X056/301

NATIONAL QUALIFICATIONS 2000

THURSDAY, 25 MAY
9.00 AM – 10.10 AM

MATHEMATICS
HIGHER
Paper 1
(Non-calculator)

Read Carefully

1 **Calculators may NOT be used in this paper.**

2 There are three Sections in this paper.

Section A assesses the compulsory units Mathematics 1 and 2.
Section B assesses the optional unit Mathematics 3.
Section C assesses the optional unit Statistics.

Candidates must attempt **all** questions in Section A (Mathematics 1 and 2) **and**

either Section B (Mathematics 3)

or Section C (Statistics).

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FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}|\,|\boldsymbol{b}| \cos \theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$

or $\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

Table of standard derivatives and integrals:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

Statistics:

Sample standard deviation: $s = \sqrt{\frac{1}{n-1}\Sigma(x_i - \bar{x})^2} = \sqrt{\frac{1}{n-1}\left(\Sigma x_i^2 - \frac{1}{n}\left(\Sigma x_i\right)^2\right)}$ where n is the sample size.

Sums of squares and products: $S_{xx} = \Sigma\left(x_i - \bar{x}\right)^2 = \Sigma x_i^2 - \frac{1}{n}\left(\Sigma x_i\right)^2$

$$S_{yy} = \Sigma\left(y_i - \bar{y}\right)^2 = \Sigma y_i^2 - \frac{1}{n}\left(\Sigma y_i\right)^2$$

$$S_{xy} = \Sigma\left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right) = \Sigma x_i y_i - \frac{1}{n}\Sigma x_i \Sigma y_i$$

Linear regression: The equation of the least squares regression line of y on x is given by $y = \alpha + \beta x$, where estimates for α and β, a and b, are given by: $a = \bar{y} - b\bar{x}$

$$b = \frac{\Sigma\left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right)}{\Sigma\left(x_i - \bar{x}\right)^2} = \frac{S_{xy}}{S_{xx}}$$

Product moment correlation coefficient r: $r = \frac{\Sigma\left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right)}{\sqrt{\Sigma\left(x_i - \bar{x}\right)^2 \Sigma\left(y_i - \bar{y}\right)^2}} = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}}$

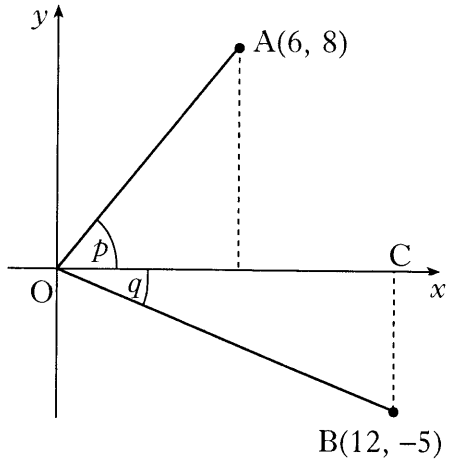
SECTION A (Mathematics 1 and 2)

Marks

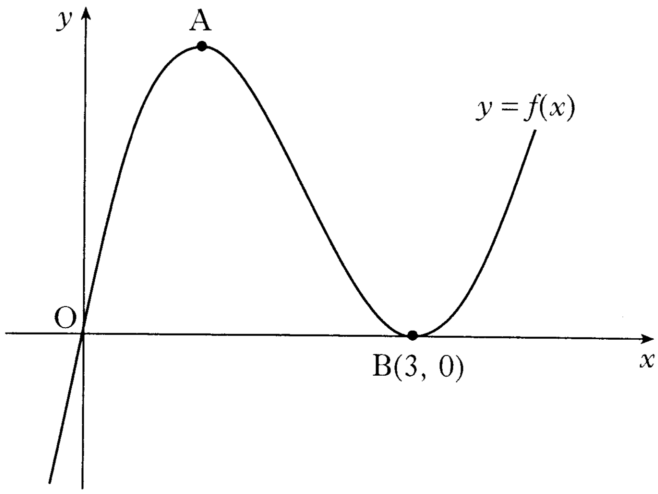
ALL candidates should attempt this Section.

A1. On the coordinate diagram shown, A is the point (6, 8) and B is the point (12, −5). Angle AOC = p and angle COB = q.

Find the exact value of $\sin(p + q)$. **4**



A2. A sketch of the graph of $y = f(x)$ where $f(x) = x^3 - 6x^2 + 9x$ is shown below. The graph has a maximum at A and a minimum at B(3, 0).



(*a*) Find the coordinates of the turning point at A. **4**

(*b*) Hence sketch the graph of $y = g(x)$ where $g(x) = f(x + 2) + 4$.

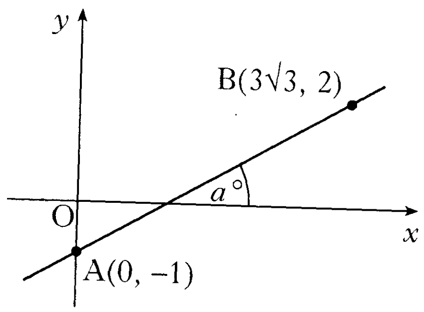
Indicate the coordinates of the turning points. There is no need to calculate the coordinates of the points of intersection with the axes. **2**

(*c*) Write down the range of values of k for which $g(x) = k$ has 3 real roots. **1**

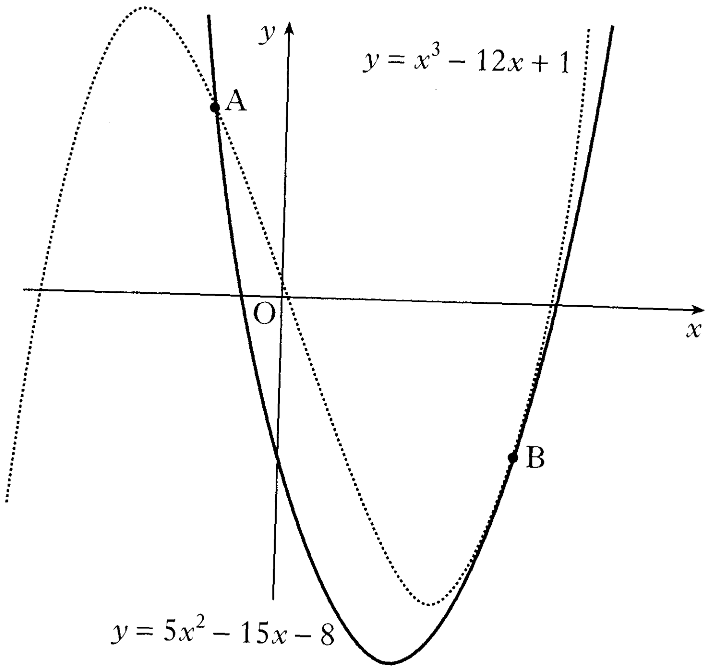
[Turn over

Marks

A3. Find the size of the angle $a°$ that the line joining the points A(0, –1) and B($3\sqrt{3}$, 2) makes with the positive direction of the x-axis. 3



A4. The diagram shows a sketch of the graphs of $y = 5x^2 - 15x - 8$ and $y = x^3 - 12x + 1$. The two curves intersect at A and touch at B, ie at B the curves have a common tangent.



(*a*) (i) Find the x-coordinates of the points on the curves where the gradients are equal. 4

(ii) By considering the corresponding y-coordinates, or otherwise, distinguish geometrically between the two cases found in part (i). 1

(*b*) The point A is (–1, 12) and B is (3, –8).

Find the area enclosed between the two curves. 5

Marks

A5. Two sequences are generated by the recurrence relations $u_{n+1} = au_n + 10$ and $v_{n+1} = a^2v_n + 16$.

The two sequences approach the same limit as $n \to \infty$.

Determine the value of a and evaluate the limit. **5**

A6. For what range of values of k does the equation $x^2 + y^2 + 4kx - 2ky - k - 2 = 0$ represent a circle? **5**

[*END OF SECTION A*]

Candidates should now attempt

EITHER Section B (Mathematics 3) on *Page six*

OR Section C (Statistics) on *Pages seven and eight*

[Turn over

SECTION B (Mathematics 3)

Marks

ONLY candidates doing the course Mathematics 1, 2 and 3 should attempt this Section.

B7. VABCD is a pyramid with a rectangular base ABCD.

Relative to some appropriate axes,

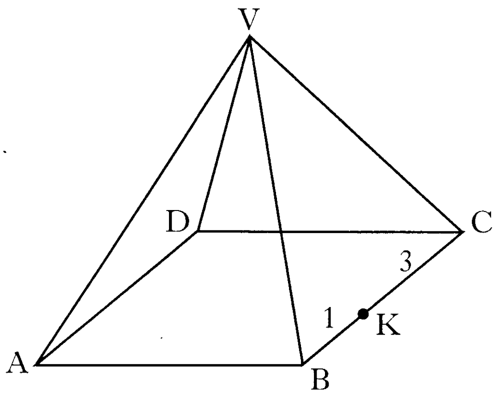
$\overrightarrow{VA}$ represents $-7\boldsymbol{i} - 13\boldsymbol{j} - 11\boldsymbol{k}$

$\overrightarrow{AB}$ represents $6\boldsymbol{i} + 6\boldsymbol{j} - 6\boldsymbol{k}$

$\overrightarrow{AD}$ represents $8\boldsymbol{i} - 4\boldsymbol{j} + 4\boldsymbol{k}$.

K divides BC in the ratio 1:3.

Find $\overrightarrow{VK}$ in component form.



3

B8. The graph of $y = f(x)$ passes through the point $\left(\frac{\pi}{9}, 1\right)$.

If $f'(x) = \sin(3x)$, express y in terms of x. **4**

B9. Evaluate $\log_5 2 + \log_5 50 - \log_5 4$. **3**

B10. Find the maximum value of $\cos x - \sin x$ and the value of x for which it occurs in the interval $0 \le x \le 2\pi$. **6**

[*END OF SECTION B*]

SECTION C (Statistics)

Marks

ONLY candidates doing the course Mathematics 1, 2 and Statistics should attempt this Section.

C7. The random variable X represents the number of faulty components in a circuit board. X has the following probability distribution:

$$P(X = x) = \begin{cases} \frac{1}{4}k(4 - x) & \text{for } x = 0,1,2,3 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant.

(*a*) Find the value of k. **2**

(*b*) Find the expected value and variance of X. **3**

C8. A class of 31 students estimated the length of a line which was 88 millimetres long. The distribution of male and female estimates is recorded in the back-to-back stem-and-leaf diagram below.

Male ($n = 13$)							Female ($n = 18$)							
						5	6							
8	8	6	4	4	4	6	1	3	4	4	7	9		
7	7	3	2	0	0	7	0	2	2	3	3	3	7	7
					0	8	1	1						
						9	7							

9 | 7 means 97 millimetres

(*a*) Write down the median value and the semi-interquartile range for the distribution of female estimates. **2**

(*b*) (i) Determine any possible outliers within the female distribution. **2**

(ii) Draw a boxplot to illustrate the distribution of the female estimates. **1**

(*c*) The distribution of the male estimates has a median value of 70 mm and a semi-interquartile range of 5 mm. Compare the distributions of the male and female estimates. **2**

[Turn over for Question C9 on *Page eight*

Marks

C9. A trial consists of tossing two unbiased coins. Let the random variable X represent the number of heads obtained.

(*a*) Tabulate the probability distribution of X after one trial. **2**

(*b*) A calculator was used to produce the list of random numbers below.

0·667	0·013	0·600	0·277	0·011
0·921	0·836	0·255	0·726	0·247
0·101	0·731	0·222	0·594	0·820
0·934	0·492	0·095	0·402	0·646

Use these random numbers to simulate 10 trials of this random experiment.

Explain your working. **2**

[*END OF SECTION C*]

[*END OF QUESTION PAPER*]

X056/302

NATIONAL QUALIFICATIONS 2000

THURSDAY, 25 MAY
10.30 AM – 12.00 NOON

MATHEMATICS
HIGHER
Paper 2

Read Carefully

1 **Calculators may be used in this paper.**

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Section A assesses the compulsory units Mathematics 1 and 2.
Section B assesses the optional unit Mathematics 3.
Section C assesses the optional unit Statistics.

Candidates must attempt **all** questions in Section A (Mathematics 1 and 2) **and**

either Section B (Mathematics 3)

or Section C (Statistics).

3 Full credit will be given only where the solution contains appropriate working.

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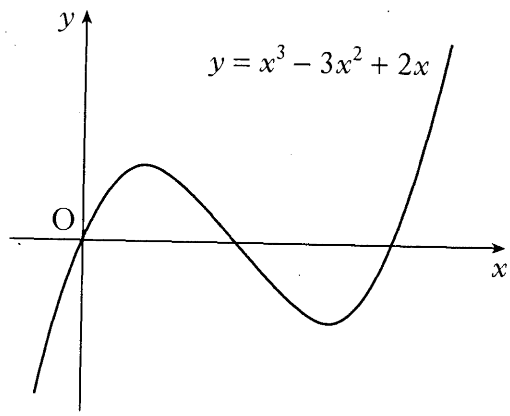
 Marks

ALL candidates should attempt this Section.

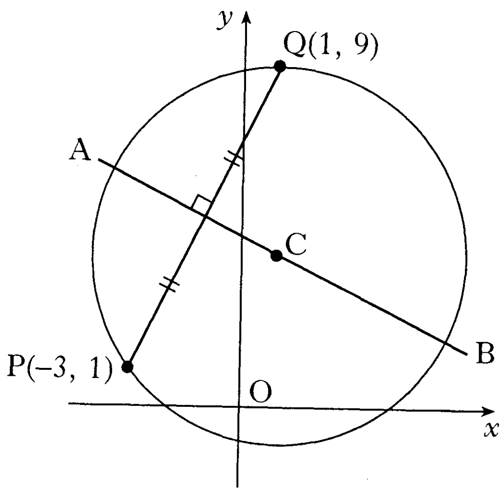
A1. The diagram shows a sketch of the graph of $y = x^3 - 3x^2 + 2x$.



(*a*) Find the equation of the tangent to this curve at the point where $x = 1$. **5**

(*b*) The tangent at the point (2, 0) has equation $y = 2x - 4$. Find the coordinates of the point where this tangent meets the curve again. **5**

A2. (*a*) Find the equation of AB, the perpendicular bisector of the line joining the points P(–3, 1) and Q(1, 9). **4**



(*b*) C is the centre of a circle passing through P and Q. Given that QC is parallel to the y-axis, determine the equation of the circle. **3**

(*c*) The tangents at P and Q intersect at T.

Write down

(i) the equation of the tangent at Q

(ii) the coordinates of T. **2**

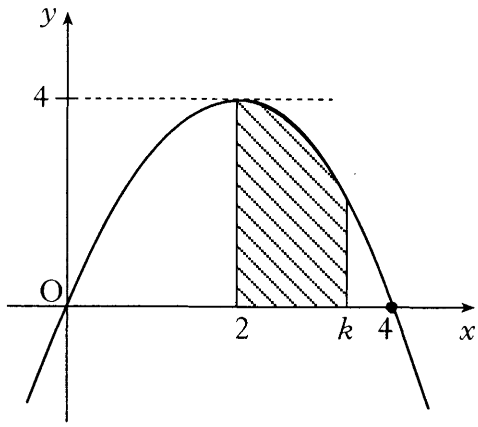
A3. $f(x) = 3 - x$ and $g(x) = \frac{3}{x}$, $x \neq 0$.

(*a*) Find $p(x)$ where $p(x) = f(g(x))$. **2**

(*b*) If $q(x) = \frac{3}{3-x}$, $x \neq 3$, find $p(q(x))$ in its simplest form. **3**

A4. The parabola shown crosses the x-axis at (0, 0) and (4, 0), and has a maximum at (2, 4).

The shaded area is bounded by the parabola, the x-axis and the lines $x = 2$ and $x = k$.



(*a*) Find the equation of the parabola. **2**

(*b*) Hence show that the shaded area, A, is given by

$$A = -\frac{1}{3}k^3 + 2k^2 - \frac{16}{3}.$$ **3**

A5. Solve the equation $3\cos 2x° + \cos x° = -1$ in the interval $0 \leq x \leq 360$. **5**

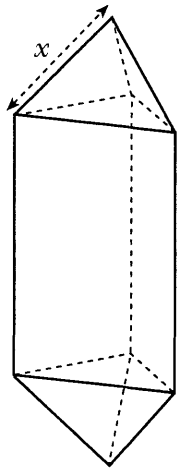
A6. A goldsmith has built up a solid which consists of a triangular prism of fixed volume with a regular tetrahedron at each end.

The surface area, A, of the solid is given by

$$A(x) = \frac{3\sqrt{3}}{2}\left(x^2 + \frac{16}{x}\right)$$

where x is the length of each edge of the tetrahedron.

Find the value of x which the goldsmith should use to minimise the amount of gold plating required to cover the solid. **6**



[*END OF SECTION A*]

Candidates should now attempt
EITHER Section B (Mathematics 3) on *Pages five and six*
OR Section C (Statistics) on *Pages seven and eight*

SECTION B (Mathematics 3)

Marks

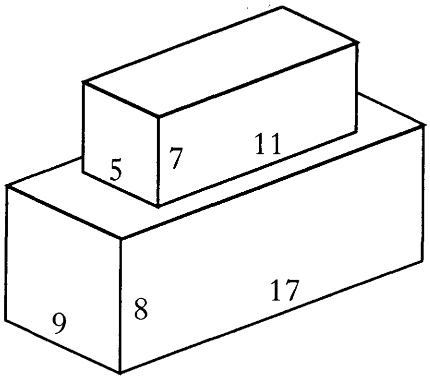
ONLY candidates doing the course Mathematics 1, 2 and 3 should attempt this Section.

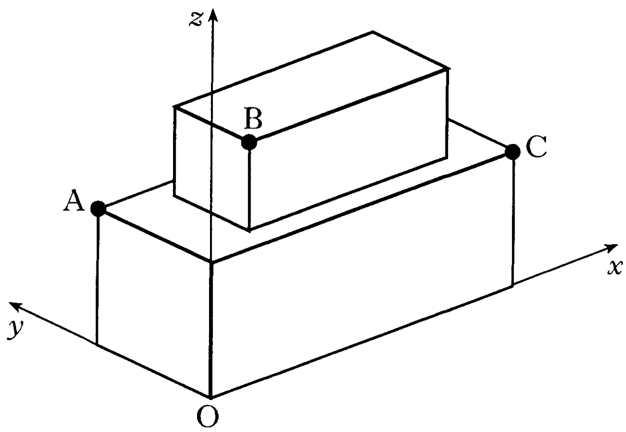
B7. For what value of t are the vectors $u = \begin{pmatrix} t \\ -2 \\ 3 \end{pmatrix}$ and $v = \begin{pmatrix} 2 \\ 10 \\ t \end{pmatrix}$ perpendicular? **2**

B8. Given that $f(x) = (5x-4)^{\frac{1}{2}}$, evaluate $f'(4)$. **3**

B9. A cuboid measuring 11 cm by 5 cm by 7 cm is placed centrally on top of another cuboid measuring 17 cm by 9 cm by 8 cm.

Coordinate axes are taken as shown.





(*a*) The point A has coordinates (0, 9, 8) and C has coordinates (17, 0, 8).
Write down the coordinates of B. **1**

(*b*) Calculate the size of angle ABC. **6**

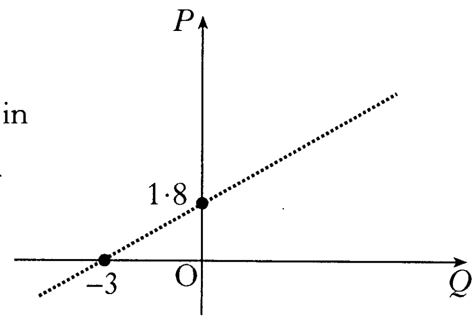
[Turn over

Marks

B10. Find $\int \frac{1}{(7-3x)^2}\,dx.$ 2

B11. The results of an experiment give rise to the graph shown.

(*a*) Write down the equation of the line in terms of P and Q. 2



It is given that $P = \log_e p$ and $Q = \log_e q$.

(*b*) Show that p and q satisfy a relationship of the form $p = aq^b$, stating the values of a and b. 4

[*END OF SECTION B*]

ONLY candidates doing the course Mathematics 1, 2 and Statistics should attempt this Section.

C7. A scientific researcher wishes to investigate the relationship between the amount of time that a rat remains conscious and the amount of anaesthetic administered. An experiment was carried out on 10 rats, similar in size and weight, using known amounts of anaesthetic. The results are shown in the table below.

Dose (ml)	x	0·30	0·35	0·40	0·45	0·50	0·55	0·60	0·65	0·70	0·75
Time remaining conscious (s)	y	10·9	11·5	9·9	9·3	7·4	6·9	7·1	4·1	4·5	2·3

A scattergraph shows that a linear model is appropriate.
The following summary statistics were calculated.

$$\sum y = 73\cdot9, \quad \sum y^2 = 630\cdot69 \quad \text{and} \quad \sum xy = 34\cdot735$$

(*a*) (i) Determine the equation of the least squares regression line of y on x. **5**

(ii) Use the regression equation to predict the time a rat will remain conscious if it is given a dose of 0·62 ml of anaesthetic. **1**

(*b*) Calculate the product moment correlation coefficient and comment on your answer. **3**

C8. A recent survey was carried out on the amount of time, measured in hours, that a typical Scottish family spend watching TV on any given day of the year. The amount of time was found to be modelled by a continuous random variable X with the following probability density function.

$$f(x) = \begin{cases} \frac{1}{50}(10 - x) & 0 \le x \le 10 \\ 0 & \text{otherwise} \end{cases}$$

(*a*) Find $P(5 < X < 10)$. **2**

(*b*) Calculate the median value m. **4**

[Turn over for Question C9 on *Page eight*

Marks

C9. A Parent-Teacher Association has 12 members consisting of 9 parents and 3 teachers. A group of 5 office bearers is to be selected. Each member of the Association has an equal chance of being chosen as an office bearer.

(*a*) How many different ways are there of choosing 5 office bearers from 12 members? **2**

(*b*) How many different ways are there of choosing 5 office bearers, 4 of whom must be parents and the other a teacher? **2**

(*c*) Hence calculate the probability that the 5 office bearers will consist of 4 parents and 1 teacher. **1**

[*END OF SECTION C*]

[*END OF QUESTION PAPER*]