Advanced Higher Unit 1 **Binomial Theorem Homework Outcome 1a**

1. Expand and simplify (a) $(2y-3)^3$ (b) $\left(x^3-\frac{3}{x}\right)^5$

2. (a) For what value of $n \neq 3$ is $\binom{8}{n}=\binom{8}{3}$

(b) For what values of p and q is $\binom{7}{3}+\binom{7}{4}=\binom{p}{q}$

3. Consider the expansion $(x+y)^{20}$

(a) The term in x^5 is of the form $\binom{p}{q}x^5y^r$ find p, q and r

(b) What values of s and t is $\binom{20}{14}$ the coefficient of x^s and y^t

(c) Evaluate (i) $\binom{20}{14}$ (ii) $^{20}C_{11}$

4. Find the coefficient of x^{10} in the expansion $\left(3x^2+\frac{4}{x^3}\right)^{10}$

5. Evaluate the term in y^3 in the expansion $\left(y-\frac{5}{y}\right)^7$

6. Find the coefficient of x^4 in the expansion $(1+x)^2(1+2x)^3$

7. (a) Simplify $\binom{n+1}{3}\div\binom{n}{2}$

(b) Find n given that $\binom{n}{2}=55$

8. Show that in the expansion of $\left(x-\frac{2}{x}\right)^{10}$ the term which is independent of x has the value -8064

Advanced Higher Unit 1 **Partial Fractions Homework Outcome 1b**

1. Express in partial fractions:

(a) $\dfrac{5}{(x-2)(x+3)}$

(b) $\dfrac{5x-8}{x(x-4)}$

(c) $\dfrac{2x^2+1}{x(x+1)^2}$

(d) $\dfrac{x+12}{2x^2+3x-2}$

(e) $\dfrac{3x^2+8x-1}{(x-2)(x^2+x+3)}$

(f) $\dfrac{x^3+2x^2-16x+19}{x^2+4x-5}$

(g) $\dfrac{x^2+3x}{x^2+x-12}$

(h) $\dfrac{x^4+x^3-x^2+1}{(x-1)(x^2+1)}$

2. The cubic polynomial $c(x)$ is defined by $c(x)=x^3-x^2-x-2$.

(a) Find a real root of the equation $c(x)=0$ and hence factorise $c(x)$ into the product of a linear factor $l(x)$ and a quadratic factor $q(x)$.

(b) Show that $c(x)$ cannot be written as the product of three linear factors with real coefficients.

(c) Hence express $\dfrac{5x+4}{x^3-x^2-x-2}$ in partial fractions.

3. The function f is defined by $f(x)=\dfrac{2(5x^2-15x+9)}{x(2x-3)^2}$ for all real numbers x, except for $x=0$ and $x=\dfrac{3}{2}$.

(a) Find the values of the constants A, B and C for which

$$f(x)=\frac{A}{x}+\frac{B}{2x-3}+\frac{C}{(2x-3)^2}.$$

(b) Hence find the value of $f'(1)$.

1. Differentiate each function with respect to x, simplifying your answers as far as possible.

(a) $f(x)=\dfrac{1}{\left(x^2+3x+5\right)^2}$ (b) $y=\tan^3 x$ (c) $y=\sin^2 x\cos^3 x$ (d) $y=\dfrac{x+2}{\sqrt{x+1}}$

(e) $y=x^2e^{4x}$ (f) $y=\dfrac{\ln(2x+1)}{x^2}$ (g) $f(x)=\cot(3x)$ (h) $y=2x^2\sec x$

(i) $y=\ln\left(\dfrac{\sin x}{1+\cos x}\right)$ (j) $y=\operatorname{cosec}^2 x$ (k) $y=\dfrac{x^3}{e^{3x}}$ (l) $f(x)=\dfrac{x^2\ln x}{x+1}$

2. Find the equation of the tangent to the curve with equation $y=x\ln(x)$ at the point where $x=e$.

3. Part of a journey an object made was observed.
The displacement, s metres, of the object traveling in a straight line at time t seconds is given by: -

$$s=\frac{t^3}{3}+t^2-8t+10$$

(a) How far from its origin was the object when observation was started?

(b) At what time was the object stationary?

(c) Comment on the motion of the object when $t=5$ seconds.

(d) Does the object ever reach a constant velocity or decelerate during its journey? Justify your answer.

4. If $y=\dfrac{\sin x}{x^2}$, prove that $x^2\dfrac{d^2y}{dx^2}+4x\dfrac{dy}{dx}+(x^2+2)y=0$

5. Given that $y = (x+2)e^{x+1}$, find expressions for $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ and $\frac{d^3y}{dx^3}$.

Write down an expression for the nth derivative $\frac{d^ny}{dx^n}$, and find the value of $\frac{d^{10}y}{dx^{10}}$ when $x = 2$, in terms of e.

6. The function f is defined by

$$f(x) = x^3e^{-x}$$

Find the stationary values of f(x) and determine their nature.

State the interval on which the gradient of the function is negative.

7. A farmer ploughed a square field, ABCD, of side 132 metres.

There is a path along the perimeter of the field which the farmer can walk along at a speed of 8km/h. He can walk across the ploughed field at a speed of 5km/h.

In order to get from A to the opposite corner C, the farmer starts walking along the perimeter path from A to B. When he reaches a point P he leaves the path and heads directly for C, across the ploughed field.

What is the distance AP, if he takes the least possible time in getting from A to C by the route described?

1. Find each of these indefinite integrals.

a) $\int\left(e^{3x} - \frac{6}{e^{2x}}\right)dx$ b) $\int\frac{2}{4x+1}dx$ c) $\int\frac{x}{x^2-2}dx$

2. Express $\int_1^5 \frac{1}{3x+1}dx$ in the form $k\ln 2$ for some real number k, and state the value of k.

3. Evaluate the following integrals using a suitable substitution.

a) $\int \sin x \cos^3 x dx$

b) $\int\frac{x}{\sqrt{x^2+1}}dx$

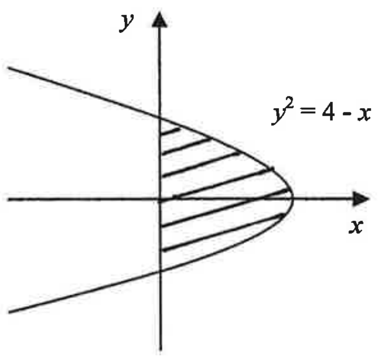
Evaluate the following integrals using the given substitution.

c) $\int_3^4 (x+3)\sqrt[3]{x^2+6x-1}dx$ $x^2 + 6x - 1 = t$

d) $\int_0^{\pi/4}\frac{\sec^2 x}{1+\tan x}dx$ $1 + \tan x = u$

4. Show that $\int_1^{\sqrt{3}}\frac{1}{\sqrt{4-x^2}}dx = \frac{\pi}{6}$ using the substitution $x = 2\sin t$

5. Calculate the shaded area in the diagram.



6. A particle starts from rest and its acceleration a after t seconds is given by $a = 6t + 2$. Determine the velocity and the distance from the start after 3 seconds.

7. The section of the curve $y = x^3$ between $y = 0$ and $y = 8$ is rotated about the $y-$axis to form a volume of revolution. Calculate this volume.

Advanced Higher Unit 1 Properties of Functions Homework Outcome 4

1. For the following functions, write down suitable domains and their corresponding ranges.

   (a) $f(x)=\dfrac{3}{(x+1)^2}$ (b) $f(x)=\sqrt{2x-1}$

2. (a) Sketch the function $f(x)=|x^2+x-6|$

   (b) Identify all local extrema and determine their nature if $f(x)$ is defined on the domain [-4, 2).

   (c) Identify any global maxima and minima if different from (b).

3. Sketch each of the curves below, showing clearly any points of intersection with the coordinate axes, any asymptotes and stationary points.

   (a) $y=\dfrac{2x+5}{x-3}$ (b) $y=\dfrac{x^2-4x+1}{x-4}$

4. Determine whether the following functions are odd, even or neither. State what significance this has for the graph of each function.

   (a) $f(x)=x+2\sin x$ (b) $f(x)=3x^2-3+\dfrac{4}{x^4}$

   (c) $f(x)=2x^3-3x^2+x$ (d) $f(x)=\dfrac{2}{x^3}-\dfrac{4}{x}$

5. A function f is defined by

$$f(x)=\frac{x^2+6x+12}{x+2} \qquad x\neq -2$$

   (a) Express $f(x)$ in the form $ax+b+\dfrac{b}{x+2}$ stating the values of a and b.

   (b) Write down an equation for each of the two asymptotes.

   (c) Determine the coordinates and nature of the stationary points.

   (d) Sketch the graph of f.

   (e) State the range of values of k such that the equation $f(x) = k$ has no solution.

1. Use Gaussian elimination to solve each system of equations below for x, y and z.

(a)
$$x + y - z = 11$$
$$x + 2y + z = 7$$
$$2x + 3y + 2z = 12$$

(b)
$$x + y = 2$$
$$x + 2z = 1$$
$$2y - 3z = 4$$

2. For the system of equations

$$x - y + z = 1$$
$$x + y + 2z = 0$$
$$2x - y + az = 2$$

Use Gaussian elimination to express the value of z in terms of a.
Write down the value of a for which there is no solution.

Hence write down the values of x, y and z when $a = 3$.

3. Use Gaussian Elimination to obtain solutions of the equations

$$2x - y + 2z = 1$$
$$x + y - 2z = 2$$
$$x - 2y + 4z = -1$$

4. The parabola with equation $y = ax^2 + bx + c$, where a, b and c are constants, passes through the point (1, -16).

(a) Write down an equation in a, b and c.
(b) Given that the parabola also passes through the points (2, -18) and (-3, 32), write down a further two equations in a, b and c.
(c) Use Gaussian elimination to find the values of a, b and c, and write down the equation of the parabola.
(d) Sketch the parabola, showing clearly the points of intersection with the coordinate axes and the turning point.
(e) Find the area enclosed between the parabola and the x-axis.

Advanced Higher Unit 1 **Homework Revision Outcome 1 – Outcome 5**

1. Obtain partial fractions for $\dfrac{x^2-2x}{(x+2)(x^2+4)}$

2. A car manufacturer is planning future production patterns. Based on estimates of time, cost and labour, he obtains a set of three equations for the numbers x, y and z of three new types of car. These equations are:

$$\begin{aligned} x + 2y + z &= 60 \\ 2x + 3y + z &= 85 \\ 3x + y + (\lambda + 2)z &= 105 \end{aligned}$$

where the **integer** λ is a parameter such that $0 < \lambda < 10$.

(a) Use Gaussian elimination to find an expression for z in terms of λ.

(b) Given that z must be a positive integer, what are the possible values of z?

(c) Find the corresponding values of x and y for each value of z.

3. Let the function f be given by

$$f(x) = \frac{2x^3 - 7x^2 + 4x + 5}{(x-2)^2}, \quad x \neq 2$$

(a) The graph of $y = f(x)$ crosses the y-axis at (0, a). State the value of a.

(b) For the graph of $f(x)$

(i) Write down the equation of the vertical asymptote.

(ii) Show algebraically that there is a non-vertical asymptote and state its equation.

(c) Find the coordinates and nature of the stationary point of $f(x)$.

(d) Show that $f(x) = 0$ has a root in the interval $-2 < x < 0$.

(e) Sketch the graph of $y = f(x)$. (You must include on your sketch the results obtained in the first four parts of this question).

4. Obtain the derivative of the function

$$f(x) = x\ln x \quad x > 0$$

Hence, or otherwise, find the indefinite integral $\int \ln x \, dx$.

Work out the **exact** value of

$$\int_2^4 \ln x \, dx$$

expressing your answer in terms of ln 2

5. Find the stationary value of the function $\frac{x-1}{e^x}$ and determine its nature.

Find also the coordinates of the point of inflexion on the graph of this function.

6. By using appropriate trigonometric formulae show that:-

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{\operatorname{cosec}^2 x - 1} dx = \frac{3-1}{\sqrt{3}} - \frac{\pi}{6}$$

7. Shown are the middle three terms in the expansion of $\left(\frac{1}{2}x + \frac{2}{x}\right)^{20}$

$\left(\frac{1}{2}x + \frac{2}{x}\right)^{20} = \ldots + 41990x^2 + a + \frac{671840}{x^2} + \ldots$, where a is a constant.

Determine the value of a.

8. Using the substitution $u = 1 - \cos x$ find $\int_{\frac{\pi}{3}}^{\pi} \frac{\sin x}{(1-\cos x)^2} dx$

Advanced Higher Unit 1 **Specimen Unit Assessment (1)**

Outcome 1

1 Expand $(x+y)^4$ **2**

2 Express $\dfrac{5x+11}{(x-2)(x+5)}$ in partial fractions **3**

Outcome 2

3 Differentiate the following functions with respect to x.

(a) $f(x) = 2x^3e^x$ **2**

(b) $f(x) = \dfrac{x^2+3}{5x+4}$ **2**

(c) $f(x) = \ln(\sin x)$ **2**

Outcome 3

4 Find (a) $\displaystyle\int \frac{4x^3}{x^4-8}\,dx$ (b) $\displaystyle\int e^{3x}\,dx$ **2, 2**

5 By using the substitution $u = \cos x$, find $\displaystyle\int \sin x \cos^3 x\,dx$ **3**

Outcome 4

6 $f(x) = \dfrac{x^2-3x+11}{x-2}, x \neq 2, x \in \mathbb{R}$

(a) Write down the equation of the vertical asymptote of the graph of $y = f(x)$ **1**

(b) Show that the graph has a non-vertical asymptote and find its equation. **2**

(c) Sketch the graph of $y = f(x)$ showing clearly its intersections with the axes, and its turning points with appropriate justification. **6**

Outcome 5

7 Use Gaussian elimination to solve the following system of equations.

$$\begin{aligned} x + y + z &= 4 \\ x - 2y - z &= 3 \\ 2x + y + z &= 7 \end{aligned}$$

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Advanced Higher Unit 1 **Specimen Unit Assessment (2)**

Outcome 1

1 Expand $(x-y)^5$ **2**

2 Express $\dfrac{4x+13}{(x+2)(x+3)}$ in partial fractions **3**

Outcome 2

3 Differentiate the following functions with respect to x.

(a) $f(x)=2x^5\ln x$ **2**

(b) $f(x)=\dfrac{3x-1}{2x+5}$ **2**

(c) $f(x)=\exp(\cos x)$ **2**

Outcome 3

4 Find (a) $\displaystyle\int\frac{3x^2}{x^3-18}\,dx$ (b) $\displaystyle\int e^{4x}\,dx$ **2, 2**

5 By using the substitution $u=\sin x$, find $\displaystyle\int\sin^5 x\cos x\,dx$ **3**

Outcome 4

6 $f(x)=\dfrac{x^2-3x+6}{x-2}, x\neq 2, x\in\mathbb{R}$

(a) Write down the equation of the vertical asymptote of the graph of $y = f(x)$ **1**

(b) Show that the graph has a non-vertical asymptote and find its equation. **2**

(c) Sketch the graph of $y = f(x)$ showing clearly its intersections with the axes, and its turning points with appropriate justification. **6**

Outcome 5

7 Use Gaussian elimination to solve the following system of equations.

$$\begin{aligned} x + y \phantom{{}+z} &= 3 \\ 2y - z &= 3 \\ 2x + y + z &= 4 \end{aligned}$$

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