HIGHER MATHS REVISION - DO THE BASICS

TRY TO MAKE SURE YOU HAVE WRITTEN YOUR ANSWERS AS CLEARLY AND FULLY AS POSSIBLE :)

1. $m = \frac{1--7}{6-2} = \frac{8}{4} = 2$

$y - b = m(x-a)$

$y - 1 = 2(x-6)$

$y - 1 = 2x - 12$

$y = 2x - 11$

2. $m = \tan 50$

$= 1.191...$

$= 1.2$

3. a) $m = \frac{1}{2}$

b) $m_{line} = -3$

$m_{\perp} = \frac{1}{3}$

4a) $y = \sin x - 1$

b) $y = \sin 3x$

5 a) $f(g(x)) = f(3x-1) = (3x-1)^2 - 1$ $\left[= 9x^2 - 6x \right]$

b) $k(h(x)) = k(4x) = \cos 4x$

6. $\frac{1+x^4}{x^2} = \frac{1}{x^2} + \frac{x^4}{x^2} = x^{-2} + x^2$

$\frac{dy}{dx} = -2x^{-3} + 2x = \frac{-2}{x^3} + 2x$

7. $y = x^2 - 6x + 8$

$\frac{dy}{dx} = 2x - 6 \implies m = 2x - 6$

sub in $x = 5$

$m = 10 - 6$

$m = 4$

8. $y = \frac{1}{3}x^3 - x + 1$ ← here for y-values

$\frac{dy}{dx} = x^2 - 1$

For SPs $\frac{dy}{dx} = 0$ ← MUST WRITE

$x^2 - 1 = 0$

$(x+1)(x-1) = 0$ ← here for nature table

$x = -1 \quad x = 1$

$y = \frac{-5}{3} \quad y = \frac{1}{3}$

working for y-values...

$y = \frac{1}{3}(-1)^3 - (-1) + 1$

$= -\frac{1}{3} + 1 + 1$

$= -\frac{5}{3}$

$y = \frac{1}{3}(1)^3 - 1 + 1$

$= \frac{1}{3}$

SPs are: $\left(-1, -\frac{5}{3}\right)$ and $\left(1, \frac{1}{3}\right)$

Nature:

x	$\xrightarrow{-2}$	-1	$\xrightarrow{0}$	1	$\xrightarrow{2}$
$\frac{dy}{dx}$	$+$	0	$-$	0	$+$
slope	/	—	\	—	/

So SPs: $\left(-1, -\frac{5}{3}\right)$ is a max TP.

$\left(1, \frac{1}{3}\right)$ is a min TP.

MUST WRITE

9. $(x+2)$ factor $\Rightarrow$ $x=-2$ root

$$\begin{array}{r|rrrr} -2 & 1 & 4 & 1 & -6 \\ & & -2 & -4 & 6 \\ \hline & 1 & 2 & -3 & \boxed{0} \end{array}$$

$(x+2)$ is a factor because rem $= 0$

MUST WRITE!

10. $3x^2+5x+1=0$

b^2-4ac

$=5^2-4\times3\times1$

$=25-12$

$=13$

Because $b^2-4ac>0$ there are two real distinct roots

11. $2\sin 2x=\sqrt{3}$ $\quad 0\le x<\pi$

$\sin 2x = \frac{\sqrt{3}}{2}$

$2x=\sin^{-1}\frac{\sqrt{3}}{2}$ (can do non calc)

$2x=\frac{\pi}{3}$ and $\frac{2\pi}{3}$ (sin quadrant: $\pi -$)

$x=\frac{\pi}{6}$ and $\frac{2\pi}{6}$

NOTE:

(Period of sin wave usually 2π so $\sin 2x \Rightarrow$ period is π
Add π on to both answers to get answers on second wave BUT
question states $0\le x<\pi$ so don't have to do that this time)

12a) $\log_p 6+\log_p 3 = \log_p 18$

b) $2\log_2 6-\log_2 9 = \log_2 6^2-\log_2 9 = \log_2 36-\log_2 9 = \log_2\frac{36}{9}=\log_2 4 = 2$

because $2^2=4$

13. $k\cos(x-\alpha) = k\cos x\cos\alpha + k\sin x\sin\alpha$

$\qquad\qquad\qquad\qquad 2\cos x \quad + \quad 5\sin x$

so $k\sin\alpha = 5$

$k\cos\alpha = 2$

$k^2 = 5^2 + 2^2 = 29$

$k = \sqrt{29}$

MUST SHOW THIS!

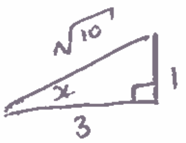
$\tan\alpha = \dfrac{\cancel{k}\sin\alpha}{\cancel{k}\cos\alpha} = \dfrac{5}{2}$

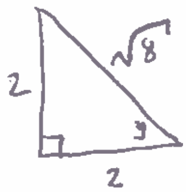
$\alpha = \tan^{-1}\dfrac{5}{2}$

$= 68.2°$

$\Rightarrow \quad 2\cos x + 5\sin x = \sqrt{29}\cos(x-68.2)°$.

14. a)





so $\cos x = \dfrac{A}{H} = \dfrac{3}{\sqrt{10}}$

$\sin x = \dfrac{O}{H} = \dfrac{1}{\sqrt{10}}$

use triangle with x in it

$\cos y = \dfrac{A}{H} = \dfrac{2}{\sqrt{8}}$

$\sin y = \dfrac{O}{H} = \dfrac{2}{\sqrt{8}}$

use triangle with y in it.

b) $\sin(x+y) = \sin x\cos y + \cos x\sin y$

$= \dfrac{1}{\sqrt{10}} \times \dfrac{2}{\sqrt{8}} + \dfrac{3}{\sqrt{10}} \times \dfrac{2}{\sqrt{8}}$

$= \dfrac{2}{\sqrt{80}} + \dfrac{6}{\sqrt{80}}$

$= \dfrac{8}{\sqrt{80}}$ as required.

15. a) $\cos x \cos 30 - \sin x \sin 30$

$\cos A \cos B - \sin A \sin B = \cos(A+B)$

$\Rightarrow \cos(x+30)$

b) $\cos x \cos 30 - \sin x \sin 30 = \frac{1}{4}$

$\cos(x+30) = \frac{1}{4}$

$(x+30) = \cos^{-1} \frac{1}{4}$

$(x+30) = 75.5$ and 284.5 (cos quadrant $360-$)

$x = 75.5 - 30$ and $284.5 - 30$

$x = 45.5$ and 254.5

16a)(i) $\vec{EF} = \underline{f} - \underline{e} = \begin{pmatrix} -1 \\ 8 \\ -1 \end{pmatrix} - \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \\ 1 \end{pmatrix}$

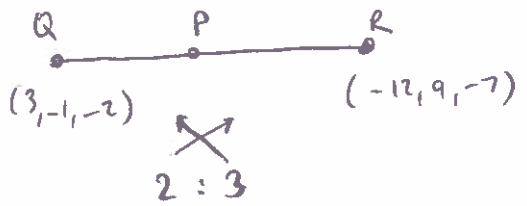
(ii) $\vec{FG} = \underline{g} - \underline{f} = \begin{pmatrix} -5 \\ 16 \\ 1 \end{pmatrix} - \begin{pmatrix} -1 \\ 8 \\ -1 \end{pmatrix} = \begin{pmatrix} -4 \\ 8 \\ 2 \end{pmatrix} = 2\begin{pmatrix} -2 \\ 4 \\ 1 \end{pmatrix}$

Because $\vec{EF}$ and $\vec{FG}$ are multiples of $\begin{pmatrix} -2 \\ 4 \\ 1 \end{pmatrix}$ this means they are parallel.

They also have common point F $\Rightarrow$ the points E, F and G are collinear

THIS ANSWER MUST INCLUDE ALL OF THIS !!!

b)



$x: \frac{3\times3 + 2\times-12}{5} = \frac{9-24}{5} = \frac{-5}{5} = -3$

$y: \frac{3\times-1 + 2\times9}{5} = \frac{-3+18}{5} = \frac{15}{5} = 3$

$z: \frac{3\times-2 + 2\times-7}{5} = \frac{-6-14}{5} = \frac{-20}{5} = -4$

$P = (-3, 3, -4)$

17a) $\vec{ST} \cdot \vec{TU} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 2 \\ 0 \end{pmatrix} = 2 \times -2 + 3 \times 2 + -1 \times 0$

$= -4 + 6 + 0$

$= 2$

b) $\cos TSU = \dfrac{ST \cdot TU}{|ST||TU|}$

Use correct notation !!

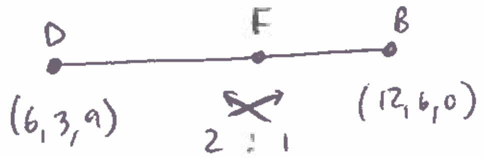
$|ST| = \sqrt{2^2 + 3^2 + (-1)^2} = \sqrt{4 + 9 + 1} = \sqrt{14}$

$|TU| = \sqrt{(-2)^2 + 2^2 + 0^2} = \sqrt{4 + 4} = \sqrt{8}$

$= \dfrac{2}{\sqrt{14} \times \sqrt{8}}$

$= 0.1889\ldots$

$TSU = 79.11°$

18 a) 

$x: \dfrac{1 \times 6 + 2 \times 12}{3} = \dfrac{6 + 24}{3} = \dfrac{30}{3} = 10$

$y: \dfrac{2 \times 6 + 1 \times 3}{3} = \dfrac{12 + 3}{3} = \dfrac{15}{3} = 5$ $\Rightarrow F = (10, 5, 3)$

$z: \dfrac{2 \times 0 + 1 \times 9}{3} = \dfrac{0 + 9}{3} = \dfrac{9}{3} = 3$

b) $\vec{AF} = \underline{f} - \underline{a} = \begin{pmatrix} 10 \\ 5 \\ 3 \end{pmatrix} - \begin{pmatrix} 12 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 5 \\ 3 \end{pmatrix}$

19 a) $\sin(p+q) = \sin p\cos q + \cos p\sin q$

$\sin p = \frac{O}{H} = \frac{15}{17}$ $\quad$ $\cos p = \frac{A}{H} = \frac{8}{17}$

$\sin q = \frac{O}{H} = \frac{6}{10}$ $\quad$ $\cos q = \frac{A}{H} = \frac{8}{10}$

MAKE SURE YOU USE TRIANGLES CAREFULLY.

$\Rightarrow \sin(p+q) = \frac{15}{17} \times \frac{8}{10} + \frac{8}{17} \times \frac{6}{10}$

$= \frac{120}{170} + \frac{48}{170}$

$= \frac{168}{170}$

$= \frac{84}{85}$ as required.

b) (i) $\cos(p+q) = \cos p\cos q - \sin p\sin q$

$= \frac{8}{17} \times \frac{8}{10} - \frac{15}{17} \times \frac{6}{10}$

$= \frac{64}{170} - \frac{90}{170}$

$= \frac{-26}{170}$

$= \frac{-13}{85}$

(ii) $\tan(p+q) = \frac{\sin(p+q)}{\cos(p+q)}$ because $\tan x = \frac{\sin x}{\cos x}$

$= \frac{84}{85} \div \frac{-13}{85}$

$= \frac{84}{85} \times \frac{85}{-13}$

$= \frac{7140}{-1105} = \frac{1428}{-221}$ (don't need to simplify)

20. $\log_2(x+1) - 2\log_2 3 = 3$

$\log_2(x+1) - \log_2 3^2 = 3\log_2 2$ (Because $\log_2 2 = 1$)

$\cancel{\log_2} \dfrac{x+1}{9} = \cancel{\log_2} 2^3$

$\dfrac{x+1}{9} = 8$

$x+1 = 72$

$x = 71$

21. $kx^2 + kx + 6 = 0$

Equal roots $\Rightarrow$ $b^2 - 4ac = 0$ MUST STATE THIS

$k^2 - 4 \times k \times 6 = 0$

$k^2 - 24k = 0$

$k(k-24) = 0$

$k = 0$ $\quad$ $k = 24$

$\uparrow$

can't be 0 $\quad$ so $\underline{\underline{k = 24}}$ (state answer)

22. $2x^2 + 12x + 1$ (complete the square)

$2 \mid x^2 + 6x + \frac{1}{2}$

$(x+3)^2 + \frac{1}{2} - 9$

$(x+3)^2 - \frac{17}{2}$

$= 2(x+3)^2 - 17$

23 a) $(x-1)$ factor $\Rightarrow$ $x=1$ root.

$$\begin{array}{c|cccc} 1 & 1 & 3 & 1 & -5 \\ & & 1 & 4 & 5 \\ \hline & 1 & 4 & 5 & \boxed{0} \end{array}$$

So $x^3+3x^2+x-5 = (x-1)(x^2+4x+5)$ doesn't factorise further

b) This isn't basic...

$y = x^4 + 4x^3 + 2x^2 - 20x + 3$

"Stationary Point" in question tells you to differentiate...

$$\frac{dy}{dx} = 4x^3+12x^2+4x-20$$

For SPs $\frac{dy}{dx} = 0$

$4x^3+12x^2+4x-20=0$

$4(x^3+3x^2+x-5)=0$ NOTICE THIS IS SAME AS PART (a)!!!

$4(x-1)(x^2+4x+5) = 0$

$x=1$ NO SOLUTIONS (from part (a))

So SP is at $x=1$ (question doesn't ask for y so don't bother).

Nature:

x	$\xrightarrow{0}$	1	$\xrightarrow{2}$
$\frac{dy}{dx}$	$-$	0	$+$
slope	\	__	/

$\Rightarrow$ $x=1$ is a min TP.

24. (i) $(x-4)$ factor $\Rightarrow$ $x=4$ root.

4	1	−5	2	8
		4	−4	−8
	1	−1	−2	$\boxed{0}$

$x-4$ is a factor because remainder $=0$

MUST WRITE!

(ii) x^3-5x^2+2x+8

$=(x-4)(x^2-x-2)$

$=(x-4)(x-2)(x+1)$

(iii) $(x-4)(x-2)(x+1)=0$ SHOW YOU KNOW MUST $=0$ TO SOLVE.

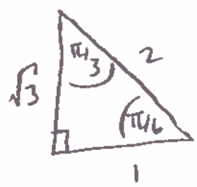
$x=4$ $x=2$ $x=-1$

25. $k\cos(x+\alpha) = k\underline{\cos x}\cos\alpha - k\underline{\sin x}\sin\alpha$ (MUST HAVE k's)

$\underline{\cos x} - \sqrt{3}\,\underline{\sin x}$

$\left.\begin{array}{l} k\sin\alpha = \sqrt{3} \\ k\cos\alpha = 1 \end{array}\right\}$ signs match so don't have to worry about negatives.

$\tan\alpha = \dfrac{k\sin\alpha}{k\cos\alpha} = \dfrac{\sqrt{3}}{1} \Rightarrow \dfrac{\pi}{6} = \alpha$ (do without calc)



$k^2 = \sqrt{3}^2 + 1^2$

$k = \sqrt{4} = 2$

$\alpha = \dfrac{\pi}{6}$

$k=2$

26. a) (i) $f(g(x)) = f(x+4) = (x+4)^2+3$ $[= x^2+8x+19]$

(ii) $g(f(x)) = g(x^2+3) = x^2+3+4$ $[= x^2+7]$

b) $f(g(x)) + g(f(x)) = x^2+8x+19+x^2+7 = \underline{2x^2+8x+26}$

want to show $b^2-4ac<0$ for no real roots but don't know this yet.

Start with: b^2-4ac

$= 8^2 - 4\times2\times26$

$= 64 - 208$

Because $b^2-4ac<0$ there are no real roots.

MUST WRITE.