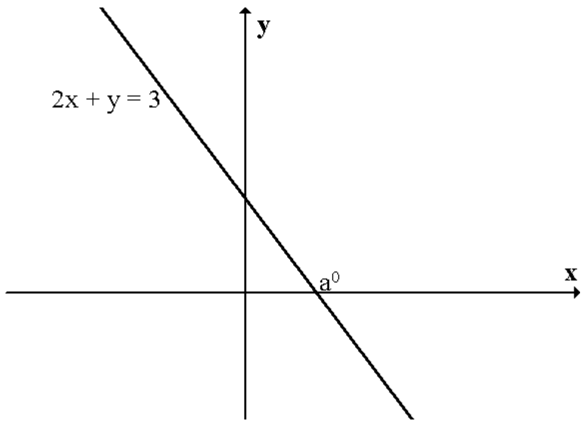
Higher Mathematics – Revision

Equation of a Line:

1. Find the equation of the line parallel to the line $3x + 2y – 10 = 0$ which passes through the point (-1,4).

2. Find the equation of the line through the point (2,-5) perpendicular to the line AB where A is (4,1) and B is (6,-3).

3. Find, a^0, the angle the line $2x + y = 3$ makes with the positive direction of the x-axis.



4. Triangle ABC has vertices A(2,-5), B(8,1) and C(7,2). Find the equation of the median from C.

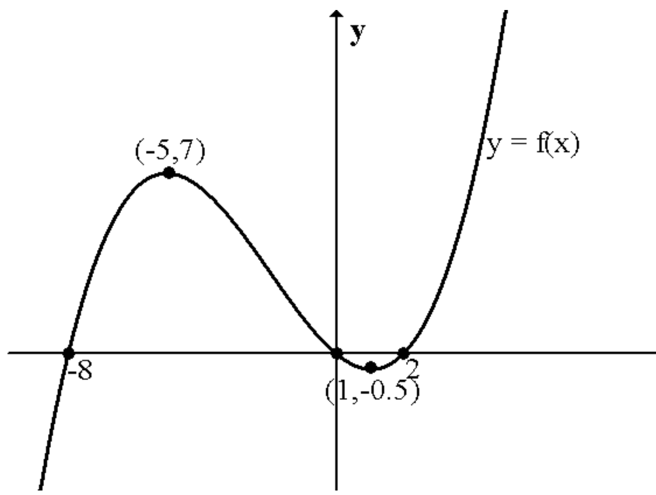
5. A is the point (2,-1), B is (10,-5) and C is (6,2).

   (a) Find the equation of the perpendicular bisector of AB.
   (b) Find the equation of the altitude from B to AC.
   (c) Find the point of intersection of these lines.

Graphical Functions:

6. The diagram opposite shows the graph of $y = f(x)$.

   (a) Sketch the graph of $y = -f(x) + 3$
   (b) Sketch the graph of $y = -3(x – 2)$



Composition of Functions:

7. The functions f(x) and g(x) are defined on suitable domains with

$$f(x) = \frac{3x - 4}{x} \quad \text{and} \quad g(x) = \frac{4}{3 - x}$$

   (a) Find a formula for g(f(x)).
   (b) State the connection between f(x) and g(x).

8. $f(x) = x^2 - x - 12$ and $g(x) = 3x + 1$

(a) Find a formula for $f(g(x))$.
(b) Solve $f(g(x)) = 0$.
(c) State a suitable domain for the function $h(x)$ where $h(x) = \dfrac{1}{f(g(x))}$

Recurrence Relations:

9. $u_{n+1} = 0.6u_n + 20 \qquad u_o = 40$

(a) Find n such that $u_n > 49$
(b) Explain why u_{n+1} has a limit and find the exact value of this limit.

10. A recurrence relation is defined as $u_n = au_{n-1} + b$.
The first three terms of this relation are 160, 200 and 230.
Find the values of a and b.

11. A recurrence relation is $u_{n+1} = 0.5u_n + 10$.
Given $u_3 = 30$, find the value of u_1.

12. Two sequences are defined by the recurrence relations

$$u_{n+1} = 0.4u_n + p \qquad v_{n+1} = 0.6v_n + q$$

If both sequences have the same limit, express p in terms of q.

13. A patient is injected with 60 ml of an antibiotic drug. Every 4 hours 30% of the drug passes out of her bloodstream. To compensate for this an extra 20ml of antibiotic is given every 4 hours.

(a) Find a recurrence relation for the amount of drug in the patient's bloodstream.
(b) Calculate the amount of antibiotic remaining in the bloodstream after one day.

Differentiation:

14. $f(x) = \dfrac{x^2 - 1}{\sqrt{x}}$. Find $f'(4)$.

15. $s = 3u(u^2 + 1)$. Find the rate of change of s when $u = \frac{4}{3}$

16. Find the equation of the tangent to the curve $y = \dfrac{x^2(x^2 - 2)}{x}$ at the point where $x = 2$.

17. A tangent to the curve $y = x^4 - 2x$ has gradient -6. Find the equation of this tangent.

18. Show that the curve $y = x^3 - 6x^2 + 12x + 3$ is never decreasing.

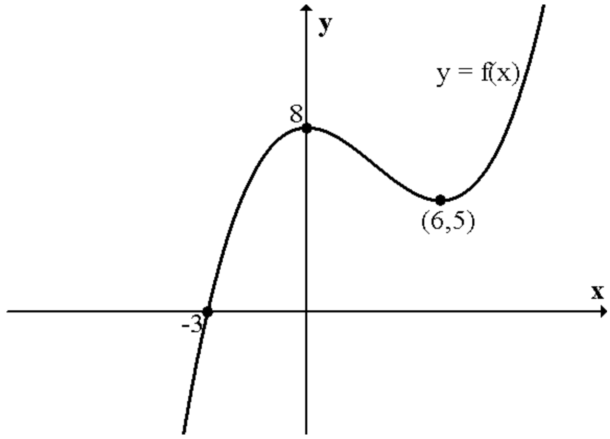
19. Find the values of x for which the curve $f(x) = 2x^3 - 6x^2 - 48x + 5$ is strictly increasing.

20. $f(x)= x^4 - 4x^3 + 5$.
Find the stationary points of f(x) and determine their nature.

21. Find the maximum and minimum values of $f(x) = 2x^3 - 3x^2 - 12x$ in the range $-3 \le x \le 3$.

22. Shown opposite is the graph of y = f(x).

Sketch the graph of $y = f'(x)$.



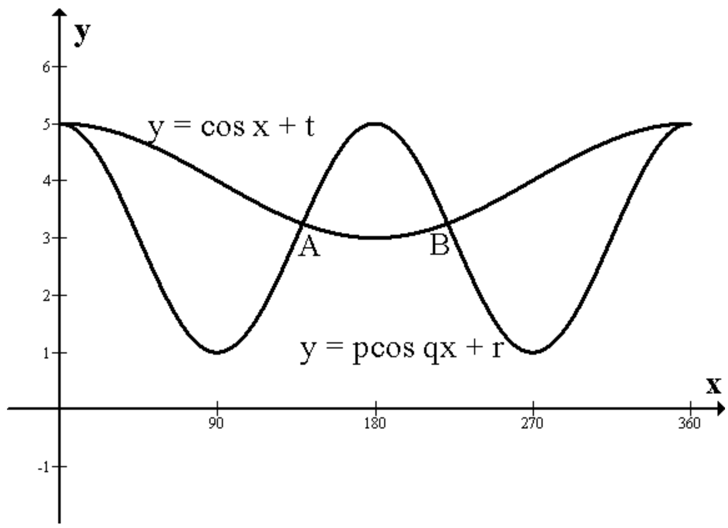
Trigonometry:

23. Solve the equations

(a) $3\tan^2 x - 1 = 0 \qquad 0 \le x \le 2\pi$
(b) $4\cos(2x - 30) + 4 = 2 \qquad 0 \le x \le 360$
(c) $3\sin 2x = 2\cos x \qquad 0 \le x \le 360$

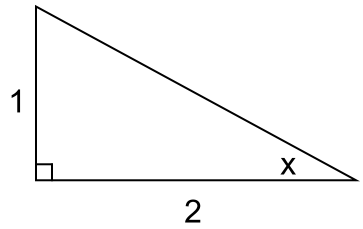
24. The diagram opposite shows the graphs of y = p cos qx + r and y = cos x + t.

(a) Write down the values of p, q, r and t.
(b) Find the coordinates of A and B.



25. (a) Express 4sin x + 3cos x in the form ksin(x + a) where k > 0 and $0 \le a \le 360$
(b) Solve the equation 4sin x + 3cos x = 3 $\quad 0 \le x \le 360$
(c) Find the minimum value of 4sin x + 3cos x and the value of x for which it occurs in the range $0 \le x \le 360$
(d) Sketch the graph of y = 4sin x + 3cos x for $0 \le x \le 360$

26. $\tan x = \frac{1}{2}$. Find the exact value of



(a) $\sin 2x$
(b) $\cos 2x$
(c) $\tan 2x$

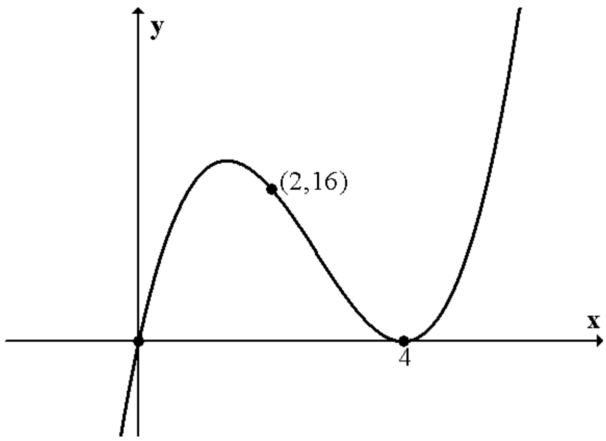
27. $\cos x = \frac{3}{5}$ and $\sin y = \frac{5}{13}$. Find the exact value of $\cos(x + y)$.

Polynomials:

28. $f(x) = 2x^3 - 3x^2 - 2x + 3$. Show that $(x - 1)$ is a factor of $f(x)$.
Find the other factors of $f(x)$.

29. A function is defined as $f(x) = x^3 + 2x^2 - 5x - 6$. Given -1 is a root of $f(x)$, find the other roots.

30. The function shown in the graph opposite crosses the x-axis at 0 and 4 and the point (2,16) lies on the graph.
Find the equation of this function.



31. -3 is a root of $2x^3 - 3x^2 + px + 30 = 0$.
Find p and hence find the other roots of $2x^3 - 3x^2 + px + 30 = 0$.

32. $(x - 2)$ and $(x + 4)$ are both factors of $x^3 - 2x^2 - px + q$.
Find the values of p and q.

Quadratics/Discriminant:

33. (a) Express $x^2 - 8x + 1$ in the form $(x + a)^2 + b$.
(b) Sketch the graph of $y = x^2 - 8x + 1$, showing clearly its turning point.

34. (a) Express $f(x) = 3x^2 + 12x - 2$ in the form $f(x) = a(x + b)^2 + c$.
(b) Hence, or otherwise, write down the turning point of $f(x)$ stating whether this turning point is a maximum or minimum.

35. State the nature of the roots of

(a) $3x^2 - 2x - 5 = 0$ (b) $x^2 + 3x + 7 = 0$

36. The roots of the equation $(x + 1)(x + k) = -4$ are equal. Find k.

37. The roots of the equation $x^2 + kx - 3k = 4k - 7$ are real. Find k.

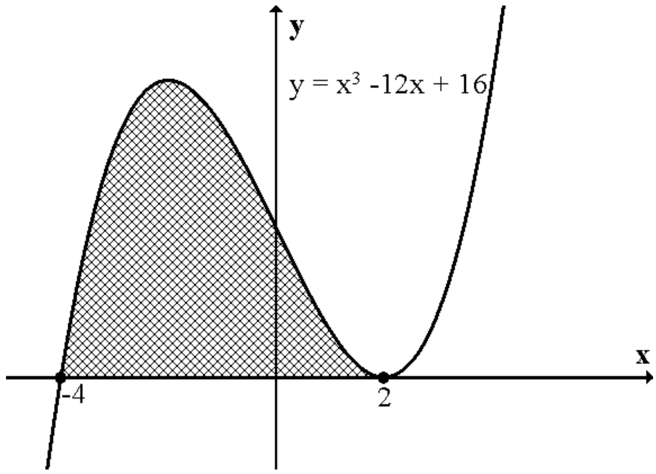
38. Show that $y = 2x^3 + x^2 + 9x + 1$ has no stationary points.

Integration:

39. Find (a) $\int \frac{x^3 - 1}{x^2}\, dx$ (b) $\int_1^4 \sqrt{x}\left(\sqrt{x} - x\right)\, dx$

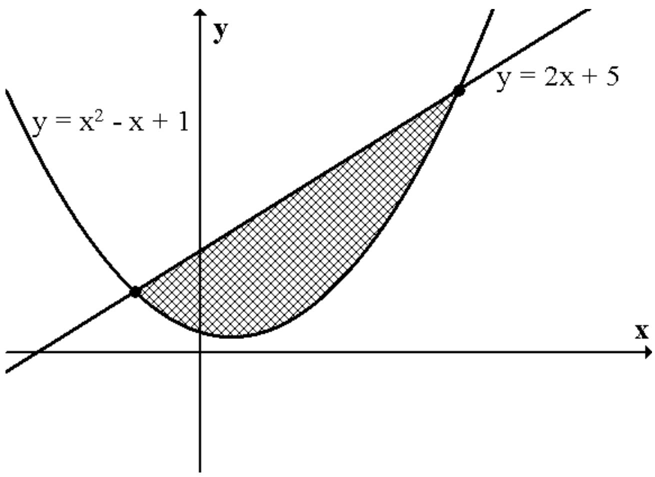
40. $\frac{dy}{dx} = 3x^2 - 4x + 1$. Find a formula for y given y = 2 when x = -1.

41. Calculate the shaded area in the diagram shown opposite.



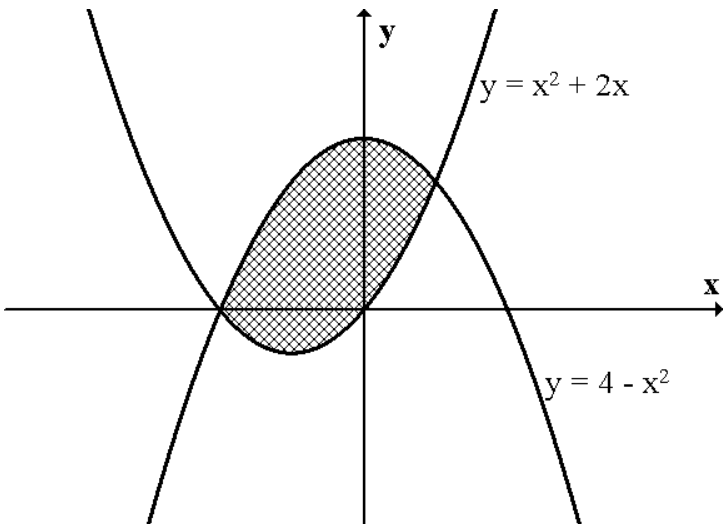
42. The diagram shows the line $y = 2x + 5$ and the curve $y = x^2 - x + 1$.

Calculate the shaded area.



43. The diagram shows the parabolas $y = x^2 + 2x$ and $y = 4 - x^2$.

Calculate the area enclosed by these two parabolas.



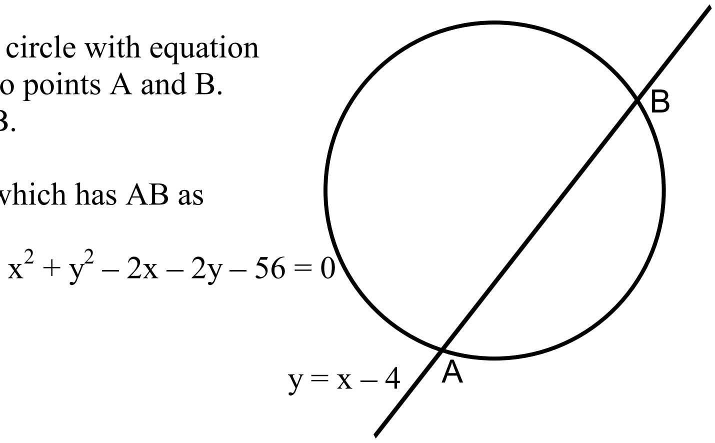
Circles:

44. A circle has equation $x^2 + y^2 - 6x + 2y - 35 = 0$.
Find the equation of the tangent to this circle at the point (-3,2).

45. Find the equation of the circle which has PQ as diameter where P is (-2,2) and Q is (6,10).

46. (a) The line $y = x - 4$ intersects the circle with equation $x^2 + y^2 - 2x - 2y - 56 = 0$ at two points A and B. Find the coordinates of A and B.

(b) Find the equation of the circle which has AB as diameter.



47. Prove that the line $y = 2x + 6$ is a tangent to the circle with equation $x^2 + y^2 - 8x + 2y - 28 = 0$ and find the point of contact.

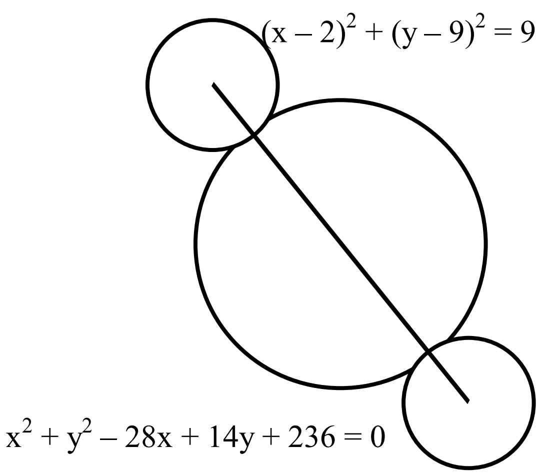
48. The line $y = x - 2$ intersects the circle $x^2 + y^2 - 4x + 2y - 20 = 0$ at the points S and T. Find the coordinates of S and T.

49. Three circles touch externally as shown. The centres of the circles are collinear and the equations of the two smaller circles are

$$(x - 2)^2 + (y - 9)^2 = 9 \quad \text{and}$$

$$x^2 + y^2 - 28x + 14y + 236 = 0$$

Find the equation of the larger circle.



Vectors:

50. $\mathbf{u} = 2\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ and $\mathbf{v} = 3\mathbf{i} + \mathbf{k}$.

(a) Find the vector $3\mathbf{u} + \mathbf{v}$
(b) Find the magnitude of vector $2\mathbf{v} - \mathbf{u}$.
(c) Find a unit vector parallel to the vector $2\mathbf{v} - \mathbf{u}$.

51. A is the point (-1,2,0), B is (3,0,6) and C is (9,-3,15).
Show that A, B and C are collinear stating the ratio of AB:BC.

52. The points P(0,5,9), Q(2,3,4) and R(6,u,v) are collinear.
Find the values of u and v.

53. The points P, Q, R and S are (3,1,-2), (-2,-4,8) (0,-2,4) and (4,2,-4) respectively.

(a) T divides PQ in the ratio 2:3. Find the coordinates of T.
(b) Show that R, T and S are collinear.

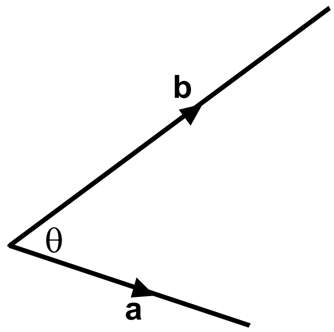
54. $\mathbf{u} = 3\mathbf{i} - 2\mathbf{j} - 4\mathbf{k}$ and $\mathbf{v} = 4\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$.

Show that the vectors **u** and **v** are perpendicular.

55. A triangle ABC has vertices A(2,1,-6), B(4,0,-1) and C(-5,2,3).

Show that triangle ABC is right-angled at B.

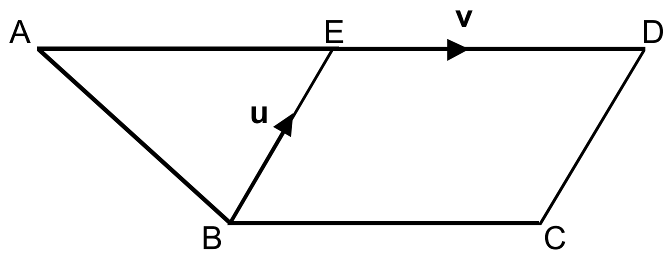
56. Calculate the angle between the vectors $\mathbf{a} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$ and $\mathbf{b} = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$



57. P is the point (4,0,-3), Q is (6,1,-1) and R is (14,0,-5).

Calculate the size of angle PQR.

58. In the diagram opposite, BCDE is a parallelogram and AE = ED.
$\overrightarrow{BE} = \mathbf{u}$ and $\overrightarrow{ED} = \mathbf{v}$



Find, in terms of **u** and **v**

(i) $\overrightarrow{BD}$ (ii) $\overrightarrow{AC}$

(iii) $\overrightarrow{AF}$ where F divides AC in the ratio 2:1

(iv) $\overrightarrow{FD}$

Further Calculus:

59. Differentiate

(a) $f(x) = (x^2 - 5)^4$ (b) $y = \dfrac{2}{\sqrt{8x - 3}}$ (c) $f(x) = 2\sin 4x - 2\cos^3 x$

60. $f(x) = 2\sin^2 x$. Find the value of $f'(\frac{\pi}{4})$

61. A curve has equation $y = \sqrt[3]{3x-1}$. Find the equation of the tangent to this curve at the point where x = 3.

62. Find the equation of the tangent to the curve $y = 4\sin\left(2x - \frac{\pi}{6}\right)$ at the point where $x = \frac{\pi}{2}$.

63. Integrate

(a) $\int (4x-6)^3 \, dx$ (b) $\int 10\sqrt{1-6x} \, dx$ (c) $\int 6\cos(2x-3) \, dx$

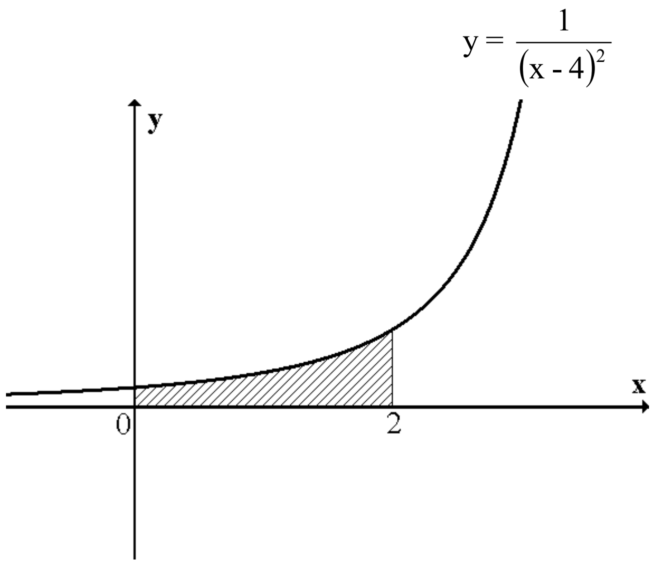
64. Evaluate $\int_1^3 \frac{8}{(2x-4)^2} \, dx$

65. $\frac{dy}{dx} = 2\cos 4x$. This curve passes through the point $\left(\frac{5}{12}\pi, \sqrt{3}\right)$. Find a formula for y.

66. The diagram shows part of the graph of

$$y = \frac{1}{(x-4)^2}$$

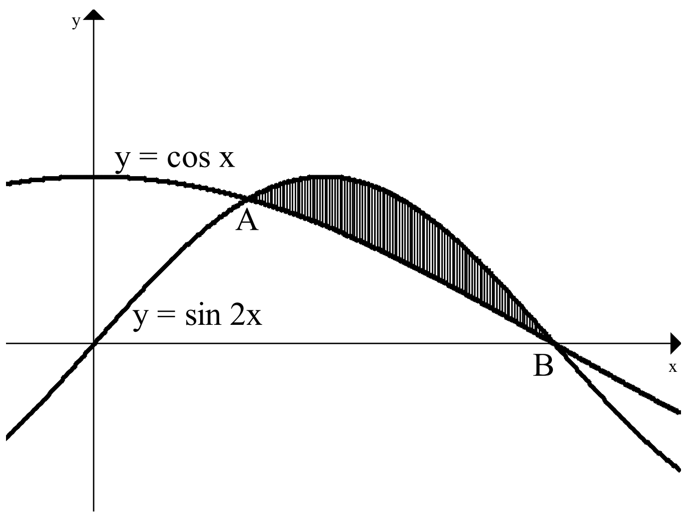
Calculate the shaded area.



67. The diagram shows part of the graphs of y = sin 2x and y = cos x.

(a) Find the x-coordinates of A and B.

(b) Calculate the shaded area.



Logarithms:

68. Simplify

(a) $\log_2 6 + \log_2 12 - \log_2 9$ (b) $\frac{3}{4}\log_{10} 16 - \frac{1}{2}\log_{10} 4 + 2\log_{10} 5$

69. Solve for $x > 0$

(a) $\log_4 x + \log_4 (3x - 2) = 2$ (b) $\log_3 (x^2 + x - 2) - \log_3 (x^2 - 4) = 1$

70. A curve has equation $y = \log_2 (x + 4) - 3$.
Find where this curve cuts the x and y axes.

71. The mass, M grams, of a radioactive isotope after a time of t years, is given by the formula $M = M_o e^{-kt}$ where M_o is the initial mass of the isotope.

In 4 years a mass of 20 grams of the isotope is reduced to 15 grams.

(a) Calculate k.

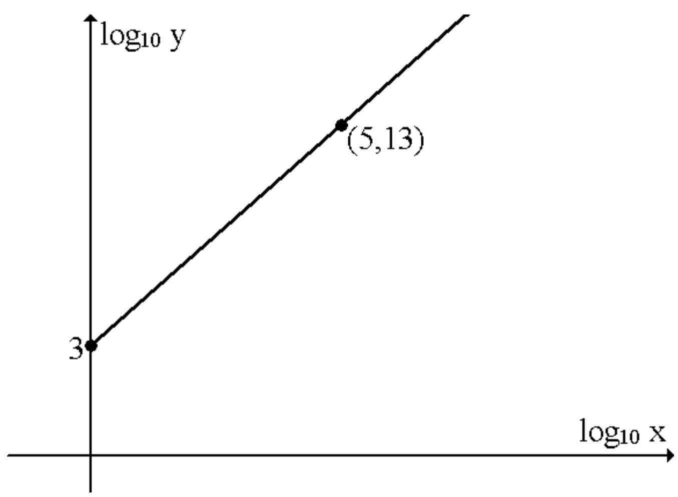
(b) Calculate the half-life of the substance i.e. the time taken for half the substance to decay.

72. Dangerous blue algae are spreading over the surface of a lake according to the formula $A_t = A_o e^{kt}$ where A_o is the initial area covered by the algae and A_t is the area covered after t days.
When first noticed the algae covered an area of 100 square metres. Two weeks later the algae covered an area of 120 square metres.

(a) Calculate the value of k.
(b) The area of algae on the lake was measured on the 1st of June and again on the 1st of July.
Calculate the percentage increase in area covered by the algae between these dates.

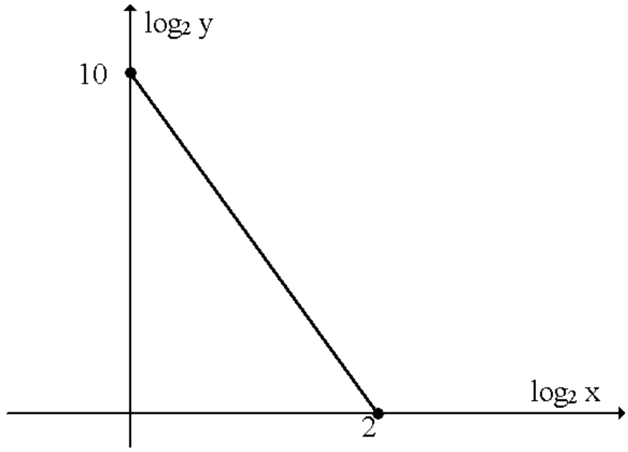
73. The graph opposite illustrates the law $y = kx^n$.

Find the values of k and n.



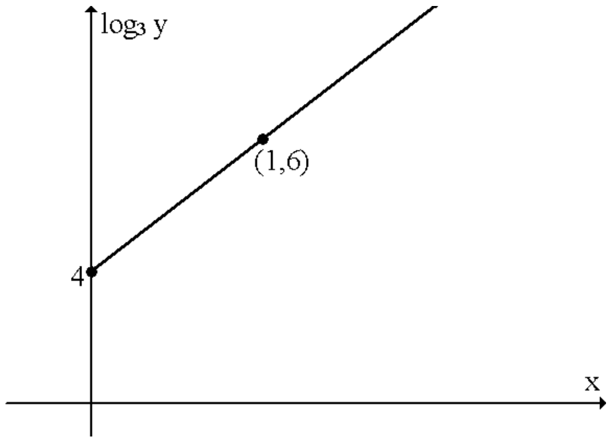
74. The graph opposite illustrates the law $y = kx^n$.

Find the values of k and n.



75. The graph opposite illustrates the law $y = ab^x$.

Find the values of a and b.



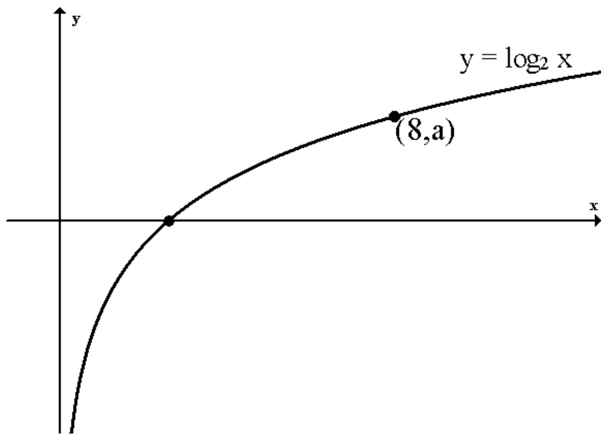
76. The diagram opposite shows the graph of $y = \log_2 x$.

(a) Find the value of a.

(b) Sketch the graph of $y = \log_2 x - 1$

(c) Sketch the graph of $y = \log_2 8x$

(d) Sketch the graph of $y = \log_2 \frac{1}{x}$



77. The diagram shows the graph of

$y = \log_b (x + a)$.

The points (5,0) and (129,3) lie on this graph.

Find the values of a and b.