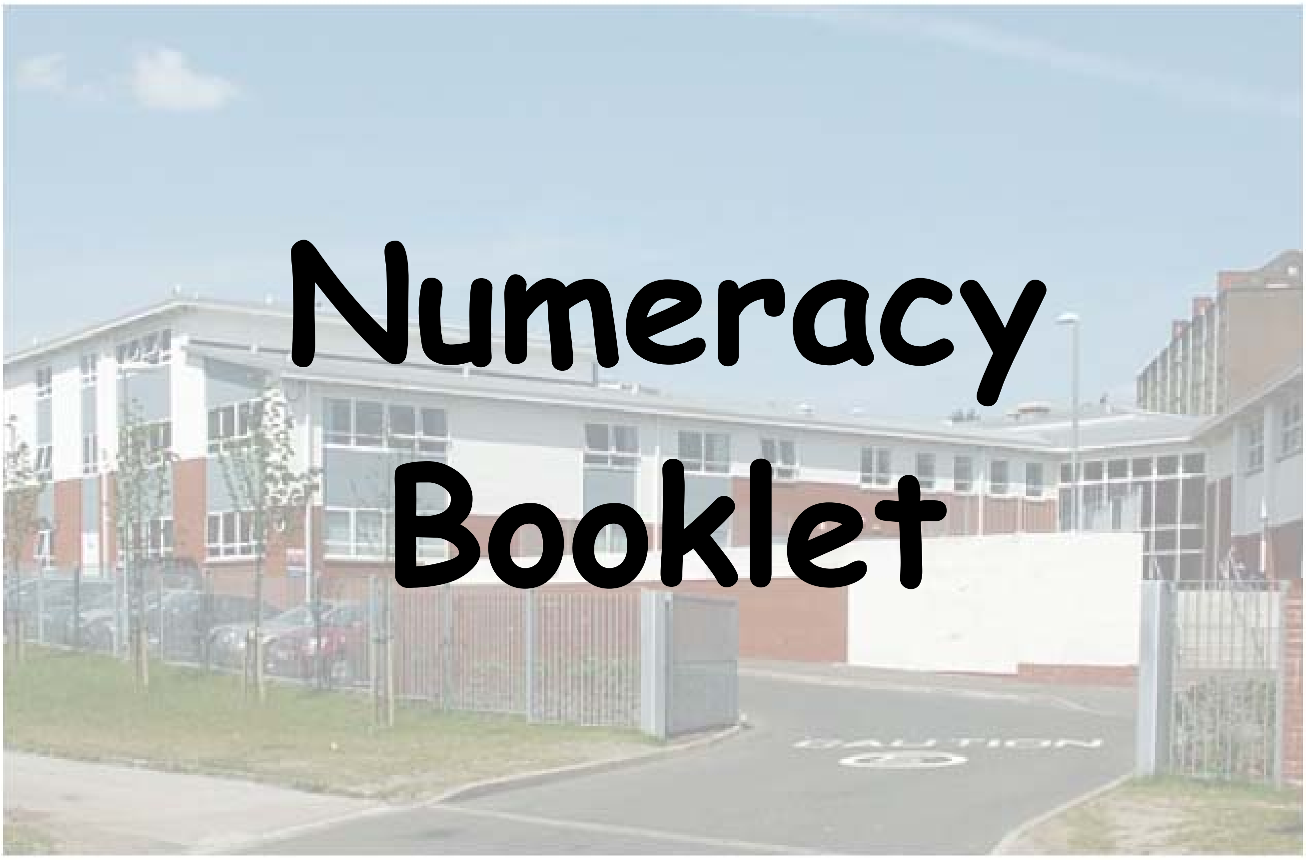
Springburn Academy



Numeracy Booklet

A guide for S1 and S2 pupils, parents and staff

Introduction

What is the purpose of the booklet?

This booklet has been produced to give guidance to pupils, parents and teachers on how certain common Numeracy topics are taught in mathematics and throughout the school. Staff from all departments have been consulted during its production and will be issued with a copy of the booklet. It is hoped that using a consistent approach across all subjects will make it easier for pupils to progress.

How can it be used?

If you are helping your child with their homework, you can refer to the booklet to see what methods are being taught in school. Look up the relevant page for a step by step guide. Pupils have been issued with their own copy and can use the booklet in school to help them solve number and information handling questions in any subject.

The booklet includes the Numeracy skills useful in subjects other than mathematics. For help with mathematics topics, pupils should refer to their mathematics textbook or ask their teacher for help. If you would like to know what your child is studying in mathematics, please ask to see the Mathematics Pupil Handbook which was issued to your child at the start of the session.

Why do some topics include more than one method?

In some cases (e.g. percentages), the method used will be dependent on the level of difficulty of the question, and whether or not a calculator is permitted.

For mental calculations, pupils should be encouraged to develop a variety of strategies so that they can select the most appropriate method in any given situation.

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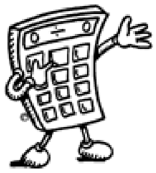
Addition

Mental strategies



There are a number of useful mental strategies for addition. Some examples are given below.

Example Calculate 54 + 27

Method 1 Add tens, then add units, then add together

50 + 20 = 70 4 + 7 = 11 70 + 11 = 81

Method 2 Split up number to be added into tens and units and add separately.

54 + 20 = 74 74 + 7 = 81

Method 3 Round up to nearest 10, then subtract

54 + 30 = 84 but 30 is 3 too much so subtract 3;
84 - 3 = 81

Written Method

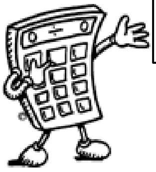
When adding numbers, ensure that the numbers are lined up according to place value. Start at right hand side, write down units, carry tens.

Example Add 3032 and 589

$$\begin{array}{r}3032\\ +5\underset{1}{8}9\\ \hline 1\\ \hline\end{array} \longrightarrow \begin{array}{r}3032\\ +\underset{1}{5}\underset{1}{8}9\\ \hline 21\\ \hline\end{array} \longrightarrow \begin{array}{r}3032\\ +\underset{1}{5}\underset{1}{8}9\\ \hline 621\\ \hline\end{array} \longrightarrow \begin{array}{r}3032\\ +\underset{1}{5}\underset{1}{8}9\\ \hline 3621\\ \hline\end{array}$$

2 + 9 = 11 3+8+1=12 0+5+1=6 3 + 0 = 3

Subtraction



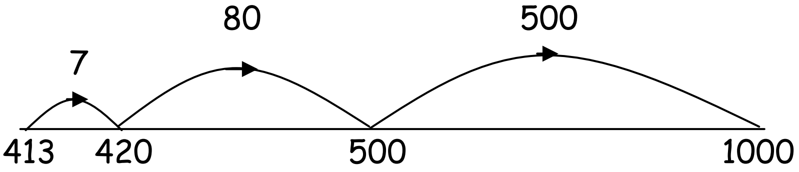
We use decomposition as a written method for subtraction (see below). Alternative methods may be used for mental calculations.

Mental Strategies

Example Calculate 1000 - 413

Method 1 Count on

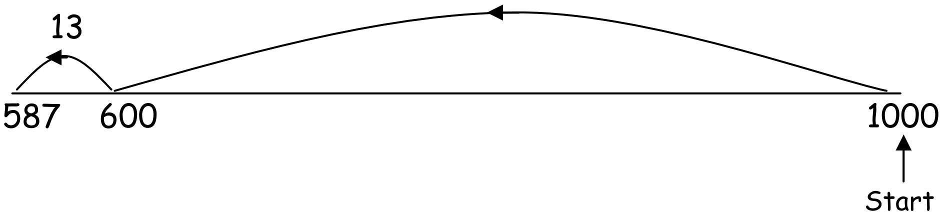
Count on from 413 until you reach 1000. This can be done in several ways e.g.



Method 2 Break up the number being subtracted

e.g. subtract 400, then subtract 13

1000 - 400 = 600
600 - 13 = 587



Written Method

Example 1 4590 – 386

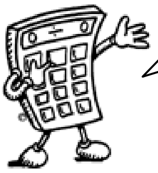
$$\begin{array}{r} 45\overset{8}{\cancel{9}}{}^{1}0 \\ -\ 386 \\ \hline 4204 \\ \hline \end{array}$$

We do not "borrow and pay back".

Example 2 Subtract 365 from 4000

$$\begin{array}{r} \overset{3}{\cancel{4}}\,{}^{1}\overset{9}{\cancel{0}}\,{}^{1}\overset{9}{\cancel{0}}\,{}^{1}0 \\ -\ \ 3\ 6\ 5 \\ \hline 3\ 6\ 3\ 5 \\ \hline \end{array}$$

Multiplication 1



It is essential that you know all of the multiplication tables from 1 to 10. These are shown in the tables square below.

x	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Mental Strategies

Example Find 39 x 6

Method 1

30 x 6
= 180

9 x 6
= 54

180 + 54
= 234

Method 2

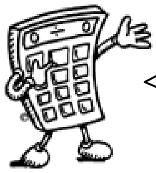
40 x 6
=240

40 is 1 too many
so take away 6x1

240 - 6
= 234

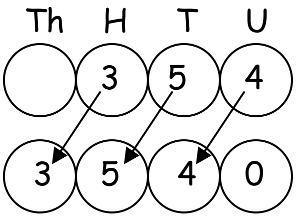
Multiplication 2

Multiplying by multiples of 10 and 100

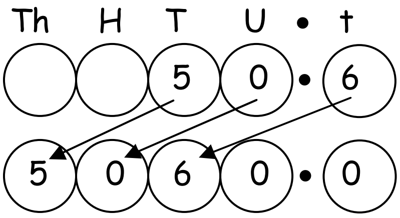


To multiply by **10** you move every digit ***one*** place to the left.
To multiply by **100** you move every digit ***two*** places to the left.

Example 1 (a) Multiply 354 by 10 (b) Multiply 50.6 by 100



354 x 10 = 3540



50.6 x 100 = 5060

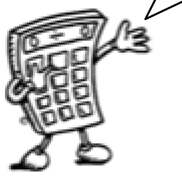
(c) 35 x 30

To multiply by 30, multiply by 3, then by 10.

35 x 3 = 105
105 x 10 = 1050

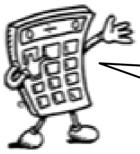
so 35 x 30 = 1050

We may also use these rules for multiplying decimal numbers.



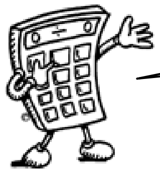
Example 2 (a) 2.36 x 20 (b) 38.4 x 50

2.36 x 2 = 4.72	38.4 x 5 = 192.0
4.72 x 10 = 47.2	192.0x 10 = 1920
so 2.36 x 20 = 47.2	so 38.4 x 50 = 1920



In these examples it is in fact the numbers that are moving but it '<u>appears</u>' that the decimal point moves.

Division



You should be able to divide by a single digit or by a multiple of 10 or 100 without a calculator.

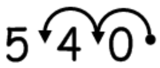
Written Method

Example 1 42 ÷ 6 = 7

Example 2 540 ÷ 100 = 5.40

Mental Strategy

The decimal point appears to move 2 places to the left



Example 3 There are 192 pupils in first year, shared equally between 8 classes. How many pupils are in each class?

$$\begin{array}{r} 2\,4 \\ 8\,\overline{)\,1\,9^{3}2} \end{array}$$

There are 24 pupils in each class

Example 4 Divide 4.74 by 3

$$\begin{array}{r} 1\,.\,5\,8 \\ 3\,\overline{)\,4\,.\,{}^{1}7{}^{2}4} \end{array}$$

When dividing a decimal number by a whole number, the decimal points must stay in line.

Example 5 A jug contains 2.2 litres of juice. If it is poured evenly into 8 glasses, how much juice is in each glass?

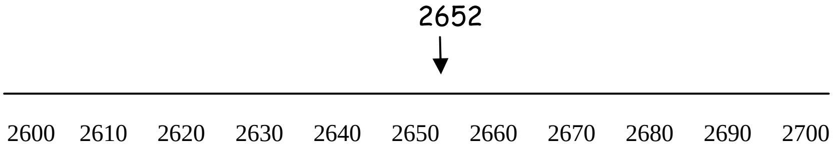
$$\begin{array}{r} 0\,.\,2\,7\,5 \\ 8\,\overline{)\,2\,.{}^{2}2{}^{6}0{}^{4}0} \end{array}$$

Each glass contains 0.275 litres

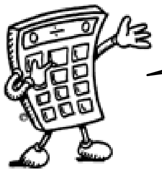
If you have a remainder at the end of a calculation, add a zero onto the end of the decimal and continue with the calculation.

Estimation : Rounding

Numbers can be rounded to give an approximation.



26**5**2 rounded to the nearest 10 is 2650.
2**6**52 rounded to the nearest 100 is 2700.



When rounding numbers which are exactly in the middle, convention is to **round up**.
78**6**5 rounded to the nearest 10 is 78**7**0.

The same principle applies to rounding decimal numbers.

In general, to round a number, we must first identify the place value to which we want to round. We must then look at the next digit to the right (the "check digit") - if it is 5 or more round up.

Example 1 Round 46 753 to the nearest thousand.

6 is the digit in the thousands column - the check digit (in the hundreds column) is a 7, so round up.

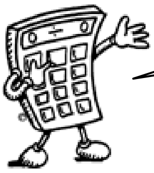
4**6** 753
= 47 000 to the nearest thousand

Example 2 Round 1.57359 to 2 decimal places

The second number after the decimal point is a 7 - the check digit (the third number after the decimal point) is a 3, so round down.

1.57359
= 1.5**7** to 2 decimal places

Time 1



Time may be expressed in 12 or 24 hour notation.

12-hour clock

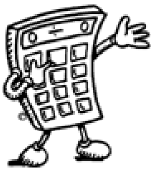
Time can be displayed on a clock face, or digital clock.

5.15am
or
5.15pm

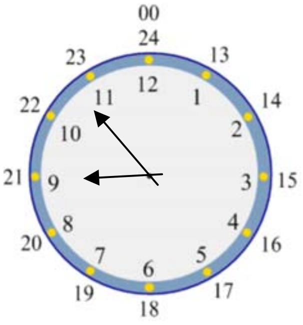
These clocks both show fifteen minutes past five, or quarter past five.

When writing times in 12 hour clock, we need to add a.m. or p.m. after the time.
a.m. is used for times between midnight and 12 noon (morning)
p.m. is used for times between 12 noon and midnight (afternoon / evening).

24-hour clock



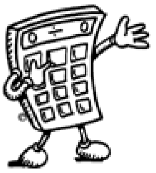
In 24 hour clock, the hours are written as numbers bet ween 00 and 24. Midnight is expressed as 00 00, or 24 00. After 12 noon, the hours are numbered 13, 14, 15 … etc.



Examples

9.55 am → 0955 hours
3.35 pm → 1535 hours
12.20 am → 0020 hours
0216 hours → 2.16 am
2045 hours → 8.45 pm

Time 2



It is essential to know the number of months, weeks and days in a year, and the number of days in each month.

Time Facts

In 1 year, there are: 365 days (366 in a leap year)
52 weeks
12 months

The number of days in each month can be remembered using the rhyme:

"30 days hath September,
April, June and November,
All the rest have 31,
Except February alone,
Which has 28 days clear,
And 29 in each leap year."

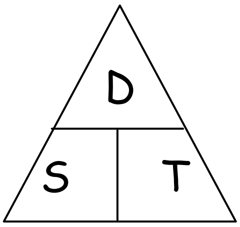
Distance, Speed and Time.

For any given journey, the distance travelled depends on the speed and the time taken. If speed is constant, then the following formulae apply:

$$\text{Distance} = \text{Speed} \times \text{Time} \quad \text{or} \quad D = S\,T$$

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} \quad \text{or} \quad S = \frac{D}{T}$$

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}} \quad \text{or} \quad T = \frac{D}{S}$$



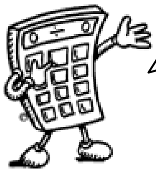
Example Calculate the speed of a train which travelled 450 km in 5 hours

$$S = \frac{D}{T}$$

$$S = \frac{450}{5}$$

$$S = 90 \text{ km/h}$$

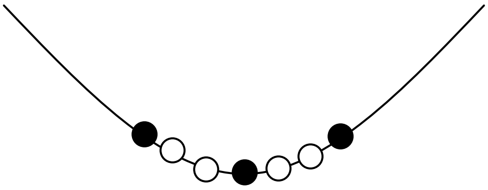
Fractions 1



Addition, subtraction, multiplication and division of fractions are studied in mathematics.
However, the examples below may be helpful in all subjects.

Understanding Fractions

Example
A necklace is made from black and white beads.

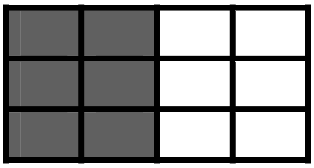


What fraction of the beads are black?

There are 3 black beads out of a total of 7, so $\frac{3}{7}$ of the beads are black.

Equivalent Fractions

Example
What fraction of the flag is shaded?



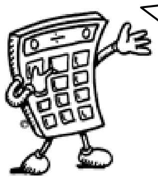
6 out of 12 squares are shaded. So $\frac{6}{12}$ of the flag is shaded.

It could also be said that $\frac{1}{2}$ the flag is shaded.

$\frac{6}{12}$ and $\frac{1}{2}$ are **equivalent fractions**.

Fractions 2

Simplifying Fractions



The top of a fraction is called the **numerator**, the bottom is called the **denominator**.
To simplify a fraction, divide the **numerator** and **denominator** of the fraction by the same number.

Example 1

(a)

$$\frac{20}{25} \underset{\div 5}{\overset{\div 5}{=}} \frac{4}{5}$$

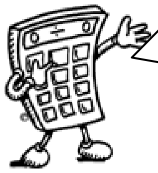
(b)

$$\frac{16}{24} \underset{\div 8}{\overset{\div 8}{=}} \frac{2}{3}$$

This can be done repeatedly until the numerator and denominator are the smallest possible numbers - the fraction is then said to be in it's **simplest form**.

Example 2 Simplify $\frac{72}{84}$ $\frac{72}{84} = \frac{36}{42} = \frac{18}{21} = \frac{6}{7}$ (simplest form)

Calculating Fractions of a Quantity



To find the fraction of a quantity, divide by the denominator.
To find $\frac{1}{2}$ divide by 2, to find $\frac{1}{3}$ divide by 3, to find $\frac{1}{7}$ divide by 7 etc.

Example 1 Find $\frac{1}{5}$ of £150

$\frac{1}{5}$ of £150 = £150 ÷ 5 = £30

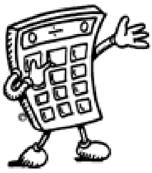
Example 2 Find $\frac{3}{4}$ of 48

$\frac{1}{4}$ of 48 = 48 ÷ 4 = 12

so $\frac{3}{4}$ of 48 = 3 x 12 = 36

To find $\frac{3}{4}$ of a quantity, start by finding $\frac{1}{4}$

Percentages 1



Percent means out of 100.
A percentage can be converted to an equivalent fraction or decimal.

36% means $\frac{36}{100}$

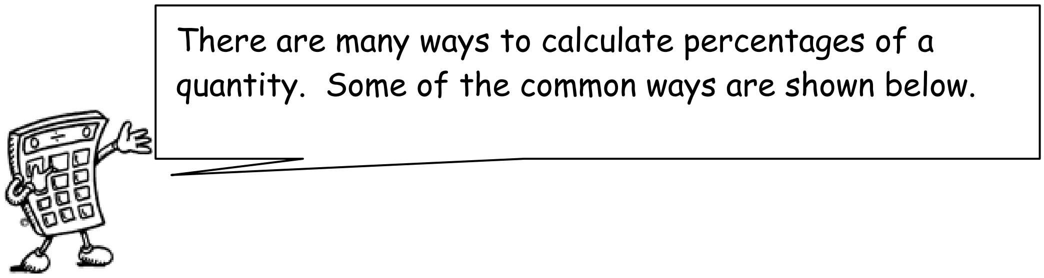
36% is therefore equivalent to $\frac{9}{25}$ and 0.36

Common Percentages

Some percentages are used very frequently. It is useful to know these as fractions and decimals.

Percentage	**Fraction**	**Decimal**
1%	$\frac{1}{100}$	**0.01**
10%	$\frac{1}{10}$	**0.1**
20%	$\frac{1}{5}$	**0.2**
25%	$\frac{1}{4}$	**0.25**
$33^1/_3$%	$\frac{1}{3}$	**0.333…**
50%	$\frac{1}{2}$	**0.5**
$66^2/_3$%	$\frac{2}{3}$	**0.666…**
75%	$\frac{3}{4}$	**0.75**

Percentages 2



Non- Calculator Methods

Method 1 Using Equivalent Fractions

Example Find 25% of £640

$$25\% \text{ of } £640 = \frac{1}{4} \text{ of } £640 = £640 \div 4 = £160$$

Method 2 Using 1%

In this method, first find 1% of the quantity (by dividing by 100), then multiply to give the required value.

Example Find 9% of 200g

$$1\% \text{ of } 200g = \frac{1}{100} \text{ of } 200g = 200g \div 100 = 2g$$

so 9% of 200g = 9 x 2g = 18g

Method 3 Using 10%

This method is similar to the one above. First find 10% (by dividing by 10), then multiply to give the required value.

Example Find 70% of £35

$$10\% \text{ of } £35 = \frac{1}{10} \text{ of } £35 = £35 \div 10 = £3.50$$

so 70% of £35 = 7 x £3.50 = £24.50

Percentages 3

Calculator Method

To find the percentage of a quantity using a calculator, change the percentage to a decimal, then multiply.

Example 1 Find 17.5% of £600

$$= \frac{17.5}{100} \times 600$$

$$= £105$$

Example 2 House prices increased by 19% over a one year period. What is the new value of a house which was valued at £236000 at the start of the year?

$$\text{Increase} = \frac{19}{100} \times £236000$$

$$= £44840$$

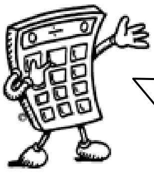
Value at end of year = original value + increase

= £236000 + £44840

= £280840

The new value of the house is £280840

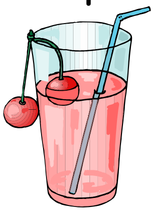
Ratio 1



When quantities are to be mixed together, the ratio, or proportion of each quantity is often given. The ratio can be used to calculate the amount of each quantity, or to share a total into parts.

Writing Ratios

Example 1



To make a fruit drink, 4 parts water is mixed with 1 part of cordial.
The ratio of water to cordial is 4:1
(said "4 to 1")
The ratio of cordial to water is 1:4.

Order is important when writing ratios.

Example 2

In a bag of balloons, there are 5 red, 7 blue and 8 green balloons.

The ratio of red : blue : green is 5 : 7 : 8

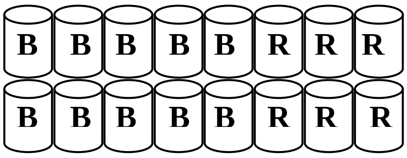
Simplifying Ratios

Ratios can be simplified in much the same way as fractions.

Example 1

Purple paint can be made by mixing 10 tins of blue paint with 6 tins of red. The ratio of blue to red can be written as 10 : 6

It can also be written as 5 : 3, as it is possible to split up the tins into 2 groups, each containing 5 tins of blue and 3 tins of red.



Blue : Red = 10 : 6
= 5 : 3

To simplify a ratio, divide each figure in the ratio by a common factor.

Ratio 2

Simplifying Ratios (continued)

Example 2

Simplify the ratio:

(a) 24:36
= 2:3

Divide each figure by 12

Example 3

Concrete is made by mixing 20 kg of sand with 4 kg cement. Write the ratio of sand : cement in its simplest form

Sand : Cement = 20 : 4
= 5 : 1

Using ratios

The ratio of fruit to nuts in a chocolate bar is 3 : 2. If a bar contains 15g of fruit, what weight of nuts will it contain?

	Fruit	Nuts	
	3	2	
x5			x5
	15	**10**	

So the chocolate bar will contain 10g of nuts.

Ratio 3

Sharing in a given ratio

Example

Lauren and Sean earn money by washing cars. By the end of the day they have made £90. As Lauren did more of the work, they decide to share the profits in the ratio 3:2. How much money did each receive?

Step 1 Add up the numbers to find the total number of parts

3 + 2 = 5

Step 2 Divide the total by this number to find the value of each part

90 ÷ 5 = £18

Step 3 Multiply each figure by the value of each part

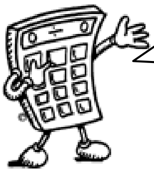
3 x £18 = £54
2 x £18 = £36

Step 4 Check that the total is correct

£54 + £36 = £90 √

Lauren received £54 and Sean received £36

Proportion



Two quantities are said to be in direct proportion if when one doubles the other doubles.
We can use proportion to solve problems.

It is often useful to make a table when solving problems involving proportion.

Example 1
5 adult tickets for the cinema cost £27.50. How much would 8 tickets cost?

Find the cost of 1 ticket →

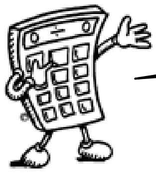
Tickets	Cost
5	£27.50
1	£5.50
8	£44.00

Working:

$$\begin{array}{r} \text{£}5.50 \\ 5\overline{)\text{£}27.50} \end{array} \qquad \begin{array}{r} \text{£}5.50 \\ {}_{4}\times 8 \\ \hline \text{£}44.00 \\ \hline \end{array}$$

The cost of 8 tickets is £44

Information Handling : Tables



It is sometimes useful to display information in graphs, charts or tables.

Example 1 The table below shows the average maximum temperatures (in degrees Celsius) in Barcelona and Edinburgh.

	J	F	M	A	M	J	J	A	S	O	N	D
Barcelona	13	14	15	17	20	**24**	27	27	25	21	16	14
Edinburgh	6	6	8	11	14	17	18	18	16	13	8	6

The average temperature in June in Barcelona is 24°C

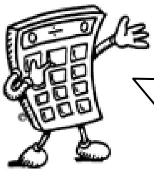
Frequency Tables are used to present information. Often data is grouped in intervals.

Example 2 Homework marks for Class 4B

27 30 23 24 22 35 24 33 38 43 18 29 28 28 27
33 36 30 43 50 30 25 26 37 35 20 22 24 31 48

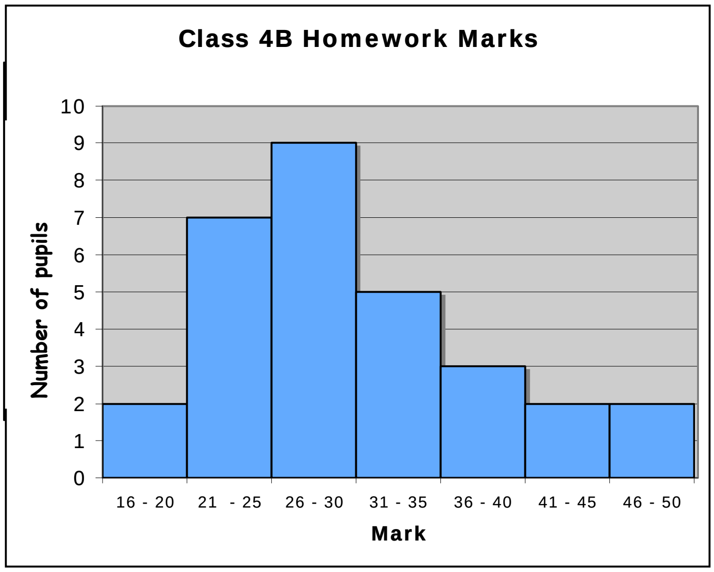
Mark	Frequency
16 - 20	2
21 - 25	7
26 - 30	9
31 - 35	5
36 - 40	3
41 - 45	2
46 - 50	2

Information Handling : Bar Graphs

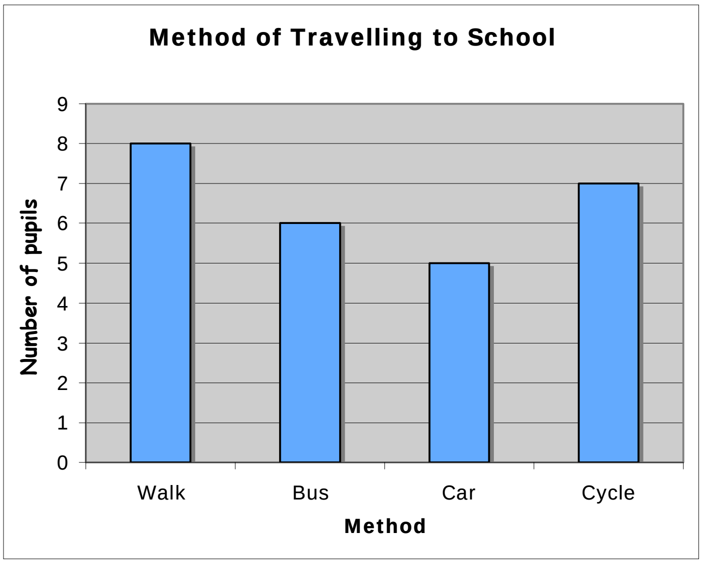


Bar graphs are often used to display data. The horizontal axis should show the categories or class intervals, and the vertical axis the frequency. All graphs should have a title, and each axis must be labelled.

Example 1 The graph below shows the homework marks for Class 4B.

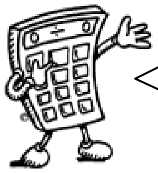


Example 2 How do pupils travel to school?



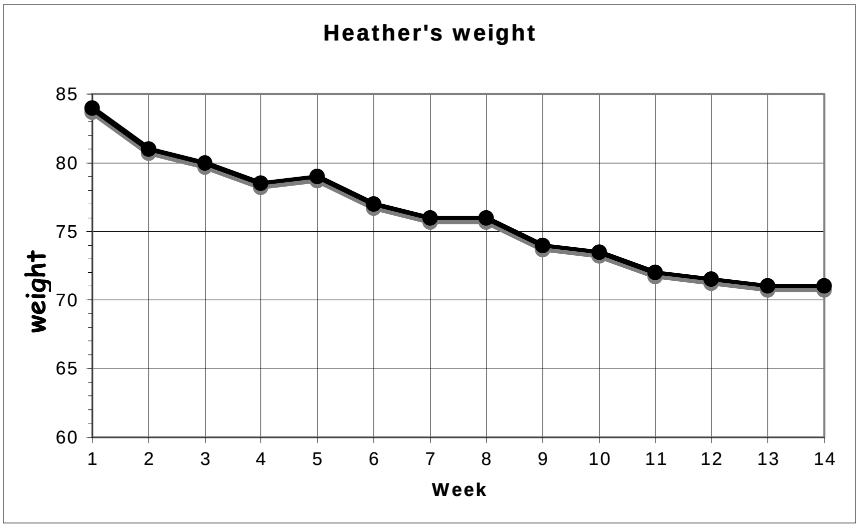
When the horizontal axis shows categories, rather than grouped intervals, it is common practice to leave gaps between the bars.

Information Handling : Line Graphs



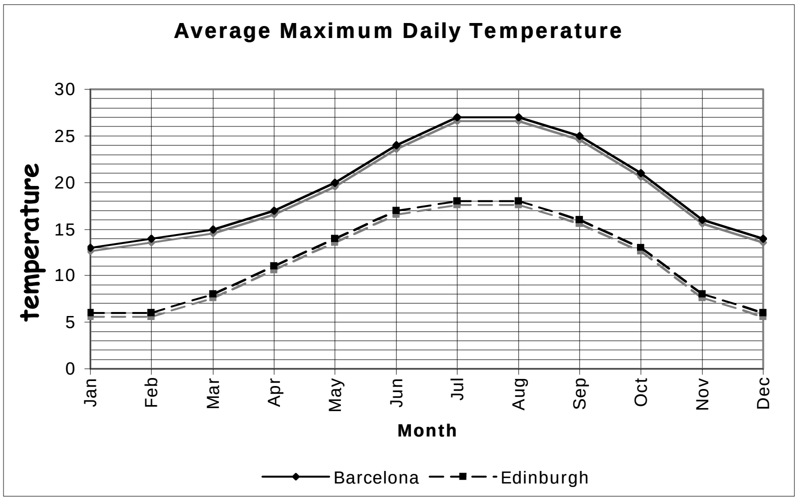
Line graphs consist of a series of points which are plotted, then joined by a line. All graphs should have a title, and each axis must be labelled. The trend of a graph is a general description of it.

Example 1 The graph below shows Heather's weight over 14 weeks as she follows an exercise programme.

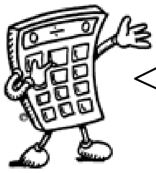


The trend of the graph is that her weight is decreasing.

Example 2 Graph of temperatures in Edinburgh and Barcelona.



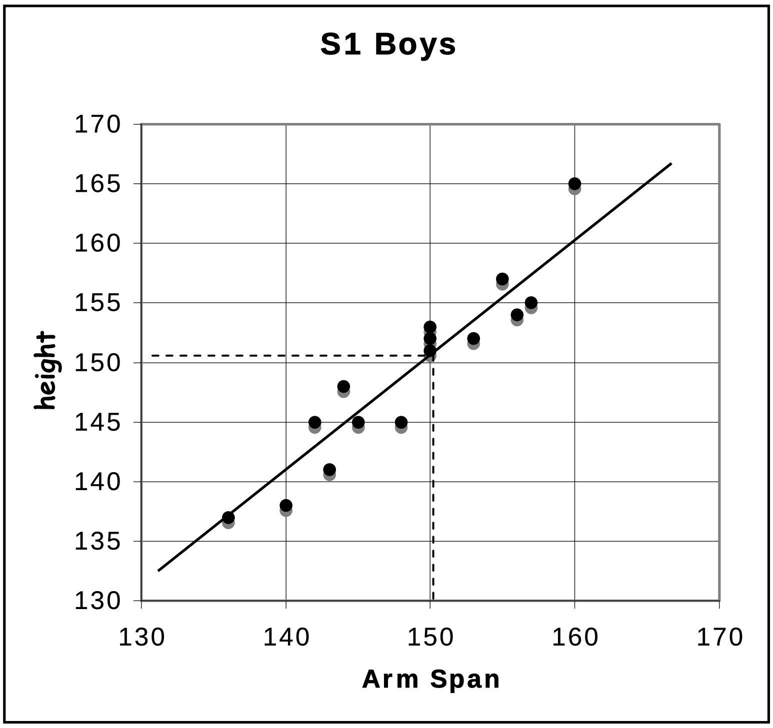
Information Handling : Line of Best Fit



A **line of best fit** is used to display the relationship between two variables.
A pattern may appear on the graph. This is called a **correlation**.

Example The table below shows the height and arm span of a group of first year boys. This is then plotted as a series of points on the graph below.

Arm Span (cm)	150	157	155	142	153	143	140	145	144	150	148	160	150	156	136
Height (cm)	153	155	157	145	152	141	138	145	148	151	145	165	152	154	137

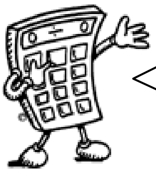


The graph shows a general trend, that as the arm span increases, so does the height. This graph shows a positive correlation.

The line drawn is called the line of best fit. This line can be used to provide estimates. For example, a boy of arm span 150cm would be expected to have a height of around 151cm.

Note that in some subjects, it is a requirement that the axes start from zero.

Information Handling : Averages



To provide information about a set of data, the average value may be given. There are 3 ways of finding the average value – the mean, the median and the mode.

Mean

The mean is found by adding all the data together and dividing by the number of values.

Median

The median is the middle value when all the data is written in numerical order (if there are two middle values, the median is half-way between these values).

Mode

The mode is the value that occurs most often.

Range

The range of a set of data is a measure of spread.

Range = Highest value – Lowest value

Example Class 1A4 scored the following marks for their homework assignment. Find the mean, median, mode and range of the results.

7, 9, 7, 5, 6, 9, 10, 9, 9, 4, 9, 5, 7, 10

$$\text{Mean} = \frac{7+9+7+5+6+9+10+9+9+4+9+5+7+10}{14}$$

$$= \frac{106}{14} = 7.571...$$

Mean = 7.6 to 1 decimal place

Ordered values: 4, 5, 5, 6, 7, 7, $\boxed{7,\ 9,}$ 9, 9, 9, 9, 10, 10

Median = 8

9 is the most frequent mark, so Mode = 9

Range = 10 – 4 = 6

Mathematical Dictionary (Key words):

Add; Addition (+)	To combine 2 or more numbers to get one number (called the sum or the total) Example: 12+76 = 88
a.m.	(ante meridiem) Any time in the morning (between midnight and 12 noon).
Approximate	An estimated answer, often obtained by rounding to nearest 10, 100 or decimal place.
Calculate	Find the answer to a problem. It doesn't mean that you must use a calculator!
Data	A collection of information (may include facts, numbers or measurements).
Denominator	The bottom number in a fraction (the number of parts into which the whole is split).
Difference (-)	The amount between two numbers (subtraction). Example: The difference between 50 and 36 is 14 50 – 36 = 14
Division (÷)	Sharing a number into equal parts. 24 ÷ 6 = 4
Double	Multiply by 2.
Equals (=)	Makes, or has the same amount as.
Equivalent fractions	Fractions which have the same value. Example: $\frac{6}{12}$ and $\frac{1}{2}$ are equivalent fractions
Estimate	To make an approximate or rough answer, often by rounding.
Evaluate	To work out the answer.
Even	A number that is divisible by 2. Even numbers end with 0, 2, 4, 6 or 8.
Frequency	How often something happens. In a set of data, the number of times a number or category occurs.
Maximum	The largest or highest number in a group.
Mean	The arithmetic average of a set of numbers (see p25)
Median	Another type of average - the middle number of an ordered set of data (see p25)
Minimum	The smallest or lowest number in a group.
Minus (-)	To subtract.

Mode	Another type of average – the most frequent number or category (see p25)
Multiply (x)	To combine an amount a particular number of times. Example: 6 x 4 = 24
Negative Number	A number less than zero. Shown by a minus sign. Example: -5 is a negative number.
Numerator	The top number in a fraction.
Odd Number	A number which is not divisible by 2. Odd numbers end in 1, 3, 5, 7 or 9.
Operations	The four basic operations are addition, subtraction, multiplication and division.
Place value	The value of a digit dependent on its place in the number. Example: in the number 1573.4, the 5 has a place value of 100.
p.m.	(post meridiem) Any time in the afternoon or evening (between 12 noon and midnight).
Remainder	The amount left over when dividing a number.
Share	To divide into equal groups.
Sum	The total of a group of numbers (found by adding).
Total	The sum of a group of numbers (found by adding).