x+3)$

2. $a = 4$, $b = 2$, $c = 1$ 3. $42\frac{2}{3}$ units^2

4.(a) $y = 4x - 2$ (b) Prove clearly that $(2, 6)$ is the only point of intersection + Clear conclusion.

5.



6. $6 - \sqrt{3}$

7.(a) $\sqrt{29} \sin(x + 68 \cdot 2^\circ)$

(b) $P(201 \cdot 8^\circ, -\sqrt{29})$

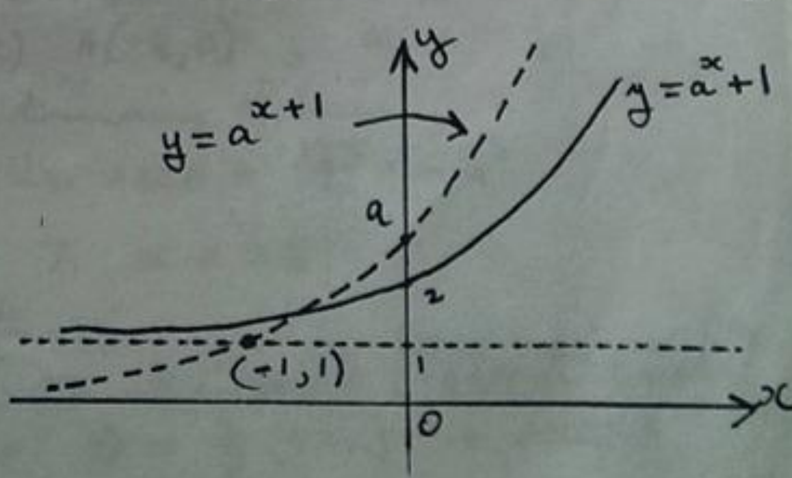
8 (a) PROOF (b) $x = 60$ cm [TABLE OF VALUES REQUIRED]

9. $\theta = 56 \cdot 6^\circ$

10. $x = 1 \cdot 23$ radians

11.(a) 2 GRAPHS : $\longrightarrow$

(b) Proof by solving the equation : $a^{x+1} = a^x + 1$



2002 HIGHER ANSWERS

[illegible]ER I

1. $3x + 2y = 12$

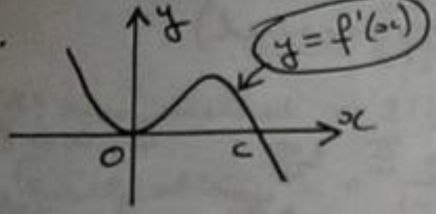
2. $Q(3, 1, -2)$
(b) Solve $2\sin 2x = 2\sin x$ to give $x = 0°, 60°, 180°, 300°, 360°$

3. (a) $f[g(x)] = \sin 2x$
$g[f(x)] = 2\sin x$

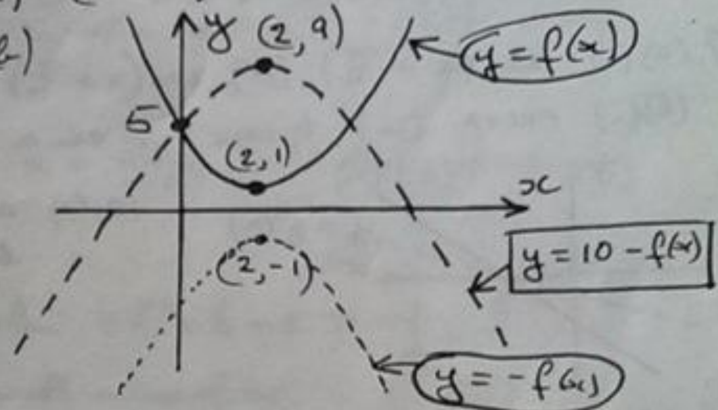
4. $(2, 4)$

5. PROOF

6.



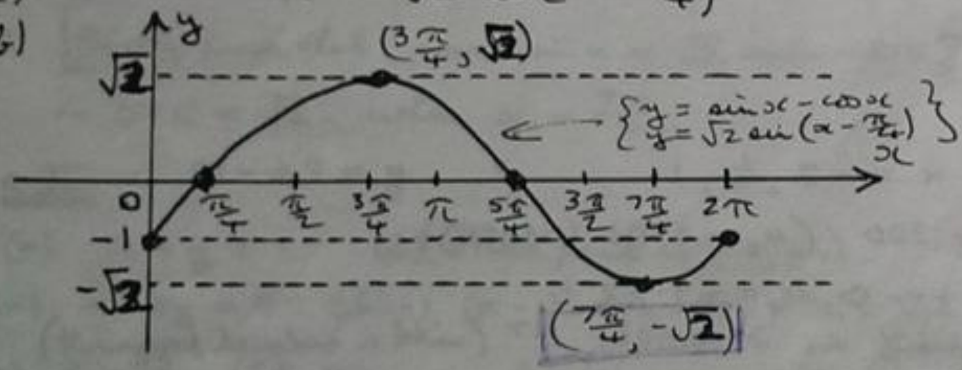
7. (a) $(x-2)^2 + 1$
(b)



(c) $-1 < x < 5$

8. (a) $y = 2\cos 2x$
(b) $B\left(\frac{7\pi}{12}, -\sqrt{3}\right)$

9(a) $\sin x - \cos x = \sqrt{2}\sin\left(x - \frac{\pi}{4}\right)$
(b)



10. (a) $-\frac{3x^2}{2(8-x^3)^{1/2}}$
(b) $-\frac{2}{3}(8-x^3)^{\frac{3}{2}} + C$

11. $y = 5x^{-2}$
ie $k = 5$
$n = -2$

PAPER II

1. (a) $y = 2$ (b) $2x + y = 2$ (c) $(0, 2)$

2. (a) $B(6, 6, 0)$ (b) $\vec{DA} = \begin{pmatrix} 3 \\ -3 \\ -8 \end{pmatrix}$ $\vec{DB} = \begin{pmatrix} 3 \\ 3 \\ -8 \end{pmatrix}$ (c) $38.7°$

3. (a) MAX S.P. $\left(\frac{1}{3}, 4\frac{17}{27}\right)$ [TABLE OF VALUES REQUIRED]
(b) $(x-2)(x-2)(2x+1)$ (c) $A\left(-\frac{1}{2}, 0\right)$; $x < -\frac{1}{2}$

4. (a) 2.5 m (b) 25% trimming of trees.

5. [OBTAIN: $\int_0^5 (5x - x^2)\,dx$] $\rightarrow$ AREA $= \frac{125}{6}$ units^2

6. $y - \sqrt{3}x = 1 - \frac{\pi\sqrt{3}}{3}$

7. $x = 2\frac{1}{3}$

8. $v = -\frac{4}{3}(4-t)^{3/2} + \frac{32}{3}$

9. PROVE: $\Delta = k(9k+8) \geq 0$ for all integers k [PROVE THIS VIA CAREFUL GRAPH]

10. (a) Use similar Δ's TWICE or $b = \frac{3}{5}(8-a)$ + PROOFS
(b) $a = 4$ [TABLE OF VALUES REQUIRED]

2001 HIGHER ANSWERS

PAPER I

1. $2x + 3y = 1$ 2. $k = \frac{1}{4}$

3.(a) PROVE THAT $\vec{AB} = \frac{3}{4}\vec{BC}$ $\therefore$ AB $\parallel^{l}$ to BC $\therefore$ A, B, C collinear ($\because$ common point)

(b) PROVE THAT $\vec{AB} \cdot \vec{DB} = 0$ $\therefore$ AB $\perp^{r}$ to DB.

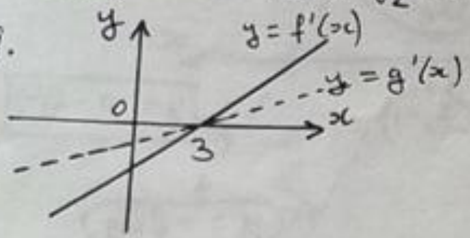
4. $(x+1)^2 - 9$ 5.(a) $30^\circ, 90^\circ, 150^\circ$ (b) $P(150^\circ, -\frac{\sqrt{3}}{2})$

6. MAX PROFIT WHEN $P = 9$ (Table of values required)

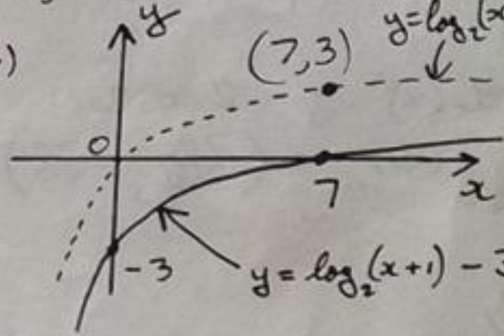
7.(a)(i) $\sin(x + \frac{\pi}{4})$ (ii) $\cos(x + \frac{\pi}{4})$ 8. $x = 81$

(b)(i) PROOF (ii) $\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x$; $x = \frac{\pi}{4}, \frac{3\pi}{4}$

9.



10.(a) $a = 1$, $b = 3$ (b)



11. (a)(i) PROOF (ii) PROVE THAT $C_1C_2 = r_1 + r_2 = 6\sqrt{2}$

(b) $y = x + 5$

(c) $x = 2 \pm 2\sqrt{3}$

PAPER II

1.(a) $k = -5$ (b) $x = -2, \frac{1}{2}, 1$ 2. $y = 2x - 12$

3.(a) $u_{n+1} = 1.015u_n - 300$ ($u_0 = 2500$ (MARCH))

(b) $u_1 = 2237.50 \dashrightarrow u_9 = -9.32$

Date of final payment is DECEMBER 1ST (with a reduced payment).

4. 71.5° 5. $10\cos(x + 36.9^\circ)$ 6. $\frac{1}{3}x^3 + \frac{4}{x} + C$

7.(a) l_1: $x = 7$ (b) l_2: $3x + 2y = 23$ (c) $(7, 1)$

(d) $(x-7)^2 + (y-1)^2 = 26$

8. $[2 \times 20\frac{1}{4}] = 40\frac{1}{2}$ units^2

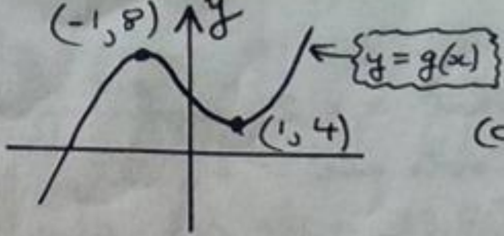
9. $k = 0.462$

10. $y = -\frac{3}{2}\cos 2x + \frac{1}{4}\sqrt{3}$

11. (a) PROOF (b) $\lambda = 2$.

2000 SQA (HIGHER) – ANSWERS

PAPER I

A1. $\frac{63}{65}$ A2.(a) $A(1,4)$ (b) 

(c) $4 < k < 8$

A3. 30°

A4(a)(i) $x = \frac{1}{3}$ or 3 (ii) at $(3,-8)$, curves have a common tangent (same gradient).
But they have equal gradients at 2 different points $\left(\frac{1}{3}, -12\frac{4}{9}\right)$ on quadratic and $\left(\frac{1}{3}, -2\frac{26}{27}\right)$ on cubic.

(b) area enclosed $= 21\frac{1}{3}$ units^2.

A5. [Firstly obtain $x = \frac{10}{1-a}$ and $x = \frac{16}{1-a^2}$; equate 2 expressions]
$a = \frac{3}{5}$. Limit $= 25$

A6. [Prove firstly that circle exists when $5k^2 + k + 2 > 0$]
Then prove that circle exist for all values of x.

B7. $\vec{VK} = \begin{pmatrix} 1 \\ -8 \\ -16 \end{pmatrix}$ B8. $y = -\frac{1}{3}\cos 3x + \frac{7}{6}$ B9. 2

B10. [Firstly prove that $\cos x - \sin x = \sqrt{2}\cos(x - 315^\circ)$]
$\therefore$ MAX $= \sqrt{2}$ when $x = \frac{7\pi}{4}$.

PAPER II

A1. (a) $x + y = 1$ (b) OTHER point is $(-1, -6)$

A2. (a) $x + 2y = 9$ (b)(i) $(x-1)^2 + (y-4)^2 = 25$ (ii) $T(-a, 9)$

A3(a) $3 - \frac{3}{x}$ (b) x A4(a) $y = x(4-x)$ (b) –

A5. 60°, 131.8°, 228.2°, 300°

A6. MIN. REQUIRED WHEN $x = 2$ {Table of values required}

B7. $t = 4$ B8. $\frac{5}{8}$

B9.(a) $B(3,2,15)$ (b) $\left[\vec{BA} = \begin{pmatrix} -3 \\ 7 \\ -7 \end{pmatrix} \vec{BC} = \begin{pmatrix} 14 \\ -2 \\ -7 \end{pmatrix}\right]$ $A\hat{B}C = 92.46^\circ$

B10. $\frac{1}{3(7-3x)} + C$

B11.(a) $P = 0.6Q + 1.8$
(b) $p = 6.05q^{0.6}$ [ie: $a = 6.05$, $b = 0.6$]

Answers to 1999 Higher

Paper I

1. (a) Verify that $x = 2$ "works" in the equation
   (b) OTHER roots are -3, $\frac{1}{2}$

2. (a) $3x - 4y = -5$ (b) $4x + 3y = 10$

3. (a) $[24 + 9] = 33$ units^2 (b) $\int_2^5 (2x+4)\,dx$ (c) 33 units^2

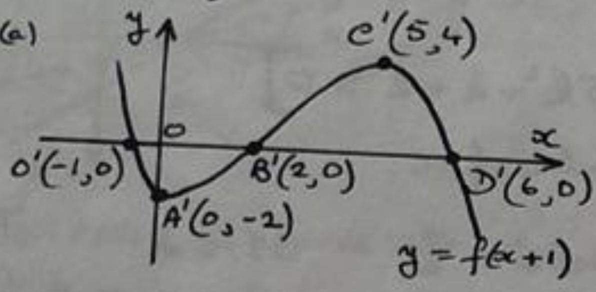
4. $(x+3)^2 + (y-4)^2 = 25$

5. $f'(-1) = 24$. 6. $\vec{CV} = \begin{pmatrix}-5\\-5\\7\end{pmatrix} \begin{pmatrix}-5\\-5\\7\end{pmatrix}$ 7. $\frac{1}{\sqrt{3}}$

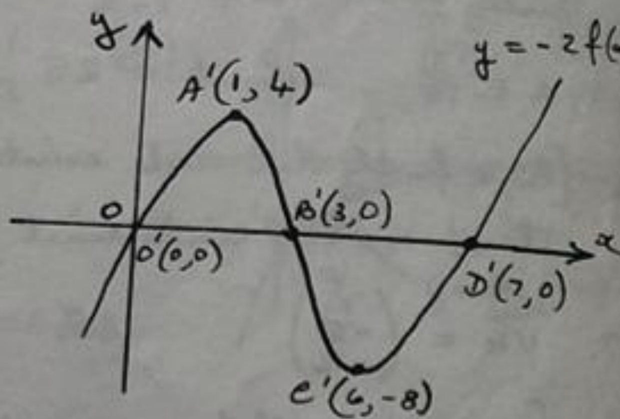
8. (a) $b^2 - 4ac = 0$ (b) Rearrange into $x^2 + 6x + 9 = 0$ and then verify that "Δ" $= 0$.

9. $10x + y = -3$

10. (a)



(b)



11. $y = \frac{1}{4}x^4 - \frac{1}{x} - \frac{1}{4}x + 3$

12. $\left[\cos x = \frac{2}{\sqrt{11}}\right] \therefore \cos 2x = -\frac{3}{11}$

13. (a) $f(x) = 2(x-1)^2 + 3$ (b) S.P. at $(1, 3)$ and it is a MINIMUM

14. $A(13 \cdot 93°, \frac{2}{3}a)$ $B(46 \cdot 07°, \frac{2}{3}a)$

15. $a = -2$ $b = 5$

16. $\left[\frac{dy}{dx} = 6x^2 + 6x + 4\right]$ and then verify that $\Delta = -60$ i.e. $\Delta < 0$ and GIVE A CLEAR CONCLUSION.

17. (a) (i) 9 (ii) 8 (iii) 6 (b) $\underset{\sim}{h} \cdot \underset{\sim}{h} = 68$; $|\underset{\sim}{h}| = 2\sqrt{17}$.

18. $p = 2q$

19. $f'(x) = -2\sin 2x$ 20. $\frac{2}{3}x^{3/2} + \frac{10}{x^{1/2}} + C$

21. (a) $f(2x) = 1 + 2x + 2x^2 + \frac{4}{3}x^3 + \frac{2}{3}x^4 + \frac{4}{15}x^5 + \cdots$
    (b) $\left[f'(2x) = 2 + 4x + 4x^2 + \frac{8}{3}x^3 + \frac{4}{3}x^4 + \cdots\right]$
    $\therefore f'(2x) = 2f(2x)$

PER II

1. (a) $3x + y = 14$ (b) $x + 2y = -2$ (c) $(6, -4)$

2. (a) $2x + y = -3$ (b) $B(0, -3)$ (c) $C(-2, 1)$ (d) $(x+1)^2 + (y+1)^2 = 5$

3. (a) $\vec{AK} = \begin{pmatrix} -5 \\ 5 \\ 11 \end{pmatrix}$ (b) $\vec{AL} = \begin{pmatrix} 2 \\ 4 \\ 9 \end{pmatrix}$ (c) $K\hat{A}L = 33.96^\circ$ (34°)

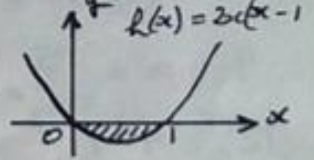
4. (a) $P(4, 0)$ (b) $x + 2y = 4$ (c) $Q\left(\frac{1}{2}, \frac{7}{4}\right)$

5. (a)(i) $8x + 9y$ (ii) $\left[A = 6xy \text{ and } y = \frac{360 - 8x}{9}\right]$ Hence required result.

   (b) [Verify firstly that $x = 22\frac{1}{2}$ gives a MAX. S.P. Table of values required]

   $x = 22\frac{1}{2}$ m $y = 20$ m MAX. AREA $= 2700$ m^2

6. (a)(i) $x^2 - 1$ (ii) $x^2 - 2x + 1$ (b) "PROOF" : GRAPH →



   (c) $\left[\text{INTEGRAL} = -\frac{1}{3}\right]$ $\therefore$ area $= \frac{1}{3}$ unit^2.

7. (a) $k = 0.07192$ (b) 51.3%

8. (a) [DRAW A PERPENDICULAR FROM P TO x-axis AND USE THAT RIGHT-ANGLED TRIANGLE]

   (b) $Q(\cos(a - 45^\circ), \sin(a - 45^\circ))$ (c) $R(\cos(a + 45^\circ), \sin(a + 45^\circ))$

   (d) $m_{QR} = -\frac{\cos a}{\sin a}$ (e) Prove also that $m_{\text{TGT AT } P} = -\frac{\cos a}{\sin a}$ and then give a clear conclusion.

9. [FIRST VERIFY THAT $2\sin x - 3\cos x = \sqrt{13}\cos(x - 146.3^\circ)$]

   $x = 100.2^\circ, 192.4^\circ$

10. (a) $h = 1$ $q = 2$

    (b) $\left[1\frac{5}{12} + 1\frac{1}{12}\right] = 2\frac{1}{2}$ units^2.

11. (a) AT P : $y = 3x + 20$ AT Q : $y = 3x - 12$

    (b) [DRAW PERP. FROM Q TO OTHER TGT. CALL THIS, SAY QR

    OBTAIN : QR: $x + 3y = -16$

    $R\left(-7\frac{3}{5}, -2\frac{4}{5}\right)$

    $QR^2 = \frac{2560}{25}$]

Required result then follows.

ANSWERS TO 1998 HIGHER

PAPER I

1. $y = 2x - 5$ 2. $(x-1)(x+2)(x-5)$

3. (a) $\begin{pmatrix} 8 \\ -5 \\ -5 \end{pmatrix}$ (b) 1 (c) 5 units 4. $3x + 4y = 26$

5. (a) $\vec{AB} = \begin{pmatrix} 6 \\ 3 \\ -6 \end{pmatrix}$ $\vec{AC} = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$ (b) $C(5, 0, 0)$ $D(7, 1, -2)$

6. (a) $x^4 + 4x^2 + 3$ (b) $(x^2+1)(x^2+3)$ 7. (a) $\frac{24}{25}$ (b) $\frac{7}{25}$ (c) $\frac{323}{325}$

8. (a) $u_n \rightarrow$ LIMIT AS $n \rightarrow \infty$ since 0.3 lies between -1 and 1 (b) $5\frac{5}{7}$

(c) (i) [$u_0 = 1$, $u_1 = 2.6$, $u_2 = 7.4$, $u_3 = 21.8$ -- (CONTINUE) .... $u_7 = 1749.8$]

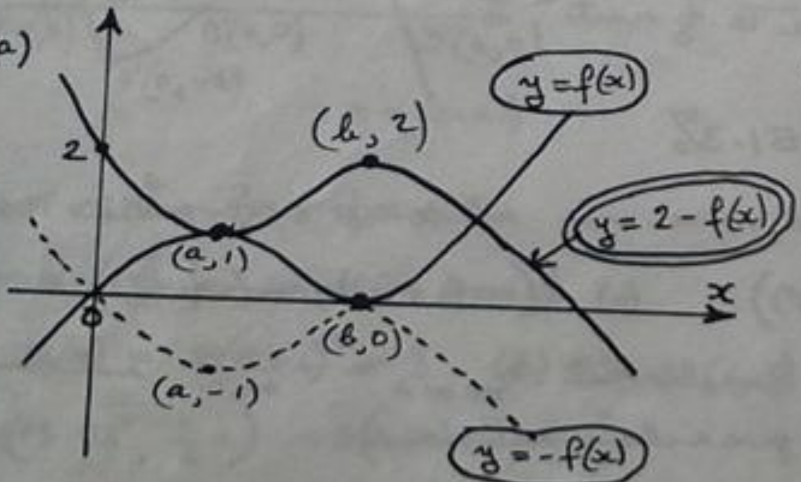
$\therefore$ SMALLEST VALUE OF n IS 7.

(ii) VALUE OF TERM $= 1749.8$.

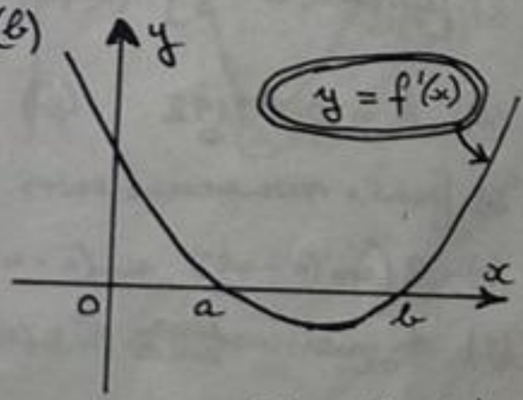
9. $x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}$ 10. $y = 2x^3 - x^2 + 5$ 11. $k = 1$

12. 9.7

13 (a)



(b)



(c) $(0, \frac{1}{2})$

14. $3x^{\frac{1}{2}} + \frac{2}{x^{1/2}}$ 15. (a) $p = 90°$ $q = 135°$ (b) $\frac{1}{2}$ unit^2

16. $\frac{3\sqrt{3}}{2}$ 17. (a) $9.8\ ms^{-1}$ (b) 2 seconds

18. (a) $\cos^2\theta + 4\cos\theta + 4 = 0$: Then prove that $\Delta = 0$
Since $\Delta = 0$, there are equal roots

(b) Prove that $\cos\theta = -2$ $\therefore$ NO SOLUTION.

19. $x = 1.544$

PAPER II

1. (a) $\vec{AB} = \begin{pmatrix} 1 \\ 7 \\ 2 \end{pmatrix}$ $\vec{AC} = \begin{pmatrix} 4 \\ 7 \\ -5 \end{pmatrix}$ (b) $51.9°$ (c) 27.43 units^2

2. (a) AND (b) $(0, -2)$ is a point of inflection
$(3, 25)$ is a maximum S.P.

(a) $2x + y = -1$ (b) 26.6°

4. (a) $a = 1$, $b = 6$ (b) 36 units^2 (c)(i) $P(5,5)$ (ii) $20\frac{5}{6}$ units^2

5. (a)(i) $A(-2,10)$ (ii) $y = 3x + 16$ (b) $B(4,28)$

6. (a) $(x - \frac{1}{2})^2 + (y - 3)^2 = \frac{13}{4}$ (b)(i) $B(8,8)$ (ii) $F(14,12)$ $C(6\frac{1}{2},7)$

(c) WORKING OUGHT TO LEAD TO:- $P = \pi\frac{\sqrt{13}}{2} + \pi(2\sqrt{13}) + \pi\left(\frac{5\sqrt{13}}{2}\right)$ etc

7. (a) $f(x) = \sqrt{13}\cos(x + 56.3^\circ)$ $k = \sqrt{13}$ $\alpha = 56.3^\circ$

(b) MAX $= \sqrt{13}$ occurs when $x = 303.7^\circ$
MIN $= -\sqrt{13}$ occurs when $x = 123.7^\circ$

(c) 0

8. (a) 1st feed = 1g 2nd feed = 1.75g 3rd feed = 2.3125g 4th feed = 2.7344g.
But $\frac{3}{4}$ of 2.3125 = 1.7344 $\therefore$ 4 feeds are required

(b)(i) $u_{n+1} = 0.75u_n + 1$ $(u_0 = 1)$ where u_n is the amount of plant food after n feeds.

(ii) Prove that the convergent value is 4
Since convergent value (4) < 5g, it can be fed indefinitely.

9. (a) $\frac{31\pi}{5}$ units^3 (b)(i) $\frac{16\pi}{3}$ units^3 (ii) $\frac{32\pi}{3}$ units^3

10. (a) [PROOF]

(b) S.A. is minimised when $r = \sqrt[3]{\frac{400}{3\pi}}$ TABLE OF VALUES REQUIRED

11. (a) PROVE that $\log_e y = bx + \log_e a$ [cf. $y = mx + c$]
$\therefore$ There is a linear relationship between $\log_e y$ and x.

(b) FIRSTLY :-

x	3.1	3.5	4.1	5.2
$\log_e y$	9.99	11.19	12.99	16.29

Use 1st/3rd points to obtain

$a = 1.99372$ $b = 3$ [using gradient and $\log_e y = 3x + \log_e a$]

ie $y = 1.99372e^{3x}$

1997 S.C.E. HIGHER I (ANSWERS)

1. $2x + y = -3$

2. Verify that $\overrightarrow{AB} = \begin{pmatrix} 1 \\ -4 \\ 2 \end{pmatrix}$ $\overrightarrow{BC} = \begin{pmatrix} 2 \\ -8 \\ 4 \end{pmatrix}$

$\therefore \overrightarrow{AB} = \frac{1}{2}\overrightarrow{BC}$

$\therefore$ AB is parallel to BC

$\therefore$ A, B, C are collinear ($\because$ of the common point B)

3. $f[g(x)] = 2(\sin x + \cos x)$

$g[f(x)] = \sin 2x + \cos 2x$

4.(a) $\overrightarrow{PQ} = \begin{pmatrix} 8 \\ -4 \\ 1 \end{pmatrix}$ (b) $PQ = 9$ units

5.(a) $\underline{\underline{a}}$ real root is $x = 2$ (b) Other roots come from $2x^2 + x + 4 = 0$

$\therefore \Delta = -31$

Since $\Delta < 0$, there are no other real roots

6. $x = -1$ 7. [You will need to construct a Δ from $\tan x = \frac{4}{3}$].

8. $\frac{dy}{dx} = 4x + 1$; then replace $\frac{dy}{dx}$ by $4x+1$ in $x\left(1 + \frac{dy}{dx}\right)$.

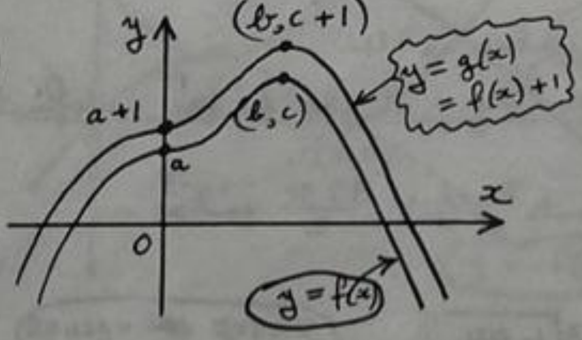
9.(a) Prove that $f(x) = 2(x+2)^2 - 11$ (b) MIN T.P. is $(-2, -11)$

10. $\frac{14}{3}$ 11. $\sqrt{29}\sin(x - 68 \cdot 2^\circ)$ 12. $(x-13)^2 + (y-4)^2 = 17$

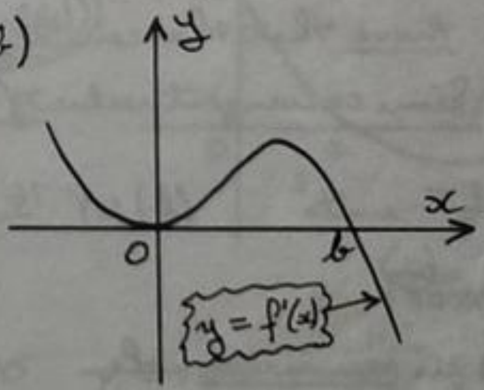
13. $\underset{\sim}{a} \cdot (\underset{\sim}{b} + \underset{\sim}{c}) = 0$; $\underset{\sim}{a}$ is perpendicular to $\underset{\sim}{b} + \underset{\sim}{c}$.

14. $c < 13$ 15. $f(x) = \frac{1}{2}\sin 2x + \frac{3}{4}$

16.(a)



(b)



(c) $g'(x)$ is the same graph as $f'(x)$

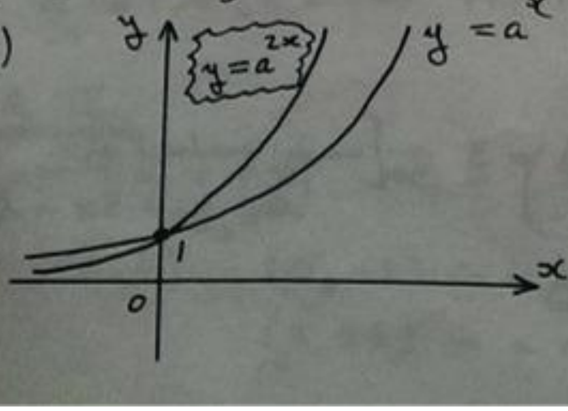
17. $A(-4\frac{1}{2}, 0)$ $B(14 \cdot 91, 8)$

18. (a) [Replace $\cos 2x$ by $1 - 2\sin^2 x$ and $\cos^2 x$ by $1 - \sin^2 x$]

(b) $x = 19 \cdot 45^\circ, 160 \cdot 55^\circ, 270^\circ$

19. (a) $t = a$; $u = 0$ (c) Point of intersection is $(1, a^2)$

(b)



20. $a = 51 \cdot 34^\circ$.

1 HIGHER II

1. (a) $A(1,3)$ $B(-3,-5)$ (b)(i) $C(-5,1)$ (ii) $x+2y=-3$

2. (a) $R(7,-1,6)$ (b) $\left[\vec{SR}=\begin{pmatrix}9\\-3\\1\end{pmatrix}\ \vec{SP}=\begin{pmatrix}1\\1\\-3\end{pmatrix}\right]$ $P\hat{S}R = 84\cdot56°$

3. (a) $\left[\begin{matrix} u_0(1^{ST}\text{JAN})=1000 & u_1(1^{ST}\text{FEB})=1105 & u_2(1^{ST}\text{MAR})=1210\cdot53 \\ u_3(1^{ST}\text{APRIL})=1316\cdot58 & u_4(1^{ST}\text{MAY})=1423\cdot16 & u_5(1^{ST}\text{JUNE})=1530\cdot28 \quad u_6(1^{ST}\text{JULY})=1537\cdot93\end{matrix}\right]$

$\therefore$ Amount on JUNE 30^{TH} = $\underline{£1537\cdot93}$

(b) [Continue until $u_{10}(1^{ST}\text{NOV})=2073\cdot94$] $\therefore$ First exceeds £2000 on 1^{ST} NOVEMBER

(c) $u_{n+1}=1\cdot005\,u_n+100$ where u_n is the amount in account n months after 1^{ST} JAN on 1^{ST} of month (where $u_0=1000$).

4. (a) $P(1,0)$ $Q(2,0)$ (b) $\left[\text{TOTAL AREA} = 1\tfrac{11}{12}+\tfrac{7}{12}\right] = \underline{\underline{2\tfrac{1}{2}\text{ units}^2}}$

5. (a) $A(0,5)$ $B(2,1)$ (b) [FIRST PROVE THAT CURVES MEET WHEN $x=0$ or 4]
$\therefore$ AREA ENCLOSED $= \underline{\underline{21\tfrac{1}{3}\text{ units}^2}}$

(c) $m=10$, $n=-1$

6. (a) $m_{TGT}=-\frac{1}{a^2}$ (b) — (c)(i) [PROVE THAT B IS $(0,\frac{2}{a})$ AND C IS $(2a,0)$]
$\therefore$ AREA OF $\Delta OBC = \underline{\underline{2\text{ units}^2}}$
(ii) Area of Δ OBC is always 2 units² regardless of position of tangent at A

7. $\dfrac{5x+1}{(x-4)(x+3)} = \dfrac{3}{x-4}+\dfrac{2}{x+3}$

8. (a) $k=-0\cdot000122$ (b) $88\cdot5\%$

9. (a) LIKELIEST : $k=3$ $q=1$ $r=-140°$ $u=230°$
(OTHERS : $k=3$ $q=1$ $r=220°$ $u=230°$
$k=-3$ $q=1$ $r=40°$ $u=230°$
$k=-3$ $q=1$ $r=-320°$ $u=230°$)

(b) $s=-0\cdot928$
$t=120\cdot53°$
[USE EQUATION WITH REPLACEMENTS FOR k, r AND q]

10. (a)(i) [USE SIMILAR Δ's ADE AND ABC] (ii) $[V=(2x)(2x)(10-\frac{5}{2}x)$ etc$]$

(b) [PROVE THAT GREATEST VOLUME OCCURS WHEN $x=\frac{8}{3}$ (not $x=0$ $\because x>0$)]
$\therefore$ DIMENSIONS ARE $\underline{\underline{\frac{16}{3}\text{ cm} \times \frac{16}{3}\text{ cm} \times \frac{10}{3}\text{ cm}}}$

11. (a)(i) — (ii) $OC = 1+2\cos k$ [and finally use $d^2 = OB^2+OC^2$]

(b)(i) $d^2 = 6+4\sqrt{2}\cos(k-\frac{\pi}{4})$ (ii) MAX. VALUE OF d^2 IS $6+4\sqrt{2}$; OCCURS WHEN $k=\frac{\pi}{4}$.

(c)(i) [REPLACE k by $\frac{\pi}{4}$ in $OB = 1+2\sin k$]; $BD = \frac{1}{2}(\sqrt{2}+2)$

(ii) $\sqrt{6+4\sqrt{2}} = 2+\sqrt{2}$

S.C.E. HIGHER ANSWERS - 1996

PAPER I

1. $3x + 2y = 17$

2. $a = 2\frac{1}{2}$

3. $\left(\frac{5\pi}{6}, 3\right)$

4. $2x + y = 10$

5. 9

6. Verify that $\overrightarrow{AB} = \begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix}$, $\overrightarrow{BC} = \begin{pmatrix} 6 \\ 3 \\ -3 \end{pmatrix}$

   $\therefore \overrightarrow{AB} = \frac{2}{3}\overrightarrow{BC}$

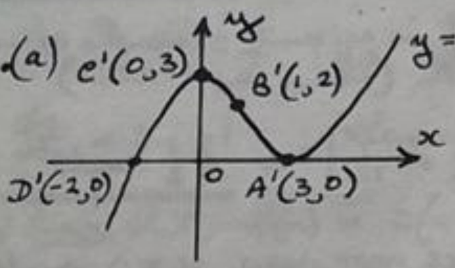
   $\therefore$ AB is parallel to BC

   $\therefore$ A, B, C are collinear [$\because$ B is a common point]

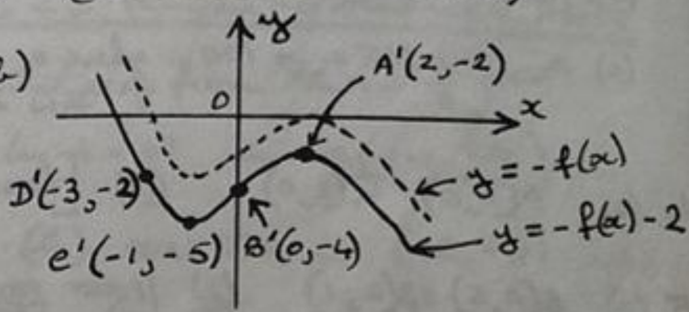
   B divides AC in the ratio $2:3$ (ie $AB : BC = 2:3$)

7. $x(x-1)(x^2+x+1)$

8. (a)



(b)



9. $f'(4) = \frac{5}{16}$

10. $0°, 120°, 180°, 240°$ [NB NOT $360°$ since $0° \leq x < 360°$]

11. (a) Limit exists because 0.3 lies between -1 and 1 (b) $7\frac{1}{7}$

12. $P\left(\frac{\pi}{6}, \frac{1}{2}\right)$ $Q\left(\frac{\pi}{2}, \frac{1}{2}\right)$

13. $\frac{dy}{dx} = \frac{-\sin x}{2\sqrt{1+\cos x}}$

14. Take 2 lines $\left\{\begin{matrix} 2x+3y=4 \\ 3x-y=17 \end{matrix}\right\}$. Verify that they intersect at $(5,-2)$. Then verify that $(5,-2)$ does NOT lie on the 3rd line $x - 3y - 10 = 0$

    $\therefore$ 3 LINES ARE NOT CONCURRENT.

15. [Verify that $BD = 5$ and $AD = 2\sqrt{6}$; then use the expansion of $\cos(x+y)$]

16. f is increasing when $x < -2$ AND $x > 3$ [TABLE OF VALUES REQUIRED]

17. $4(x+1)^2 - 9$

18. $\sin x = \frac{4\sqrt{5}}{9}$

19. (a) $k = 0.019$ (b) [FALLS TO $56.56°C$ IN NEXT 15 MINS]

    $\therefore$ IT FALLS $18.44°C$ IN NEXT 15 MINS

20. $(x-3)^2 + (y-4)^2 = 25$.

PAPER II

1. (a) S.P.'s are $(0,3)$ AND $(3,-24)$ (b) $(0,3)$ is a point of inflection

   $(3,-24)$ is a MINIMUM S.P.

2. (a) Verify that $m_{AB} m_{BC} = -1$ WITH A CLEAR CONCLUSION $\left(\begin{matrix} m_{AB} = 2 \\ m_{BC} = -\frac{1}{2} \end{matrix}\right)$

   (b)(i) AD: $x - 3y = 6$

   BE: $4x + 3y = -1$

   (ii) $M\left(1, -\frac{5}{3}\right)$

(a) $Q(2,2,9)$ $R(21,3,12)$ (b) $Q\hat{P}R = 83.4^\circ$

(a) (i) $g[f(x)] = 4x^2+4x+1+k$ (ii) $f[g(x)] = 2x^2+2k+1$

(b) (i) [ANSWER IN QUESTION] (ii) [$\Delta = 64$] $\therefore$ Roots are REAL AND DISTINCT

(iii) $k = -2$

5. [area I $= \frac{1}{2} - \frac{\sqrt{3}}{4}$; area II $= \frac{1}{2}$] $\therefore$ TOTAL AREA $= 1 - \frac{\sqrt{3}}{4}$ units^2

6. (a) Line ℓ : $y = 2x$ Parabola p : $y = 2x^2$ Circle c : $x^2+y^2=5$

(b) ℓ' : $y = 2x-4$ p' : $y = -2x^2$ c' : $(x-2)^2+y^2 = 5$

(c) $(-2,-8)$ AND $(1,-2)$

7. (a) $\sqrt{13}\cos(x - 56.3^\circ)$ (b) 138.3°, 334.3°

8. (a) $5x + y = -3$; $B(-1,2)$

(b) $\frac{4}{3}$ units^2

9. (a) $Y = 3X + 0.7$

(b) [FIRST REPLACE Y BY $\log_e y$ AND X BY $\log_e x$ TO OBTAIN :- $y = 2.014x^3$.] $\therefore \left\{ \begin{array}{l} a = 2.014 \\ b = 3 \end{array} \right\}$

10. (a) By symmetry, result needs only to be proved for one point at $(3,7)$: PROVE THAT TANGENT TO PARABOLA HAS GRADIENT $\frac{2}{3}$ AND PROVE THAT TANGENT TO CIRCLE HAS GRADIENT $-\frac{3}{2}$.

Then conclude that since product of gradients of tangents is -1, the tangents are perpendicular. $\therefore$ CURVES ARE ORTHOGONAL.

(b) (i) $2y = 3x + 23$

(ii) $x^2 + (y - 11\frac{1}{2})^2 = 29\frac{1}{4}$

11. (a) (i) $h = 5 - x - \frac{1}{2}\pi x$

(ii) [START WITH $L = 2$(area of rectangle) $+ 1$(area of $\frac{1}{2}$ circle)]

(b) $x = \dfrac{20}{8+3\pi}$ $(\doteqdot 1.1478)$

$h = \dfrac{20+5\pi}{8+3\pi}$ $(\doteqdot 2.0493)$

NB : TABLE OF VALUES IS REQUIRED

S.C.E. REVISED HIGHER — ANSWERS

1995 I

1. 4 units.

2.(a) Verify that $f(3)=0$ $\therefore (x-3)$ is a factor of $f(x)$.
(b) $(x-3)(x+4)(2x+1)$

3. $2x^3 - \frac{1}{2}x^2 + \sin x + C$

4. $k=3$

5. $4x+3y=1$

6. Prove that A_1A_2 has equation $3y = 2x+5$. Then verify that $(5,4)$ does not satisfy this equation $\therefore$ ACHILLES DOES NOT PASS OVER THE SUBMARINE.
also, prove that B_1B_2 has equation $4y = 5x-9$. Then verify that $(5,4)$ DOES satisfy this equation $\therefore$ BELLIGERENT DOES PASS OVER THE SUBMARINE.

7. $-\frac{8}{x^3} + \frac{3}{2}x^{1/2}$

8. $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

9. $(x-1)^2+(y-2)^2=18$

10.(a) $b=-10$ (b) Prove that $f'(-2)=33$. Since $f'(-2)>0$, f is increasing at P

11.(a) $\frac{x^2-2}{x^2-4}$ (b) $\{x : x \in R, x \neq 2 \text{ or } -2\}$

12. $\sin 2\alpha = \frac{3\sqrt{11}}{10}$

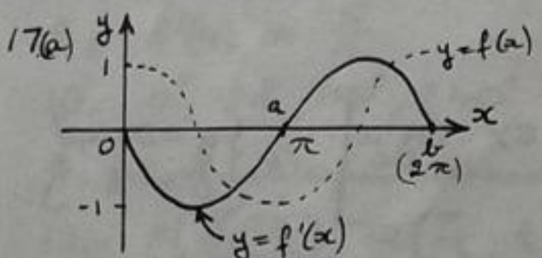
13. $k=\sqrt{29}$, $x=68{\cdot}2^\circ$

14. $f'(a)$ = gradient of line $= -\frac{1}{3}$.

15. $x = 60^\circ, 180^\circ, 300^\circ$

16. 9

17(a)



(b) $f(x) = \cos x$
$f'(x) = -\sin x$

19.8.

18.(a) $k=0{\cdot}035$ (b) $t = 8{\cdot}6$ minutes

19. [Replace $(3,0)$ and $(4,3)$ into equation]
$a=3$, $b=2$

20. $k=-5$ or 3

21.(a) Speed $= 20-10t$
Speed when thrown $= 20$ m/s
(b) 0 m/s. Ball has reached its maximum height.

1995 II

1.(a) Verify that $AB = AC = 3\sqrt{5}$ $\therefore \triangle ABC$ is isosceles.
(b)(i) [AD : $x=4$, BE : $2y-x=3$] $\therefore H(4, 3\frac{1}{2})$
(ii) Check that $HD = 1\frac{1}{2}$ units, $DA = 6$ units $\therefore$ H is $\frac{1}{4}$ the way up DA

2.(a) — (b) Tangent meets curve AGAIN at $(-2,-18)$

3. KILLPEST [$u_{n+1} = 0{\cdot}35u_n + 500$] — ie 769 pests in the long term
PESTKILL [$u_{n+1} = 0{\cdot}15u_n + 650$] — ie 765 pests in the long term
$\therefore$ PESTKILL is more effective in the long term

4.(a)(i) $f(x) = 2\sin 3x$ $\therefore a=3, b=2$ (ii) $g(x) = 3\cos 3x$ $\therefore c=3, d=3$
(b) [Let $h(x) = 2\sin 3x + 3\cos 3x = q\sin(3x+r)$]
$\therefore q=\sqrt{13}$, $r = 56{\cdot}3^\circ$ $\therefore h(x) = \sqrt{13}\sin(3x+56{\cdot}3^\circ)$.

$\left[\vec{QP} = \begin{pmatrix} 0 \\ 2 \\ -2 \end{pmatrix}, \vec{QR} = \begin{pmatrix} -2 \\ 2 \\ 0 \end{pmatrix}\right] \longrightarrow P\hat{Q}R = 60^\circ.$

(b)(i) $T(2\frac{1}{3}, 3\frac{1}{3}, 1\frac{1}{3})$ (ii) Verify that $PT = QT = RT = \frac{2}{3}\sqrt{6}$
$\therefore$ P, Q, R are equidistant from T.

(c)(i) Verify that $PA = QA = RA = \sqrt{3}$. $\therefore$ P, Q, R are equidistant from A.
(ii) By construction, T lies within ΔPQR – ie T is coplanar with P, Q, R.
Also $PA\left[\sqrt{3}\right] > PT\left[\frac{2}{3}\sqrt{6}\right]$ $\therefore$ A is not coplanar with P, Q, R.
$\therefore$ T is the centre of the circle passing through P, Q, R (AND NOT A).

6. [THE GAUSSIAN ELIMINATION METHOD AS DESCRIBED MUST BE USED.]
$\left[1^{ST} \text{ obtain } \left[\begin{array}{ccc|c} 1 & -2 & 1 & 6 \\ 0 & 7 & -4 & -11 \\ 0 & 0 & 2 & 2 \end{array}\right] \right.$ $\therefore x = 3, y = -1, z = 1$

7. $a = -\frac{1}{3}, b = \frac{1}{3}, c = 3$

8. [YOU MUST OBTAIN: $A(-12, -15)$ $r_1 = 5$ $C(24, 12)$ $r_3 = 10$
$AC = 45$ units
$r_2 = 15$ (radius of central circle) $B(4, -3)$ [QUITE HARD]]
$(x-4)^2 + (y+3)^2 = 225$

9.(a) — (b) $-\frac{3}{4}\cos x + \frac{1}{12}\cos 3x + C$

10.(a) 48 units^2 (b) —
(c)(i) — (ii) Other 2 solutions are $6+6\sqrt{3}$, $6-6\sqrt{3}$ ($\cong 16.39, -4.39$ APPROX.)
(iii) p must lie between 0 and 12 which the other 2 solutions do NOT satisfy.
p is the only valid solution to this problem.
$\therefore p = 6$ is the only valid solution to this problem.

11. (a) — (b) —
(c) $x = 4$ (not -4 since A is in the 1^{ST} quadrant)
[Table of values required to verify MAX.]
$\therefore$ MAX. LENGTH = <u>10 units</u>.

1994 I

1. $\frac{3}{4}x^4 + 2x^2 + c$ 2. $k = 3$ 3. $D(-11, 2, 3)$

4. $\vec{RS} = \begin{pmatrix}3\\3\\6\end{pmatrix}$ $\vec{ST} = \begin{pmatrix}1\\1\\2\end{pmatrix}$ $\therefore \vec{RS} = 3\vec{ST}$ $\therefore$ RS $\parallel^{\ell}$ to ST
$\therefore$ R, S, T are collinear ($\because$ S is a common point)

5. Centre $(1, 1)$ [C] $m_{BC} = \frac{1}{2}$, $m_{AC} = -2$ Since $m_{BC} m_{AC} = -1$, BC $\perp^{r}$ to AC.
But also $C\hat{B}D = C\hat{A}D = 90°$ (angle between TGT and radius)
$\therefore B\hat{D}A = 90°$ (angle sum in ABCD is 360°)
$\therefore$ Tangents at B and A are perpendicular.

6. 0 [Use $\sin(\alpha + \beta)$ TWICE and also special values]

7. $\underset{\sim}{u} + \underset{\sim}{v} = \begin{pmatrix}-2\\8\\2\end{pmatrix}$ $\underset{\sim}{u} - \underset{\sim}{v} = \begin{pmatrix}-4\\-2\\4\end{pmatrix}$. Finally verify that $(\underset{\sim}{u} + \underset{\sim}{v}).(\underset{\sim}{u} - \underset{\sim}{v}) = 0$.

8. (a) $A(6, 6)$ $B(-2, -2)$ (b) $(x-2)^2 + (y-2)^2 = 32$

9. (a) $u_2 = 4.7$ (b) [Continue sequence $\rightarrow u_7 = 10.96....$] $\therefore n = 7$ (b) 20

10. $-\frac{3}{x^4} - 3\sin 3x$ 11. $(x+4)^2 + 2$ S.P. is $(-4, 2)$

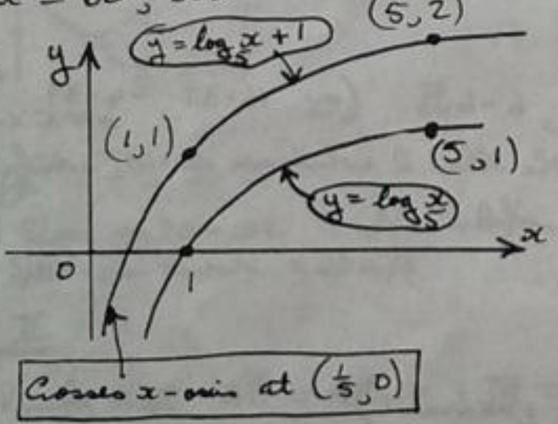
12. (a) $a = 2$, $b = 1$, $c = 2$ (b) [Solve $2 + \sin 2x = 2.5$] $x = \frac{\pi}{12}, \frac{5\pi}{12}$.

13. (a) $\sin 2\theta = \frac{24}{25}$ (b) $\sin 4\theta = \frac{336}{625}$

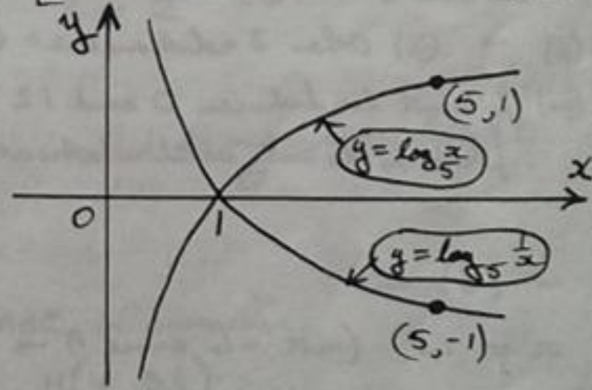
14. $m_{TGT} = 4$ [Required angle is $75.96° - 45°$] $= 30.96°$ ($\simeq 31°$)

15. $x = 60°, 300°$

16. (a)



(b) [NB $\log \frac{1}{x} = \log x^{-1} = -\log x$]



17. $\frac{dy}{dx} = 3\sin^2 x \cos x$; $\int \sin^2 x \cos x \, dx = \frac{1}{3}\sin^3 x + c$

18. [Let S/T be $(x, 0, 0)$] $S(1, 0, 0)$ $T(7, 0, 0)$

19. $f[f(x)] = \frac{x}{1 - 2x}$ 20. $x = 3.402$; $P(3.402, 42)$

1994 II

1. (a) $a = 6$ [$\therefore y = 2x^3 + x^2 - 13x + 6$] Crosses y-axis at $(0, 6)$
(b) OTHER points are $(-3, 0)$, $(\frac{1}{2}, 0)$ [APART FROM $(2, 0)$]

2. (a) $2x + y = 10$ (b) $\begin{bmatrix}2x + y = 10\\ y = \frac{1}{2}x\end{bmatrix} \rightarrow D(4, 2)$ (c) [$AD = \sqrt{5}$] Area = 5 units^2

1994

(a) $B(6,4,2)$ $C(4,3,4)$ $D(6,2,2)$

(b) Calculate mid-pt of AD to be $(4,3,4) = C$.

(c) $\left[\vec{OA} = \begin{pmatrix}2\\4\\6\end{pmatrix} \quad \vec{OB} = \begin{pmatrix}6\\4\\2\end{pmatrix}\right] \rightarrow A\hat{O}B = 44 \cdot 4^\circ$. (d) $O\hat{A}B = 67 \cdot 8^\circ$

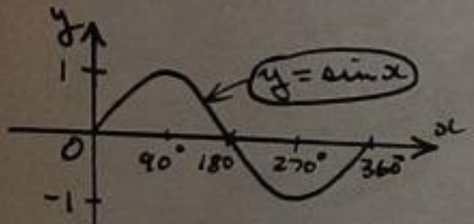
4. (a)(i) $[Q(0,3) \quad P(14,10)] \rightarrow [PQ = 7\sqrt{5} \text{ units}] = 35\sqrt{5}$ cm $= 78 \cdot 26$ cm

(ii) $[r_1 = 3 \text{ units} = 15 \text{ cm} \; ; \; r_2 = 10 \text{ units} = 50 \text{cm}]\,[78 \cdot 26 - 65]$ 13 cm 3 mm (to nearest mm.)

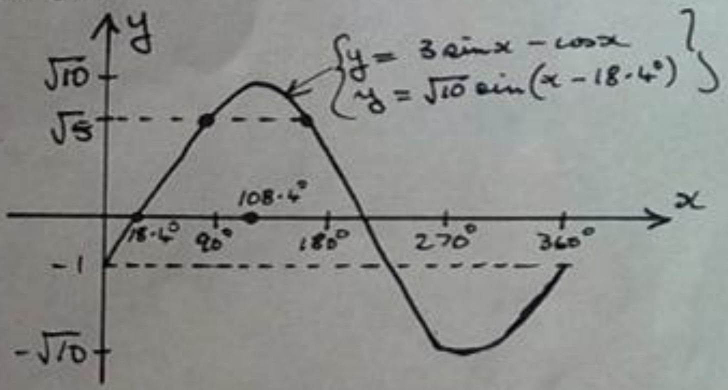
(b)(i) $m_{PB} = 1$ (ii) $[AC: \; x + y = 10]$ $A(8,2)$ $C(6,4)$

5. (a) $\sqrt{10}\sin(x - 18 \cdot 4^\circ)$ (b) $63 \cdot 4^\circ, 153 \cdot 4^\circ$

(c)



$\longrightarrow$



$3\sin x - \cos x \leq \sqrt{5} \quad (0^\circ \leq x \leq 360^\circ)$

$\therefore 0^\circ \leq x \leq 63 \cdot 4^\circ$ or $153 \cdot 4^\circ \leq x \leq 180^\circ$

6. (a) $f(0) = -1$, $f(0 \cdot 5) = 0 \cdot 75$

Since $f(0) < 0$ and $f(0 \cdot 5) > 0$, there is a root between 0 and $0 \cdot 5$

(b) $[x_1 = 0 \cdot 25, \; x_2 = 0 \cdot 32, \; x_3 = 0 \cdot 312, \; x_4 = 0 \cdot 313, \; x_5 = 0 \cdot 313]$ $0 \cdot 31$ (to 2 D.P.)

7. (a) — (b) [MIN. OCCURS WHEN $x = 4$ – TABLE REQUIRED]

MIN. AREA = 16 units^2 MAX. AREA = 24 units^2.

8. (a) — (b) $4x + 7$ (c) $6x + 5$

9. (a) $2p + q = -2$ (b) $\left[\text{When } x = 2, \; \frac{dy}{dx} = 1\right] \rightarrow p = -3$ $[q = 4]$

Equation of parabola is $y = x^2 - 3x + 4$

(c) $\Delta = -7$ $\therefore$ No real roots $\therefore$ Parabola does not cross x-axis.

10. Cross-sectional area $= 69\frac{1}{3}$ m^2

Volume $= 4160$ m^3