

Trigonometric Identities

PAGE 1: The ones you MUST know!

The expressions on this page are essential to know in the exam.

Trig Exact Value Facts

You must know all of these for substituting into expressions.

Luckily though, your calculator will help check these in an exam if you are unsure...

To help you learn them, try thinking of the graphs of sin and cos (i.e. when they cut the x-axis, and where they have maxima and minima.

$$\begin{aligned} 1 &= \cos(0) = \cos(2\pi) \\ 0 &= \sin(0) = \sin(\pi) = \sin(2\pi) \\ -1 &= \cos(\pi) \\ 0 &= \cos\left(\frac{\pi}{2}\right) = \cos\left(\frac{3\pi}{2}\right) \\ 1 &= \sin\left(\frac{\pi}{2}\right) \\ -1 &= \sin\left(\frac{3\pi}{2}\right) \end{aligned}$$

$$\begin{aligned} \sin(-\theta) &= -\sin(\theta) \\ \cos(-\theta) &= +\cos(\theta) \\ \tan(-\theta) &= -\tan(\theta) \end{aligned}$$

Identities that you MUST learn and know

(Learn them now if you don't know them already!)

$$\begin{aligned} \cos^2 A + \sin^2 A &= 1 \\ \cos^2 x &= \frac{1}{2}(1 + \cos 2x) \\ \sin^2 x &= \frac{1}{2}(1 - \cos 2x) \\ \tan A &= \frac{\sin A}{\cos A} \\ \sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\ \cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\ \tan(A \pm B) &= \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \\ \sin(2A) &= 2 \sin A \cos A \\ \cos(2A) &= \cos^2 A - \sin^2 A \end{aligned}$$

Remember that A and B can be ANY expression at all

So, for example $\sin^2(2x + 1) + \cos^2(2x + 1)$ would still be 1.

Trigonometric Identities

PAGE 2: Useful facts, but not essential to learn

You ought to be able to survive in the exam without knowing these ones off by heart,

BUT...

You should be able to derive them all if needed using the equations on Page One [there have been past exam questions asking you to do just this]

MAKE SURE YOU KNOW HOW YOU CAN GET (OR CHECK) EACH OF THEM IF YOU NEED TO

Identities

$$1 + \tan^2 A = \sec^2 A$$

$$\cot^2 A + 1 = \operatorname{cosec}^2 A$$

$$\cos(2A) = 2\cos^2 A - 1$$

$$\cos(2A) = 1 - \sin^2 A$$

$$\tan(2A) = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\sin(\cos^{-1} x) = \sqrt{1 - x^2} \quad (\text{ones like this come up a lot in differentiation.})$$

You must know how to get it, it is not enough just to learn it)

Exact Values

Technically you should know these already from Higher! I am not sure whether they could come up in the exam... Maybe better safe than sorry...

$$\sin\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$\sin\left(\frac{\pi}{6}\right) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$$

$$\sin\left(\frac{\pi}{3}\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$