

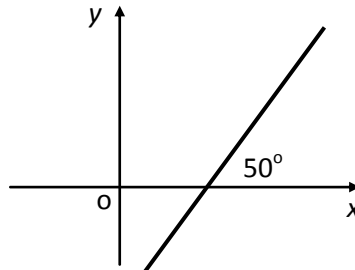
## Higher Maths Homework – Do the basics ☺

1. A line passes through the points (2,-7) and (6,1).

Find the equation of this line.

2.

A line makes an angle of  $50^\circ$  with the positive direction of the  $x$ -axis, as shown in the diagram, where the scales on the axes are equal.



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Find the gradient of the line.

- 3 (a) Write down the **gradient** of any line parallel to  $y = \frac{1}{2}x + 3$ .

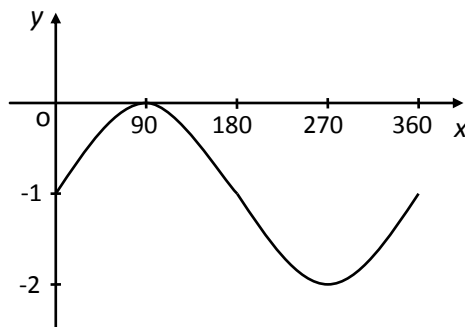
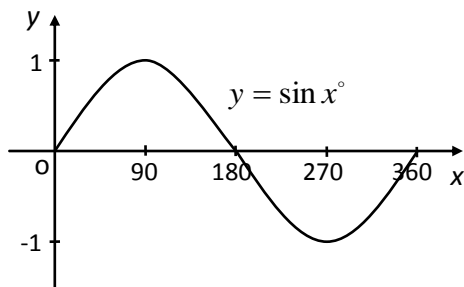
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- (b) Write down the **gradient** of a line perpendicular to  $y = -3x - 1$ .

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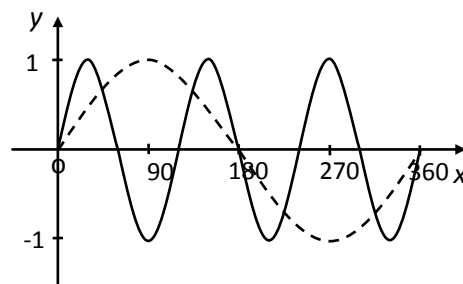
- 4 (a) The diagrams below show part of the graph of  $y = \sin x^\circ$  and the graph of a related function. Write down the equation of the related function.



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- (b) The diagram shows part of the graph of  $y = \sin x^\circ$  (drawn as a broken line) and the graph of a related function.

Write down the equation of the related graph.



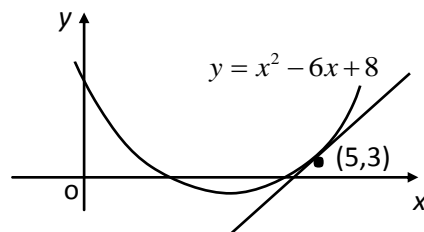
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- 5 (a) Two functions  $f$  and  $g$  are given by  $f(x) = x^2 - 1$  and  $g(x) = 3x - 1$ .  
Obtain an expression for  $f(g(x))$ . 1
- (b) Functions  $h$  and  $k$ , defined on suitable domains, are given by  $h(x) = 4x$  and  $k(x) = \cos x$ . Find  $k(h(x))$ . 1

- 6 Given  $\frac{1+x^4}{x^2}$ , find  $\frac{dy}{dx}$ . 3

- 7 The diagram shows a sketch of the curve with equation  $y = x^2 - 6x + 8$  with a tangent drawn at the point  $(5, 3)$ .

Find the gradient of this tangent.



- 8 Find the coordinates of the stationary points of the curve with equation  $y = \frac{1}{3}x^3 - x + 1$ .  
Using differentiation determine their nature. 4
- 9 Show that  $(x+2)$  is a factor of  $g(x) = x^3 + 4x^2 + x - 6$ , and express  $g(x)$  in fully factorised form. 8

- 10 Use the discriminant to determine the nature of the roots of the equation

$$3x^2 + 5x + 1 = 0.$$

- 11 Solve algebraically the equation  $2 \sin 2x = \sqrt{3}$  for  $0 \leq x < \pi$ . 4

- 12 (a) Simplify  $\log_p 6 + \log_p 3$ .

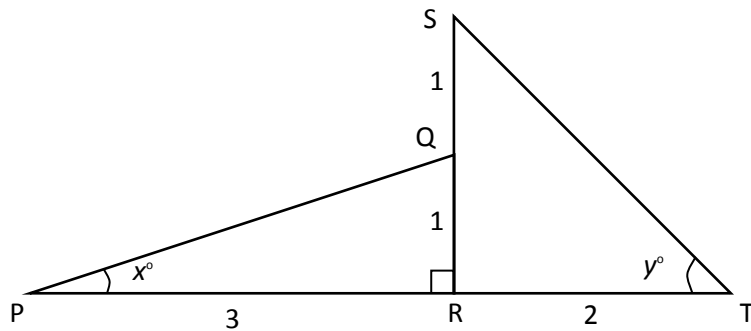
(b) Simplify  $2 \log_2 6 - \log_2 9$

- 13 Express  $5 \sin x^\circ + 2 \cos x^\circ$  in the form  $k \cos(x - a)^\circ$  where  $k > 0$  and  $0 \leq a < 360$ . 1

- 14 The diagram below shows two right-angled triangles PQR and SRT. 4

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$$SQ = QR = 1$$

$$PR = 3$$

2

(a) Write down the values of  $\cos x^\circ$  and  $\sin y^\circ$ .

(b) By expanding  $\sin(x + y)^\circ$  show that the **exact** value of  $\sin(x + y)^\circ$  is  $\frac{8}{\sqrt{80}}$ .

15 (a) Express  $\cos x^\circ \cos 30^\circ - \sin x^\circ \sin 30^\circ$  in the form  $\cos(A + B)^\circ$ .

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(b) Hence solve the equation  $\cos x^\circ \cos 30^\circ - \sin x^\circ \sin 30^\circ = \frac{1}{4}$  for  $0 < x < 360$ .

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16 (a) E, F and G have coordinates (1, 4, -2), (-1, 8, -1) and (-5, 16, 1) respectively.

→

(i) Write down the components of  $\vec{EF}$ .

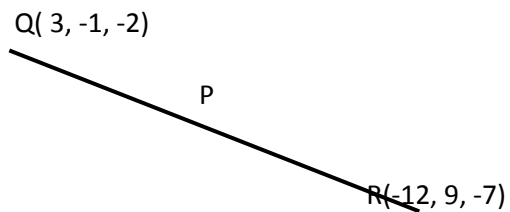
(ii) Hence show that the points E, F and G are collinear.

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(b) The point P divides QR in the ratio 2:3 as shown in the diagram.

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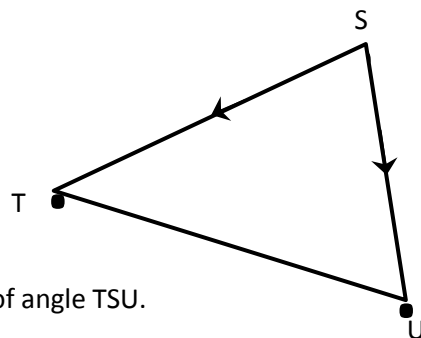
Find the coordinates of P.



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17 The diagram shows triangle STU where

$$\vec{ST} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} \text{ and } \vec{SU} = \begin{pmatrix} -2 \\ 2 \\ 0 \end{pmatrix}$$



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(a) Find the value of  $\vec{ST} \cdot \vec{SU}$ .

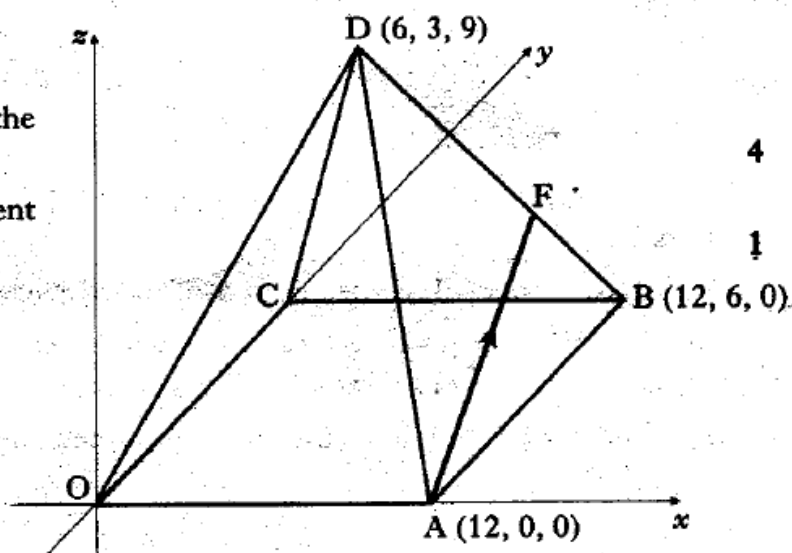
(b) Use the result of (a) to find the size of angle TSU.

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- 18 D,OABC is a pyramid. A is the point (12, 0, 0), B is (12, 6, 0) and D is (6, 3, 9).

F divides DB in the ratio 2:1.

- (a) Find the coordinates of the point F.  
 (b) Express  $\vec{AF}$  in component form.

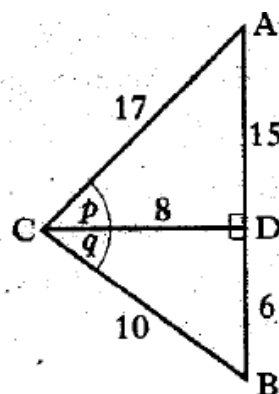


- 19 Triangles ACD and BCD are right-angled at D with angles  $p$  and  $q$  and lengths as shown in the diagram.

(a) Show that the exact value of  $\sin(p + q)$  is  $\frac{84}{85}$ .

(b) Calculate the exact values of:

- (i)  $\cos(p + q)$ ;  
 (ii)  $\tan(p + q)$ .



- 20 Solve the equation  $\log_2(x + 1) - 2\log_2(3) = 3$ .

- 21 Find the value of  $k$  such that the equation  $kx^2 + kx + 6 = 0$ ,  $k \neq 0$ , has equal roots.

- 22 Express  $2x^2 + 12x + 1$  in the form  $a(x + b)^2 + c$ .

- 23 (a) Given that  $(x - 1)$  is a factor of  $x^3 + 3x^2 + x - 5$ , factorise this cubic fully.

(b) Show that the curve with equation

$$y = x^4 + 4x^3 + 2x^2 - 20x + 3$$

has only one stationary point.

Find the  $x$ -coordinate and determine the nature of this point.

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- 24 (i) Show that  $(x - 4)$  is a factor of  $x^3 - 5x^2 + 2x + 8$ .  
(ii) Factorise  $x^3 - 5x^2 + 2x + 8$  fully.  
(iii) Solve  $x^3 - 5x^2 + 2x + 8 = 0$ . 6
- 25 The expression  $\cos x - \sqrt{3} \sin x$  can be written in the form  $k \cos(x + a)$  where  $k > 0$  and  $0 \leq a < 2\pi$ .  
Calculate the values of  $k$  and  $a$ . 4
- 26 Functions  $f$  and  $g$  are defined on the set of real numbers by
- $f(x) = x^2 + 3$
  - $g(x) = x + 4$ .
- (a) Find expressions for:
- (i)  $f(g(x))$ ;
  - (ii)  $g(f(x))$ . 3
- (b) Show that  $f(g(x)) + g(f(x)) = 0$  has no real roots. 3