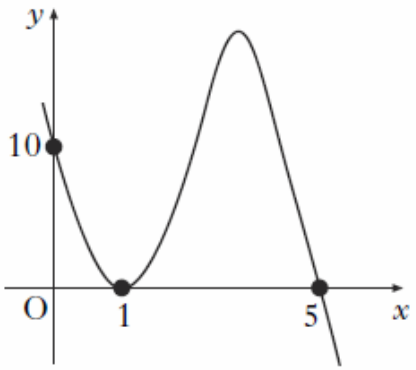
Higher : Polynomials Revision

2010 P1

16. The diagram shows the graph with equation $y = k(x-1)^2(x+t)$.



What are the values of k and t?

	k	t
A	−2	−5
B	−2	5
C	2	−5
D	2	5

Ans A

2010 P1

22. (*a*) (i) Show that $(x-1)$ is a factor of $f(x) = 2x^3 + x^2 - 8x + 5$. 5

(ii) Hence factorise $f(x)$ fully.

(*b*) Solve $2x^3 + x^2 - 8x + 5 = 0$. 1

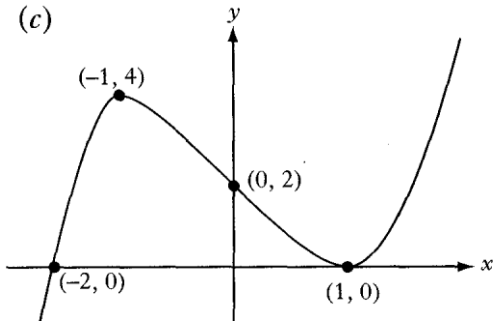
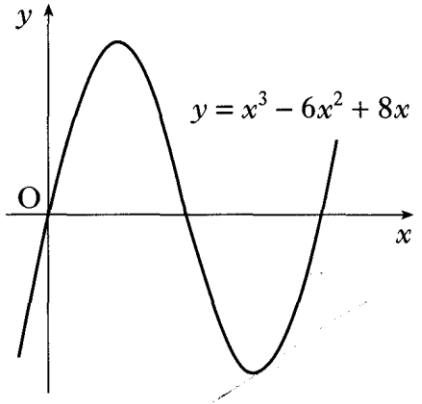
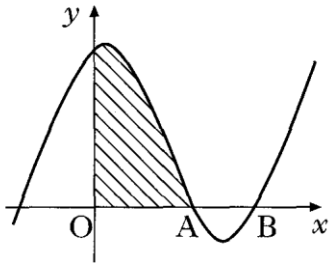
(*c*) The line with equation $y = 2x - 3$ is a tangent to the curve with equation $y = 2x^3 + x^2 - 6x + 2$ at the point G. 5

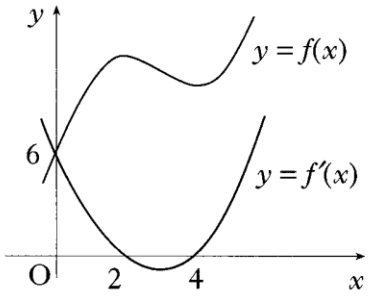
Find the coordinates of G.

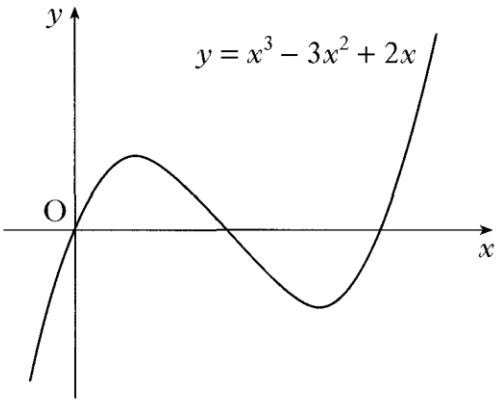
(*d*) This tangent meets the curve again at the point H. 1

Write down the coordinates of H.

Ans
(a) (x - 1)(x - 1)(2x + 5)
(b) x =1 and -2.5
(c) G(1,-1)
(d) H(-2.5,-8)

2008 P1	**21.** A function f is defined on the set of real numbers by $f(x) = x^3 - 3x + 2$. (*a*) Find the coordinates of the stationary points on the curve $y = f(x)$ and determine their nature. (*b*) (i) Show that $(x - 1)$ is a factor of $x^3 - 3x + 2$. (ii) Hence or otherwise factorise $x^3 - 3x + 2$ fully. (*c*) State the coordinates of the points where the curve with equation $y = f(x)$ meets both the axes and hence sketch the curve.	6 5 4
Ans	(*a*) (–1, 4) maximum (1, 0) minimum (*b*) (i) $x = 1$, $f(x) = 0$ so $(x - 1)$ is a factor (ii) $(x - 1)(x - 1)(x + 2)$ (*c*)  	
2008 P1	**22.** The diagram shows a sketch of the curve with equation $y = x^3 - 6x^2 + 8x$. (*a*) Find the coordinates of the points on the curve where the gradient of the tangent is –1. (*b*) The line $y = 4 - x$ is a tangent to this curve at a point A. Find the coordinates of A.  	5 2
Ans	(a) (1,3), (3,-3) (b) (1,3)	
2007 P1	**8.** The diagram shows a sketch of the graph of $y = x^3 - 4x^2 + x + 6$. (*a*) Show that the graph cuts the x-axis at (3, 0). (*b*) Hence or otherwise find the coordinates of A.  	1 3
Ans	(*a*) To cut the x-axis, $y = 0$. So $0 = x^3 - 4x^2 + x + 6$ $= (x - 3)(x^2 - x - 2)$ $= (x - 3)(x - 2)(x + 1)$ So graph cuts x-axis at $x = -1, 3, 2$. (*b*) (2,0)	

2007 P2	**10.** The diagram shows the graphs of a cubic function $y = f(x)$ and its derived function $y = f'(x)$. Both graphs pass through the point (0, 6). The graph of $y = f'(x)$ also passes through the points (2, 0) and (4, 0).  (*a*) Given that $f'(x)$ is of the form $k(x - a)(x - b)$: (i) write down the values of a and b; (ii) find the value of k.	3
Ans	(*a*) (i) $a = 2, b = 4$ (ii) $k = \frac{3}{4}$	
2005 P1	**8.** A function f is defined by the formula $f(x) = 2x^3 - 7x^2 + 9$ where x is a real number. (*a*) Show that $(x - 3)$ is a factor of $f(x)$, and hence factorise $f(x)$ fully. (*b*) Find the coordinates of the points where the curve with equation $y = f(x)$ crosses the x- and y-axes. (*c*) Find the greatest and least values of f in the interval $-2 \le x \le 2$.	5 2 5
Ans	(*a*) $(x - 3)(2x - 3)(x + 1)$ (*b*) $(-1,0), (\frac{3}{2}, 0), (3, 0)$ (*c*) greatest value = 9 least value = −35	
2005 P2	**11.** (*a*) Show that $x = -1$ is a solution of the cubic equation $x^3 + px^2 + px + 1 = 0$. (*b*) Hence find the range of values of p for which all the roots of the cubic equation are real.	1 7
Ans	(*a*) $f(-1) = -1 + p - p + 1 = 0$ (*b*) $p \le -1, p \ge 3$	
2004 P1	**2.** $f(x) = x^3 - x^2 - 5x - 3$. (*a*) (i) Show that $(x + 1)$ is a factor of $f(x)$. (ii) Hence or otherwise factorise $f(x)$ fully. (*b*) One of the turning points of the graph of $y = f(x)$ lies on the x-axis. Write down the coordinates of this turning point.	5 1
Ans	$(x + 1)(x + 1)(x - 3)$ $(-1, 0)$	

2003 P2	**1.** $f(x) = 6x^3 - 5x^2 - 17x + 6$. (*a*) Show that $(x - 2)$ is a factor of $f(x)$. (*b*) Express $f(x)$ in its fully factorised form.	4
Ans	(*b*) $(x - 2)(2x + 3)(3x - 1)$	
2002W P1	**5.** Given that $(x - 2)$ and $(x + 3)$ are factors of $f(x)$ where $f(x) = 3x^3 + 2x^2 + cx + d$, find the values of c and d.	5
Ans	$c = -19$, $d = 6$	
2001 P2	**1.** (*a*) Given that $x + 2$ is a factor of $2x^3 + x^2 + kx + 2$, find the value of k. (*b*) Hence solve the equation $2x^3 + x^2 + kx + 2 = 0$ when k takes this value.	3 2
Ans	(*a*) $k = -5$ (*b*) $x = -2, \frac{1}{2}, 1$	
2000 P2	**1.** The diagram shows a sketch of the graph of $y = x^3 - 3x^2 + 2x$. (*a*) Find the equation of the tangent to this curve at the point where $x = 1$. (*b*) The tangent at the point (2, 0) has equation $y = 2x - 4$. Find the coordinates of the point where this tangent meets the curve again.  	5 5
Ans	(*a*) $x + y = 1$ (*b*) $(-1, -6)$	
Specimen 2 P1	**1.** Show that $x = 2$ is a root of the equation $y = 2x^3 + x^2 - 13x + 6 = 0$ and hence, or otherwise, find the other roots.	4
Ans	$\begin{array}{c\|cccc} 2 & 2 & 1 & -13 & 6 \\ & & 4 & 10 & -6 \\ \hline & 2 & 5 & -3 & 0 \end{array}$ remainder $= 0 \Rightarrow x = 2$ is a root $2x^2 + 5x - 3 = 0 \Rightarrow x = \frac{1}{2}, -3$	

Specimen 1 P1	**3.** (*a*) Show that $(x - 1)$ is a factor of $f(x) = x^3 - 6x^2 + 9x - 4$ and find the other factors. (*b*) Write down the coordinates of the points at which the graph of $y = f(x)$ meets the axes.	3 1
Ans	(*a*) $f(1) = 0$, $(x - 4)$, $(x - 1)$ (*b*) $(1,0)$, $(4,0)$, $(0, -4)$	