Newbattle Community High School Maths Department

Intermediate 1 Maths Revision Notes

Units 1, 2 and 3

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Our **Values for Life**

RESPONSIBILITY – PERSEVERANCE – FOCUS – TRUST – RESPECT – HUMOUR

Use this booklet to demonstrate **responsibility** and **perseverance** by practising working independently like you will have to in an exam. This means that when you get stuck on a question, don't just leave the question blank, don't give up, and don't sit there doing nothing until your teacher manages to get to you.

Instead get in the habit of turning to this booklet to refresh your memory.

- If you have forgotten how to do a method, **examples** are given.
- If you have forgotten what a word means, use the **index** to look it up.

This booklet is for:

- Students doing an Intermediate 1 maths course, including Unit 3.

This booklet contains:

- The most important facts you need to memorise for Intermediate 1 maths.
- Examples on how to do the most common questions in each topic.
- Definitions of the key words you need to know.

Use this booklet:

- To refresh your memory of the method you were taught in class when you are stuck on a homework question or a past paper question.
- To memorise key facts when revising for the exam.

The key to revising for a maths exam is to do questions, not to read notes. **As well as using this booklet, you should also:**

- Revise by working through exercises on topics you need more practice on – such as revision booklets, textbooks, websites, or other exercises suggested by your teacher.
- Work through Past papers
- Ask your teacher when you come across a question you cannot answer
- Check the resources online at www.newbattle.org.uk/Departments/Maths/int1.html

As you get closer to the exam you should be aiming to look at this booklet less and less.

Contents

Unit 1

Unit 1 Outcome 1 – Percentages and Rounding

Percentages without a calculator

You may be asked to calculate a percentage in the non calculator paper. You need to know the following:

Percentage	Fraction	Percentage	Fraction
50%	$\frac{1}{2}$	10%	$\frac{1}{10}$
25%	$\frac{1}{4}$	1%	$\frac{1}{100}$
75%	$\frac{3}{4}$	33⅓%	$\frac{1}{3}$
		66⅔%	$\frac{2}{3}$

Examples

What is 75% of £480?

What is 33⅓% of £330?

Solution

75% of £480
= ¾ of £480
=480 ÷ 4 × 3 = **£360**

33 ⅓ % of £330
= ⅓ of £330
= £330 ÷ 3 × 1 = **£110**

Other percentages can be worked out without a calculator by finding 10% first

Examples

What is 40% of £120?

Solution

10% of £120 = £12
So 40% of £120 = 12 × 4 = **£48**

Percentages with a calculator

For every question, there are two ways of doing it. Use the one you are happiest with.

Question	Method 1 Divide and Multiply	Method 2 Decimal	Answer
27% of £360	360 ÷ 100 × 27	0.27 × 360	**£97.20**
3% of £250	250 ÷ 100 × 3	0.03 × 250	**£7.50**
17.5% of £4200	4200 ÷ 100 × 17.5	0.175 × 4200	**£735**
4.2% of £360	360 ÷ 100 × 4.2	0.042 × 360	**£15.12**

Example

A car is normally priced at £8800, but a 12% discount is being offered. What is the new price of the car?

Solution

12% of £8800 = 0.12 × 8800 [or 8800 ÷ 100 × 12] = £1056
New price = 8800 – 1056 = **£7744**

You need to be able to work out percentages when interest is paid on money in a bank account. Interest is expressed as a percentage per annum (p.a.) **Per annum** means "per year".

Example 2 (Bank Account Interest)

Vanessa puts £4800 in the bank at an interest rate of 4% per annum. How much interest would she receive after:

a) One year?
b) Five months?

Solution

a) In one year: 4% of £4800 = 0.04 × 4800 [or 4800 ÷ 100 × 4] = **£192**

b) There are twelve months in a year, so in *one month*, Vanessa will get 192÷12=£16
This means that in *five months*, she will get 16×5=**£80**

Finding the Percentage: One Number as a Percentage of Another

To find the percentage, there are three steps:

1. Write as a fraction
2. Change to a decimal by dividing
3. Change to a percentage by multiplying by 100

Example

Pete for 24 out of 32 for an exam. What is his mark as a percentage?

Solution

As a fraction, Pete got $\frac{24}{32}$.
To change this to a decimal, do 24 ÷ 32 = 0.75
To change 0.75 to a percentage, do 0.75×100=**75%**

A quick way of remembering this is to do **smaller ÷ bigger × 100** **(or top ÷ bottom × 100)**

Example 2

Out of 1250 pupils, 475 get to school by bus. What percentage is this?

Solution

As a fraction, this is $\frac{475}{1250}$.
To change this to a percentage: 475 ÷ 1250 × 100 = **38%**

More difficult questions ask you to find the percentage increase or decrease. In this questions, you always have to work out the percentage of the **original** amount.

<u>Example</u>

The temperature in an oven was 180°C. It went up to 207°C. What was the percentage increase?

Solution

Step one – what is the increase? 207 – 180 = 27°C

Step two – express as a fraction of the original amount

Original amount was 180°C, so as a fraction this is $\frac{27}{180}$.

Step three – change to a percentage: $27 \div 180 \times 100 =$ **<u>15%</u>**

Rounding

If a question asks you to round your answer, there will be a mark for this.

The question may say something like:

- "Round your answer to the nearest thousand"
- or "…giving your answer to one decimal place"

Do not let yourself lose a mark by missing these instructions!

The instruction "**round your answer to the nearest penny**" means rounding it to two decimal places (since money always has to have two decimal places).

<u>Examples</u>

4652 to the nearest ten is 4650
4652 to the nearest hundred is 4700
4652 to the nearest thousand is 5000

23.6666666 to one decimal place is 23.7
23.6666666 to two decimal places is 23.67
23.6666666 to the nearest penny is £23.67

Direct Proportion

A direct proportion question is one where you have to use the fact that numbers go up in equal amounts. The method for one of these questions is usually to find the cost for *one* first.

<u>Example</u>

John hires a car for 4 days. It costs him £90.
How much will it cost for his friend Sam to hire it for 7 days?

Solution

Step One – How much does it cost to hire the car for <u>one</u> day?

Divide: 90 ÷ 4 = 22.5, so it costs £22.50 per day.

Step Two – How much does it cost to hire the car for seven days?

Multiply: $22.50 \times 7 =$ **£157.50**.

Example 2

Jo is an electrician. She charges customers £27 for every 15 minutes she has to work. How much will they have to pay Jo for a job that lasts 3 hours?

Solution

Step One – How much does it cost for one hour?

15 minutes cost £27. There are four lots of 15 minutes in an hour,
So for one hour it costs $27 \times 4 =$ £108

Step Two – How much does it cost for three hours?

Multiply: $108 \times 3 =$ **£324**.

Unit 1 Outcome 2 – Areas and Volumes

Areas of rectangles, squares and triangles

Definition: the **area** of a 2d shape is a measure of the amount of space inside it.

These formulae are NOT given on the exam paper

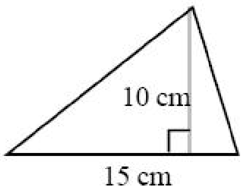
Area of a rectangle: $A = \text{Length} \times \text{Breadth}$, or $A = LB$

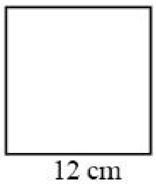
Area of a square: $A = L^2$

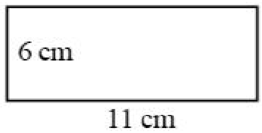
Area of a triangle: $A = \frac{1}{2}\text{Base} \times \text{Height}$, or $A = \dfrac{BH}{2}$

Examples

Find the area of these shapes







Solution

$A = \dfrac{BH}{2}$

$A = \dfrac{15 \times 10}{2} = \dfrac{150}{2} = \underline{75\text{cm}^2}$

$A = L^2$

$A = 12^2 = \underline{144\text{cm}^2}$

$A = LB$

$A = 11 \times 6 = \underline{66\text{cm}^2}$

Definition: a **composite shape** is one made by joining two or more other shapes together. In the exam, areas will always be of composite shapes, usually made up of rectangles, squares, triangles or semi-circles (for semicircles see page 11).

The method to work out the area of a composite shape is always the same:

Step one – split the shape up.

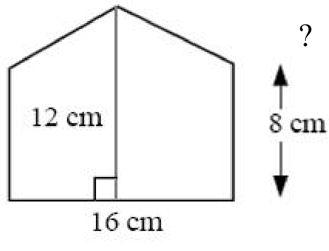
Step two – work out the area of each smaller shape separately.

Step three – either add or take away the areas:

- If the two shapes are joined together, you **add** the areas.
- If one shape if cut out of the other, you **take away** its area.

Example 1

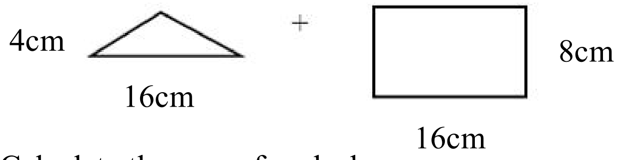
Find the area of this shape:



Solution

Step one – start by splitting the shape up into a rectangle and a triangle.

We don't know the height of the triangle yet (the length marked '?') – but from looking at the diagram, we can see that 8+'?'=12, so '?'=4cm.



Step two – Calculate the area of each shape.

$$\text{Area of triangle} = \frac{BH}{2} = \frac{16\times4}{2} = \frac{64}{2} = \underline{32\text{cm}^2}$$

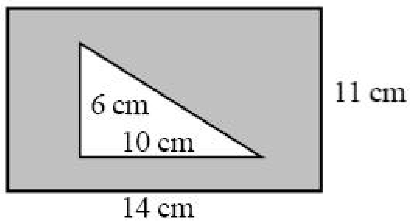
$$\text{Area of rectangle} = LB = 16\times8 = \underline{128\text{cm}^2}$$

Step three – the two shapes are joined together, so **add** the areas

Area = 32 + 128 = **<u>150cm²</u>**

<u>Example 2</u>

Find the shaded area in this shape



Solution

Step one – split the shape up into the rectangle (grey) and the triangle (white).

Step two – calculate the area of each shape

$$\text{Area of triangle} = \frac{BH}{2} = \frac{10\times6}{2} = \frac{60}{2} = \underline{30\text{cm}^2}$$

$$\text{Area of rectangle} = LB = 11\times14 = \underline{154\text{cm}^2}$$

Step three – the triangle is cut out of the rectangle, so we **take away** its area.

Area = 154 – 30 = **<u>124cm²</u>**

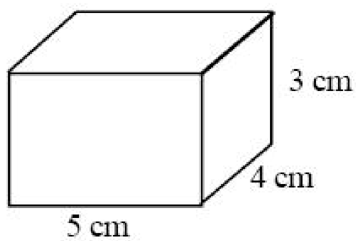
Volumes of Cubes and Cuboids

Definition: the **volume** of a 3d shape is a measure of the amount of space inside it.

This formula is <u>NOT</u> given on the exam paper
Volume of a cuboid:
$V =$ Length $\times$ Breadth $\times$ Height, or $V = LBH$

<u>Example</u>

Calculate the volume of this cuboid



Solution

$$V = LBH = 5\times4\times3 = \underline{60\text{cm}^3}$$

Example

Calculate the volume of this cube

Solution

In a cube, all the sides are the same length.

$$V = LBH$$
$$= 40 \times 40 \times 40 \quad (\underline{\text{NOT}}\ 40 \times 3)$$
$$= \underline{64000\text{cm}^3}$$

Litres and Millilitres

Some volume questions refer to litres and millilitres.

The key facts are:

1cm^3 = 1 millilitre

1000 millilitres = 1 litre

You might be told these facts on an exam paper, but you cannot guarantee this

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Example

A tank of water is shaped as shown. How many litres of water can it hold?

Solution

$$V = LBH$$
$$= 10 \times 20 \times 50$$
$$= \underline{10000\text{cm}^3}$$

10000cm^3=10000ml because millilitres and cm^3 are the same thing.
There are 1000ml in a litre, so here there are 10000÷1000=**10 litres**.

Circumference of a Circle

Definitions: the **diameter** of a circle is the distance all the way across a circle, passing through the centre. The **radius** is half of the diameter.

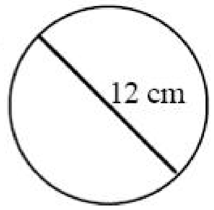
Definition: the **circumference** is the curved length around the outside of a circle. It is a special name for the perimeter of a circle.

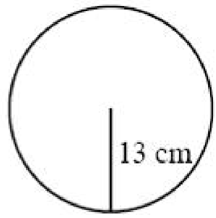
This formula is given on the exam paper

Circumference of a circle: $C = \pi d$

Examples

Find the circumferences of these two circles:





Solutions

The *diameter* is 12cm, so *d*=12

$$\begin{aligned} C &= \pi d \\ &= \pi \times 12 \quad (\text{or } 3.14 \times 12) \\ &= 37.69911184... \\ &= \underline{37.7\,\text{cm}} \ (1\text{d.p.}) \end{aligned}$$

In this circle, the *radius* is 13cm so the diameter is 26cm, i.e. *d*=26

$$\begin{aligned} C &= \pi d \\ &= \pi \times 26 \quad (\text{or } 3.14 \times 26) \\ &= 81.68140899... \\ &= \underline{81.68\,\text{cm}} \ (2\text{d.p.}) \end{aligned}$$

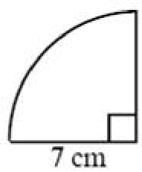
You may come across more difficult examples that involve quarter and half circles, these next two examples ask you to calculate the **perimeter** of the shapes.

Definition: the **perimeter** is the distance <u>*all the way around*</u> the shape.

The circumference refers to the curved length only. Therefore to work out the perimeter, you also need to add on any straight lengths.

Example

Find the perimeter of this shape:



Solution

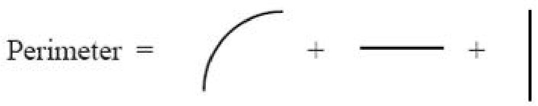
The shape is a quarter circle, so we divide by 4.

7cm is the radius, so the diameter is 14cm.

Step one – find the circumference

$$\begin{aligned} C &= \pi d \div 4 \\ &= \pi \times 14 \div 4 \quad (\text{or } 3.14 \times 14 \div 4) \\ &= 10.9955... \\ &= \underline{11.0\,\text{cm}} \ (1\text{d.p.}) \end{aligned}$$

Step two – find the perimeter by adding on the straight lengths



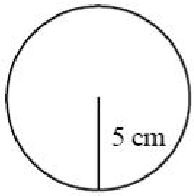
Perimeter = 11.0 + 7 + 7 = <u>**25.0cm**</u>

Area of a Circle

This formula is given on the exam paper
Area of a circle: $A = \pi r^2$

Example 1 (radius)

Find the area of this circle



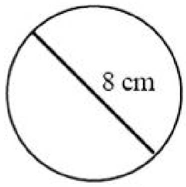
Solution

The radius of this circle is 5cm, so r=5.

$$\begin{aligned} A &= \pi r^2 \\ &= \pi \times 5^2 \quad (\text{or } 3.14 \times 5^2) \\ &= \pi \times 5 \times 5 \quad (\text{or } 3.14 \times 5 \times 5) \\ &= 78.53981634... \\ &= \underline{78.5\,\text{cm}^2} \ (1\text{d.p.}) \end{aligned}$$

Example 2 (diameter)

Find the area of this circle



Solution

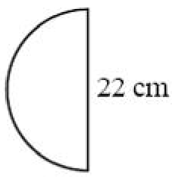
The diameter of this circle is 8cm, so the radius is 4cm, or r=4.

$$\begin{aligned} A &= \pi r^2 \\ &= \pi \times 4^2 \quad (\text{or } 3.14 \times 4^2) \\ &= \pi \times 4 \times 4 \quad (\text{or } 3.14 \times 4 \times 4) \\ &= 50.26548... \\ &= \underline{50.3\,\text{cm}^2} \ (1\text{d.p.}) \end{aligned}$$

Definition: a **semicircle** is half of a circle.

Example 3 (semicircle)

Find the area of this semicircle



Solution

22cm in this diagram is the *diameter*. This means the radius is 11cm or r=11cm.

$$\begin{aligned} A &= \pi r^2 \div 2 \\ &= \pi \times 11^2 \div 2 \quad (\text{or } 3.14 \times 11^2 \div 2) \\ &= \pi \times 11 \times 11 \div 2 \quad (\text{or } 3.14 \times 11 \times 11 \div 2) \\ &= 190.0663555... \\ &= \underline{190.1\,\text{cm}^2} \ (1\text{d.p.}) \end{aligned}$$

Unit 1 Outcome 3 – Basic Algebra

Using a Formula

Read the question carefully and substitute in the numbers you are given.
Definition: Evaluate means "do the sum"

Example

Evaluate 2*a* – 3*c* when *a*=12 and *c*=1.5

Solution

$$\begin{aligned}&2a-3c\\&=2\times 12-3\times 1.5\\&=24-4.5\\&=\underline{19.5}\end{aligned}$$

Example

Evaluate $3x^2$ when *x*=5

Solution

$$\begin{aligned}&3x^2\\&=3\times 5^2\\&=3\times 25 \qquad \text{(NOT } 15^2\text{)}\\&=\underline{75}\end{aligned}$$

Example

***S*=3*bc* – *a* .   Evaluate *S* when *a*=10, *b*=2 and *c*=7**

Solution

$$\begin{aligned}S&=3bc-a\\&=3\times 2\times 7-10\\&=42-10\\&=\underline{32}\end{aligned}$$

Evaluating a Formula Expressed in Words

When a formula is given in words, you need to read the question carefully and decide what sort of calculation to do.

Example

To cook a chicken, you need 25 minutes per kilogram, and then a further 15 minutes. I have a 6kg chicken. How long should I cook it for?

Solution

6 × 25 minutes, then add 15 minutes
= 150 minutes + 15 minutes
= **<u>165 minutes</u>** (or 2 hours 45 minutes)

Unit 1 Outcome 4 – Money

Commission

Definition: commission is extra money paid to a salesperson. The salesperson gets a percentage of whatever he or she sells. The more they sell, the more pay they get. This is an incentive to sell more items.

Example

Angie sells computers. She gets paid a monthly salary of £950 plus 3% commission on all her computer sales.
What is her monthly pay for a month in which she sells £15600?

Solution

She gets paid £950 *regardless* of how much she sells. DON'T find 3% of £950.

Commission = 3% of £15600 = 0.03 × 15600 [or 15600÷100×3] = £468
Total for month = £950 + £468 = **£1418**.

Overtime

Definitions:

- A worker's **basic hours** are the hours that they *have* to work each week (or each month etc). *e.g. John's works a basic 35 hour week.*
- **Overtime** hours are any extra hours that a worker works in addition to their basic hours. *e.g. Jacquie works a basic 28 hour week. If she works 31 hours in a week, then she has done 3 hours of overtime.*

You get paid more for each hour of overtime you work than you do for your basic hours. There are two common ways of doing this:

- Double time – where the hourly wage is doubled for overtime hours.
- Time and a half – where you get half as much again for overtime hours.

To work out overtime, you do:

- … × 2 (double time)
- … × 1.5 (time and a half)

Example

Janet works part time in a chemist. She gets £5.30 per hour, and works a basic 14 hour week, and gets time-and-a-half for overtime. How much does she get paid in a week where she works 17 hours?

Solution

Janet works 14 basic hours, and 3 overtime hours

Basic hours: 14 × £5.30 = £74.20
Overtime: 3 × £5.30 × 1.5 = £23.85

Total pay = £74.20 + £23.85 = **£98.05**

Hire Purchase (HP)

Definition: when you buy an expensive item (e.g. a holiday or a car) in a shop, you have two options:

1. Pay the whole amount at once – but not everyone has enough money to hand to do this.
2. Spread the cost by paying in monthly (or weekly) instalments. This is called **Hire Purchase** (also called HP for short).

Definitions: the **deposit** is the amount you give the shop when you first buy the item. You only pay this once. The **instalments** are the monthly (or weekly) amounts you pay every week after that.

Example

A car can be bought on Hire Purchase for a deposit of £1500 and then twelve monthly payments of £480. Calculate the total HP price.

Solution

Twelve payments of £480 = 12 × 480 = £5760
Deposit = £1500
Total price = £5760 + £1500 = **£7260**

Insurance

Definition: the **premium** is the name given to the amount you pay to an insurance company in order to be insured.

Example 1

A company offers to insure jewellery for a price of £2.30 per £100 insured. How much would it cost to insure a ring costing £1400?

Solution

£1400 ÷ £100 = 14, so there are 14 lots of £100 in £1400

£2.30 × 14 = **£32.20**

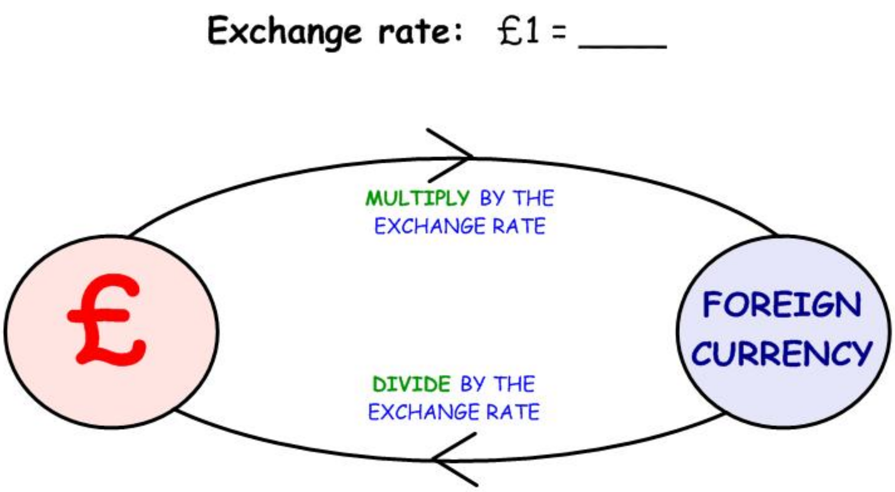
Example 2

A company offers to insure houses for £6.78 per £1000 insured. How much would it cost to insure a house costing £94500?

Solution

94500÷1000=94.5
94.5×£6.78 = **£640.71**

Exchange Rates



Example (changing into foreign money)

Janet changes £250 into Euros. The exchange rate is £1=€1.37. How much money does she get?

Solution

To change money from pounds into foreign money, we multiply.

1.37×250 = 342.5 = **€342.50**

Example (changing back into pounds)

Harry is returning from the USA with \$800. The exchange rate is £1=\$1.57. Harry changes his money back into pounds. How much money does he get?

Solution

To change money from foreign money *back into* pounds, we divide.

800÷1.57=509.554….=**£509.55** (as we always have to round money to 2 decimal places)

Unit 2

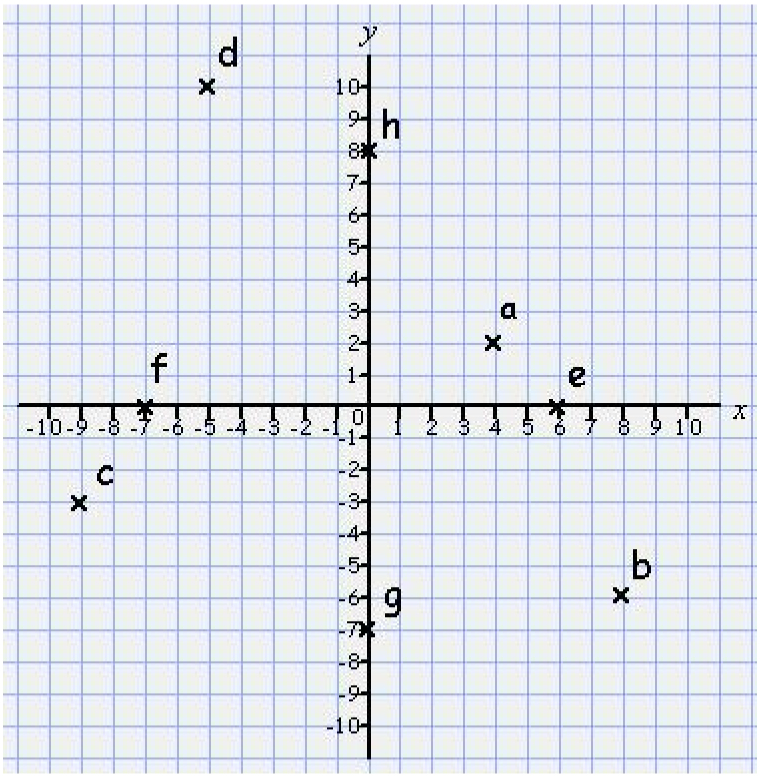
Unit 2 Outcome 1 – Integers

Definition: an **integer** is a whole number that can be either positive or negative. To work with integers, you need to be able to use the rules for doing calculations with negative numbers.

Coordinates

The basic rule for coordinates is *go along and then up* (or *"along the corridor and up the stairs"*). The first part of the coordinate tells you how many squares to count along, the second number tells you how many to count up (or down).

Example



The coordinates of the points on the grid are:

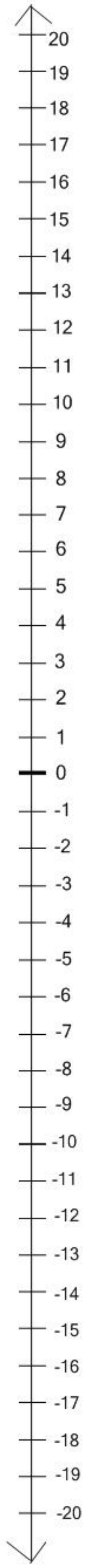
A(4 , 2)	**B**(8 , –6)	**C**(–9 , –3)	**D**(–5 , 10)
E(6 , 0)	**F**(–7 , 0)	**G**(0 , –7)	**H**(0 , 8)

Adding and Taking Away Integers

You need to be able to work with integers in everyday situations (e.g. temperature), and in sums.

Adding and taking away integers is all to do with moving up and down a number line.

In an exam, you could draw your own number line to count up and down on if it helps you.



Example

The temperature at midnight was –6°C. By midday it had risen by 23°C. What was the temperature at midday?

Solution

Start at –6°C. Count up 23°C. Answer: **17°C**.

You also need to be able to complete add and take away sums. Start at the first number, then move up if you are adding, and down if you are taking away.

Examples

–6 + 9 = start at –6 and move up 9. Answer = 3
5 – 7 = start at 5 and move down 7. Answer = –2
(–2) – 8 = start at –2 and move down 8. Answer = –10

Adding a negative number is the same as taking away. When an add and a take away sign are written next to each other, you can "get rid of" the add sign.

Examples

2 + (–6) = 2 – 6 = start at 2 and move down 6. Answer = –4
(–1) + (–7) = (–1) – 7 = start at –1 and move down 7. Answer = –8

Taking away a negative number becomes an add. When two negative signs are written next to each other without a number in between, they become an add sign

"taking away a minus makes a plus"

Examples

5 – (–2) = 5 + 2 = 7
(–7) –(–2) = (–7) + 2 = start at –7 and move up 2. Answer = –5

Multiplying and Dividing Integers

Multiplying and dividing have completely different rules to adding and taking away. To multiply and divide, you do the sum normally (as if there were no negative signs there), and then you decide whether your answer needs to be negative or positive.

When multiplying and dividing:

- If none of the numbers are negative, then the answer is **positive**.
- If one of the numbers is negative, then the answer is **negative**.
- If two of the numbers are negative, then the answer is **positive**.
- If three of the numbers are negative, then the answer is **negative**.
- and so on…

In general, if there are an odd number of negative numbers the answer is negative. If there is an even number of negative numbers, the answer is positive.

Examples (multiplying)

$(-5) \times 4 = -20$ **(one negative number means the answer is negative)**

$60 \times (-2) = -120$ **(one negative number means the answer is negative)**

$(-3) \times (-10) = +30$ (or just 30) **(two negative numbers means the answer is positive)**

$(-2) \times 3 \times (-4) = 24$ **(two negative numbers means the answer is positive)**

$(-2) \times (-3) \times (-4) = -24$ **(three negative numbers means the answer is negative)**

In particular, if you square a negative number, **the answer always has to be positive**.

Examples (squaring)

$(-6)^2 = (-6) \times (-6) = 36$ **(two negative numbers means the answer is positive)**

$(-10)^2 = (-10) \times (-10) = 100$ **(two negative numbers means the answer is positive)**

Examples (dividing)

$(-28) \div 4 = -7$ **(one negative number means the answer is negative)**

$50 \div (-5) = -10$ **(one negative number means the answer is negative)**

$(-80) \div (-10) = +8$ (or just 8) **(two negative numbers means the answer is positive)**

Unit 2 Outcome 2 – Speed, Distance and Time

Time Intervals

You need to know how to find how long something lasts for. The best way is to split each question up into smaller steps:

Example 1

How long is it from 10.45am to 2.20pm?

Solution

15 mins + 3 hrs + 20 mins = $\boxed{\text{3 hrs 35 mins}}$

10:45 am —> 11:00 am —> 2:00 pm —> 2:20 pm

Example 2

A plane leaves London at 10.50pm and arrives in New York the next morning at 6.15am. How long was the flight?

Solution

1 hr 10 mins + 6 hrs + 15 mins = $\boxed{\text{7 hrs 25 mins}}$

10:50 pm —> midnight —> 6:00 am —> 6:15 am

Distance-Time Graphs

Using a distance-time graph, a journey can be shown as a graph. In a distance time graph:

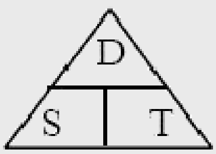
- an upward slope means they are moving away from where they started.
- an downward slope means they are coming back to where they started.
- a flat section means they are stopped for some reason.
- the steeper the line is, the faster the speed.

Speed, Distance and Time Calculations

These formulae are NOT given on the exam paper

$\text{Speed}=\frac{\text{Distance}}{\text{Time}}$ $\text{Time}=\frac{\text{Distance}}{\text{Speed}}$ $\text{Distance}=\text{Speed}\times\text{Time}$

If you have been taught it, you might also like to use the Speed, Distance and Time triangle



<u>Remember</u>

2 hours 15 minutes is not entered into the calculator as 2.15 hours. Instead it is 2.25 hours:

1 hour 30 minutes	= 1 ½ hours	= 1.5 hours
5 hours 15 minutes	= 5 ¼ hours	= 5.25 hours
45 minutes	= ¾ hour	= 0.75 hour

<u>Example</u>

I drive 90 kilometres in 2 hours and 15 minutes. Calculate my average speed

Solution

We are working out speed, so we use the formula $\text{Speed}=\frac{\text{Distance}}{\text{Time}}$

2 hours and 15 minutes is <u>not</u> 2.15 hours. It is 2 ¼ hours = 2.25 hours.

$\text{Speed}=\frac{\text{Distance}}{\text{Time}}=\frac{90}{2.25}=\underline{40\text{km/h}}$

<u>Example</u>

A bird flies for 3½ hours at an average speed of 42km/h. How far does it fly?

Solution

We are working out distance, so we use the formula $\text{Distance}=\text{Speed}\times\text{Time}$

3 ½ hours is <u>not</u> 3.30 hours. It is 3.5 hours.

$\text{Distance}=\text{Speed}\times\text{Time}=42\times3.5=\underline{147\text{km}}$

<u>Example</u>

A driver travels 127.5 miles at an average speed of 30mph. How long does it take her? Give your answer in hours and minutes.

Solution

We are working out time, so we use the formula $\text{Time}=\frac{\text{Distance}}{\text{Speed}}$

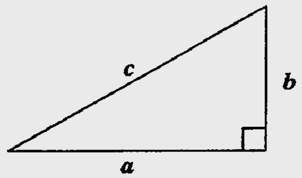
$\text{Time}=\frac{\text{Distance}}{\text{Speed}}=\frac{127.5}{30}=4.25$

4.25 hours is not 4 hours 25 minutes. It is 4 ¼ hours = **<u>4 hours 15 minutes</u>**.

Unit 2 Outcome 3 – Pythagoras' Theorem

When you know how long any two of the sides in a right-angled triangle are, you can use Pythagoras to find the length of the third side without measuring.

This formula is given on the exam paper
Theorem of Pythagoras:



$a^2 + b^2 = c^2$

Definition: the hypotenuse is the longest side in a right-angled triangle. In the diagram above, the hypotenuse is *c*. The hypotenuse is opposite the right angle.

There are three steps to any Pythagoras question:
Step One – square the length of the two lengths
Step Two – either add or take away (see below)
Step Three – square root

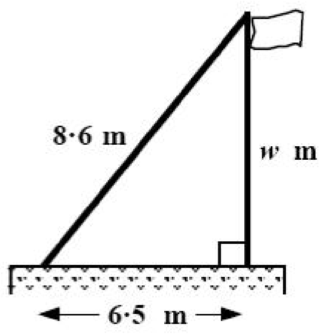
Choosing whether to add or take away:
- If you are finding the length of the longest side (the hypotenuse), you **add** the squared numbers.
- If you are finding the length of a shorter side, you **take away** the squared numbers.

Example 1 (finding the length of the hypotenuse)
Calculate the length of *x* in this triangle. Do not use a scale drawing.

Solution
We are finding the length of *s*. *s* is the hypotenuse, so we **add**.

$$s^2 = 7.8^2 + 1.3^2$$
$$s^2 = 60.84 + 1.69$$
$$s^2 = 62.53$$
$$s = \sqrt{62.53}$$
$$s = 7.90759....$$
$$s = \underline{7.91\text{cm}} \text{ (2d.p.)}$$

Example 2 (finding the length of a smaller side)
Calculate *w*, correct to 1 decimal place. Do not use a scale drawing.



Solution
We are finding the length of *w*.
w is s smaller side, so we **take away**.

$$w^2 = 8.6^2 - 6.5^2$$
$$w^2 = 73.96 - 42.25$$
$$w^2 = 31.71$$
$$w = \sqrt{31.71}$$
$$w = 5.6311....$$
$$w = \underline{5.6\,\text{cm}}\ (1\text{d.p.})$$

Exam Questions

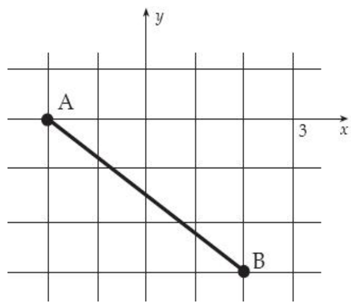
During the exam the Pythagoras question is almost always hidden, look hard because it will definitely always be in the exam – most likely in paper 2.

One way you can spot a Pythagoras question is by watching out for the phrase **"Do not use a scale drawing"**. This phrase usually indicates that the question is either Pythagoras or SOH CAH TOA.

The other main way to spot a Pythagoras question is to look for right-angled triangles. However they are not always obvious. The question below is a Pythagoras question although it does not appear to have any right-angled triangle at first.

Example

Calculate the length of the line AB. Do not use a scale drawing.

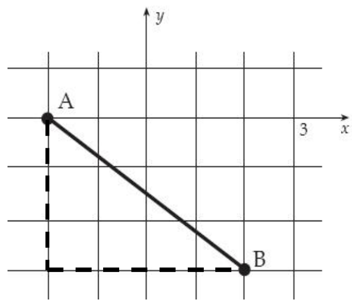


Solution

Can you see the right-angled triangle? Draw lines to complete the triangle.

The triangle has sides 3 squares and 4 squares.

Side AB is the hypotenuse, so we **add**

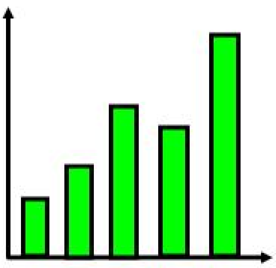


$$AB^2 = 3^2 + 4^2$$
$$AB^2 = 25$$
$$AB = \sqrt{25}$$
$$AB = \underline{5}$$

Unit 2 Outcome 4 – Graphs, Charts and Tables

Bar Graphs and Line Graphs

In an exam, you might be asked to draw a bar graph or a line graph. This means that you need to remember the difference between them:

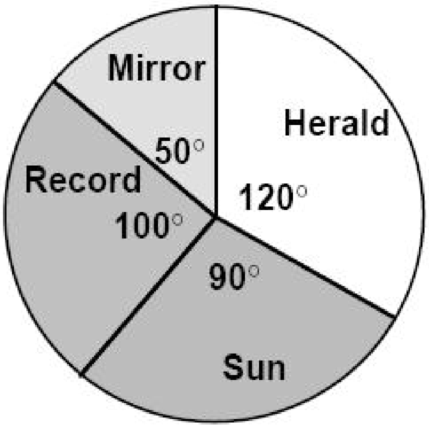


Questions will usually ask you to use the graph to make a comment. They often ask you to describe the **trend** of a graph. The trend is an overall description of what the graph is showing. For example in the two pictures above, the trend in the bar graph is that the figures are getting higher. The trend in the line graph is that the figures are getting lower.

Pie Charts

A pie chart shows how something is split up into different categories. If you know the **angle** of each slice, you can work out how the number that each slice represents, which in turn tells you the fraction.

e.g. this pie chart shows the results of a survey into newspapers that people buy:



The slice for *The Herald* is 120° out of 360°. As a fraction this is $\frac{120}{360}$ (we could simplify this if we chose to, but we don't need to).

The angles in a pie chart always add up to 360°, so the fraction will always be out of 360.

Example

1800 people were asked what newspaper they bought. The pie chart above shows the results. How many people bought *The Mirror*?

Solution

The slice for *The Mirror* is 50°, so the fraction of people who chose *The Mirror* is $\frac{50}{360}$.

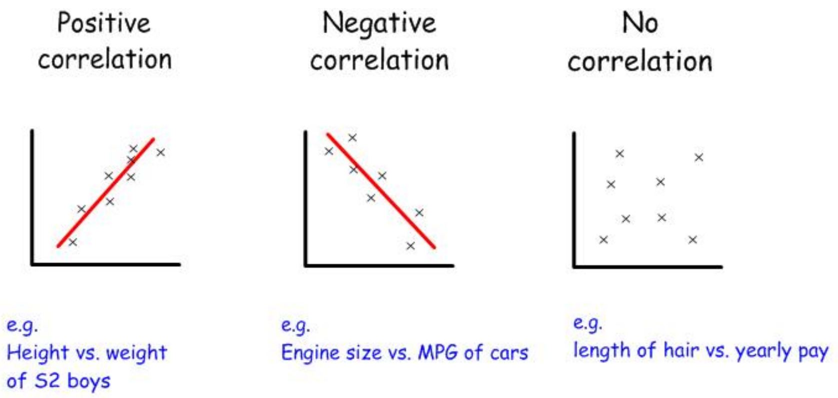
There were 1800 people in total, so we calculate $\frac{50}{360}$ of 1800.

$1800 \div 360 \times 50 = 250$, so **250 people said they bought *The Mirror***.

Scatter Graphs

A scatter graph is a way of displaying information and looking for a connection between two things.

Definition: the **correlation** between two sets of numbers refers to the relationship (if any) between the numbers. A scatter graph is good for showing correlation. Correlation can be **positive** (going up), **negative** (going down), or **none**.



Definition: a **line of best fit** is line drawn on to a scatter graph that shows the correlation of the graph. The straight lines drawn above for positive and negative correlation are examples of lines of best fit.

The line of best fit should go through the middle of the point, and should go in the same direction that the points are laid out on the page. **Do not "join the dots"!**

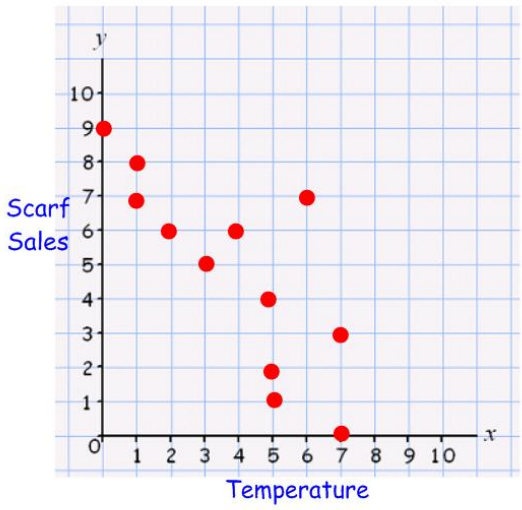
You will *always* be asked to draw the line of best fit in a scatter graph question in a maths exam. Once you have drawn the line, you will always be asked to *use it*.

Example

A gift shop records the temperature each day for 13 days. They also record how many scarves they sell each day. Show this information in a scatter graph.

	A	B	C	D	E	F	G	H	I	J	L	M
Temperature	5	4	3	5	7	5	7	1	1	6	2	0
Scarf sales	1	6	5	4	3	2	0	8	7	7	6	9

Solution



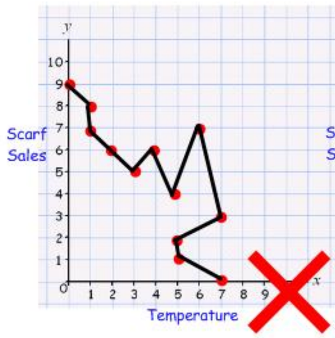
From this graph, we can see there is **negative correlation** between temperature and scarf sales.

Example

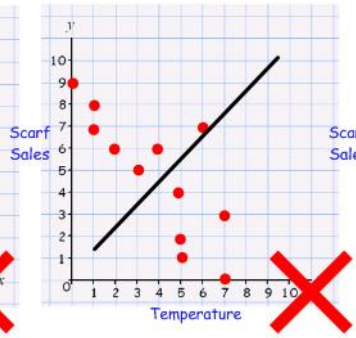
Draw a line of best fit on the scattergraph above

Solution

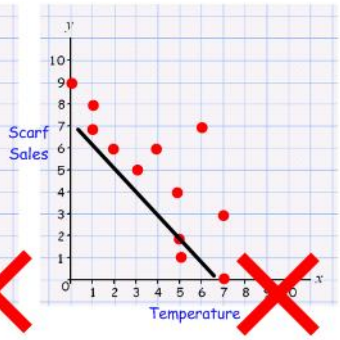
These lines of best fit would be marked **wrong**.



Joining the dots - WRONG

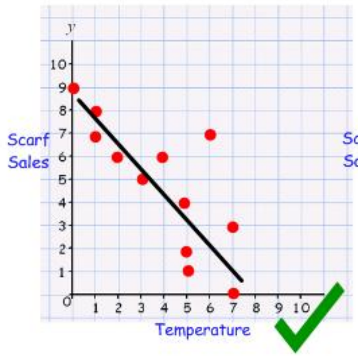


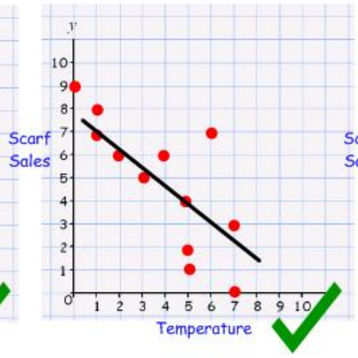
Not going in the same direction as the points - WRONG

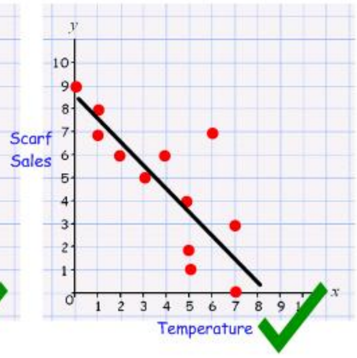


Not going through the middle of the points (too low) - WRONG

Any of these answers would be marked **correct** as they go roughly through the middle of the points, and roughly in the same direction as the points.







Example

On the next day, the temperature is 6°C. Using your line of best fit, estimate how many scarves the shop will sell.

Solution

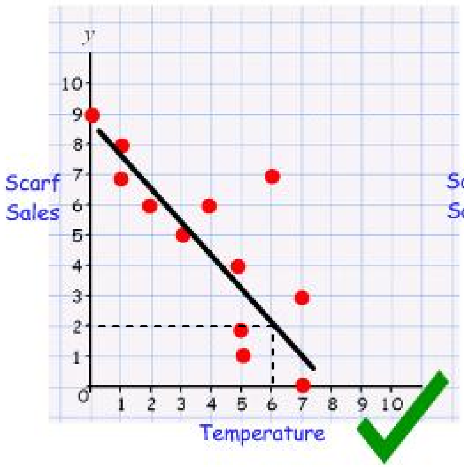
The key words here are *using your line of best fit*. If your answer matches with your line, you get the mark. If it doesn't match with your line, you don't get any marks. Simple as that.

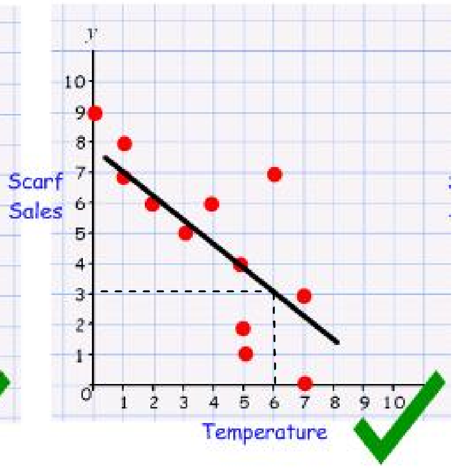
Incorrect answers:

- The answer is not 7 even though there is a point there. This is because the point is far away from the line, meaning it was unusual.

Possible Correct answers:

The correct answer will depend on your graph. You need to draw lines on your graph at 6°C, and to see where they meet the line of best fit. For the first two examples above, this would look like this:





If your line of best fit was the one on the left, your answer would be 2 scarves. If your line of best fit was the one on the right, your answer would be 3 scarves.

It does not matter that these answers are different – remember the question only asked for an *estimate*. The key thing is that it matches <u>your</u> line of best fit.

Stem and Leaf Diagrams

A Stem-and-leaf diagram is another way of showing data. The easiest way to make one is to make an unordered one first, and then to make a second, ordered diagram.

It must always have a **key**.

<u>Example:</u>

Class 1A's Test Results 56, 45, 37, 57, 82, 92, 36, 39, 54, 65, 67, 30, 48, 51, 59, 73, 86, 91, 58, 64, 37, 62

unordered stem-and-leaf diagram =>

3	7 6 9 0 7
4	5 8
5	6 7 4 1 9 8
6	5 7 4 2
7	3
8	2 6
9	2 1

5 | 7 represents 57

ordered stem-and-leaf diagram =>

3	0 6 7 7 9
4	5 8
5	1 4 6 7 8 9
6	2 4 5 7
7	3
8	2 6
9	1 2

5 | 7 represents 57

Unit 2 Outcome 5 – Statistics

Range

Definition: The **range** is the difference between the highest and the lowest numbers. It shows how spread out a list of numbers is.

This formula is NOT given on the exam paper

$$\text{Range} = \text{Highest} - \text{Lowest}$$

Example

Each day a shop records how much money it takes:
19, 42, 47, 45, 18, 36, 68, 22, 27, 35. What is the range?

Solution

Highest = 68, Lowest = 18
Range = 68 – 18 = **50**

Definition: the **mean**, **median** and **mode** are all ways of coming up with an "average value". They each have advantages and disadvantages.

Mean

If people are talking about "the average" in everyday life, they are probably referring to the mean.

This formula is NOT given on the exam paper

$$\text{Mean} = \frac{\text{Total}}{\text{How many}}$$

To find the mean:

1. add all the numbers together
2. divide by how many numbers there are

Example

For the list of numbers above, find the mean.

Solution

Step one – add all the numbers together: 19 + 42 + 47 + ….. + 35 = 359
Step two – there are ten numbers, so we divide by 10. 359÷10=35.9
Answer: the mean is **35.9**

IMPORTANT – if you type 19+42+47+45+18+36+68+22+27+35÷10 straight into a calculator, you will get the *wrong answer*. You have to either press equals before you divide, or you need to use brackets: (19+42+47+45+18+36+68+22+27+35)÷10. Always check your final answer sounds reasonable.

Mode

Definition: The **mode** is the most frequent number (the number that appears the most).

Example

Find the mode: 6 6 6 6 6 7 7 7 7 7 8 8 8 9 9 9 9 9 9 9 9 9 9 10 10 11 11 12

Solution

The mode is 9 because there are more 9s than any other number.

Median

Definition: The **median** is the middle number in the list once all the numbers have been written in order.

You can spot the median in a list, because it will have the same number of numbers before and after it. Drawing a line in the middle of the list can sometimes make this clearer. It is *essential* to put the list of numbers in order first.

Example 1

Find the median of 8, 6, 4, 2, 2, 5, 8

Solution:

Putting the numbers in order: 2, 2, 4, 5, 6, 8, 8

The arrow divides the list into two equal parts – because there are 3 numbers to the left and three numbers to the right.

The arrow goes straight through the number 5, so **the median is 5**.

Example 2

Find the median of 8, 6, 4, 3, 2, 6, 7, 8

Solution:

Putting the numbers in order: 2, 3, 4, 6, 6, 7, 8, 8

This time the arrow above is NOT dividing the list into two equal halves, as there are four numbers before the arrow, and only 3 after it. To make the two halves equal, the arrow has to go midway between two numbers:

2, 3, 4, 6, 6, 7, 8, 8

This time the arrow DOES divide the list into two equal halves, as there are four numbers either side of the arrow. The arrow is between 6 and 6. This means that **the median is 6**. (it is NOT 6.5)

Example 3

Find the median of 8, 10, 16, 19, 23, 12, 14, 16

Solution:

Putting the numbers in order: 8, 10, 12, 14, 16, 16, 19, 23

The arrow is midway between 14 and 16, because that leaves four numbers on either side. This means that the **median is 15** because 15 is midway between 14 and 16.

Mean from a Frequency Table

When you have lots of numbers, it can be confusing to give a large list of numbers.

e.g. the shoe sizes of Mr Carroll's P6 class:

2222222222333333333333444444445555556666777 8

An easier way to show this information is to use a frequency table:

Shoe Size	Frequency
2	10
3	12
4	8
5	6
6	4
7	3
8	1

From this table, you can tell the **mode** is 3 (not 12) because that is the row that has the biggest frequency.

You can also see that the **range** is 6 (highest – lowest = 8 – 6 = 2).

To work out the **median**, you need to rewrite the list of numbers back out in full:

2222222222333333333333444444445555556666777 8

Here the median is midway between 3 and 4, so the median is 3.5

However exam questions usually ask you to work out the **mean**. The table allows you to do this quickly by adding an extra column to multiply the numbers (in this case shoe size × frequency) and a **Total row**. This column will usually be added for you in the exam paper.

Example

Complete the table and use it to calculate the mean shoe size

Shoe Size	Frequency	Shoe Size × Frequency
2	10	
3	12	
4	8	
5	6	
6	4	
7	3	
8	1	
TOTAL	44	

Solution

Completing the table gives

Shoe Size	Frequency	Shoe Size × Frequency
2	10	20
3	12	36
4	8	32
5	6	30
6	4	24
7	3	21
8	1	8
TOTAL	44	171

Using the formula for the mean, $\text{Mean} = \dfrac{\text{Total}}{\text{How many}}$. The total is 171. How many is 44 because there were 44 numbers in the original list before we made the table (see previous page) and **not** 7 (the number of rows).

$$\text{Mean} = \frac{\text{Total}}{\text{How many}} = \frac{171}{44} = 3.89 \text{ (2d.p.)}$$

Always check your answer sounds reasonable

Probability

Definition: Probability is a measure of how likely an event is:

- If the event is **impossible**, the probability is 0
- If the event is **certain** to happen, the probability is 1
- If the event is between certain and uncertain, the probability is given as a fraction.

Example

In a bag, there are 20 white balls, 30 black balls and 9 blue ones. Taz picks a ball at random. What is the probability that she picks a white ball?

Solution

In total there are 20+30+9=59 balls.
20 of these are white.
So the probability is $\frac{20}{59}$.

Comparing Statistics

The **mean**, **median** and **mode** are averages. They say whether a list of numbers is higher or lower on average.

The **range** is NOT an average. Instead it is a measures of spread. It says whether a list of numbers is more or less spread out/consistent.

Example

The temperature in Aberdeen has a mean of 3°C and a range of 5. In London it has a mean of 9°C and a range of 3. Compare the temperatures in London and Aberdeen.

Solution

You would get NO MARKS (as you are stating the obvious) for:
"Aberdeen has a lower mean", "London has a higher mean", "Aberdeen has a higher range", "London has a lower range".
You WOULD get marks (as you say what the numbers MEAN) for:
"The temperature in Aberdeen is lower and the temperature is less consistent"
"The temperature in London is higher and more consistent" or similar

Unit 3

Unit 3 Outcome 1 – Algebra

Multiplying out Brackets

Multiply everything inside the brackets by the number outside the brackets.

Examples

Multiply out the brackets:

$\mathbf{3(x+5)}$ $\qquad$ $\mathbf{y(y+3)}$

Solution

$$3(x+5)$$
$$=\underline{3x+15}$$

$$y(y+3)$$
$$=\underline{y^2+3y}$$

Some times you need to simplify if there are other letters or numbers outside the brackets. You always have to simplify if you are able to, even if you are not explicitly told to. *Only* numbers inside the bracket are multiplied. Anything else that is not inside the bracket should remain unchanged until you start simplifying.

Examples

Multiply out the brackets and simplify:

$\mathbf{4(m+5)-18}$ $\qquad$ $\mathbf{4(x+5)+3(x-2)}$

Solution

$$4(m+5)-18$$
$$=4m+20-18$$
$$=\underline{4m+2}$$

$$4(x+5)+3(x-2)$$
$$=4x+20+3x-6$$
$$=\underline{7x+14}$$

However, be careful the <u>only</u> numbers (or letters) that you multiply by are ones that are right next to the bracket. Anything else that is not inside the bracket should remain unchanged until you start simplifying.

Examples

Multiply out the brackets and simplify:

$\mathbf{4+7(a+2)}$ $\qquad$ $\mathbf{2x+x(x+1)}$

Solution

$$4+7(a+2)$$
$$=4+7a+14 \quad (\underline{\textbf{NOT}}\ 11(a+2))$$
$$=\underline{18+7a} \quad (\text{or } 7a+18)$$

$$2x+x(x+1)$$
$$=2x+x^2+x \quad (\underline{\textbf{NOT}}\ 3x(x+1))$$
$$=\underline{x^2+3x} \quad (\text{or } 3x+x^2)$$

Factorising

Definition: Factorise means "put the brackets back in". You can think of it as the opposite of multiplying out the brackets.

Example

Factorise $6a + 9b$ **Factorise $3x + xy$**

Solution

Factorise $6a+9b$	Factorise $3x+xy$
The biggest common factor is 3	The common factor is x
Take 3 outside the brackets	Take x outside the brackets
$3(\quad)$	$x(\quad)$
Work out what goes inside the brackets	
Answer: $\underline{3(2a+3b)}$	Answer: $\underline{x(3+y)}$

You always need to take the ***largest possible*** number (and/or letter) outside the brackets. You can spot these questions as they will say **factorise fully** instead of just **factorise**.

Example

Factorise fully: $18x + 24$

Solution

You could answer $2(9x+12)$ or $3(6x+8)$. However you would not get full marks as the biggest number that goes into both 18 and 24 is 6. This means that 6 needs to be outside the bracket.

Answer: $\underline{6(3x+4)}$

Some expressions have letters *and* numbers as factors.

Example

Factorise fully: $6x^2 + 9x$

Solution

The largest number that goes into both 6 and 9 is 3. So 3 goes outside the brackets.
The largest letter that goes into both x^2 and x is x. So x also goes outside the brackets.

This means that $3x$ is outside the brackets: $3x(\quad)$

Answer: $\underline{3x(2x+3)}$

Equations

You need to be able solve simple equations. The method you will have been taught is to *change side and do the opposite*. You have to use a method to get the answer – ***if you just write the answer down (even if you think it is obvious) you will get no marks.***

You should always check your final answer so that you know it is correct.

Example

Solve algebraically the equation $7m - 1 = 41$

Solution

$$7m - 1 = 41$$

Step one – move the '–1' over to the other side to become '+1'

$$7m = 41 + 1$$
$$7m = 42$$

Step two – divide by the 7
Step three – write down the answer
Step four – check that 7×6–1 does equal 41.

$$7m = \tfrac{42}{7}$$
$$m = 6$$

Example

Solve algebraically the equation $8a + 4 = 34$

Solution

$$8a + 4 = 34$$

Step one – move the '+4' over to the other side to become '–4'

$$8a = 34 - 4$$
$$8a = 30$$

Step two – divide by the 8
Step three – write down the answer
Step four – check that 8×3.5+4 does equal 34.

$$8a = \tfrac{30}{8}$$
$$m = 3.75$$

Inequalities)

Inequalities (also known as Inequations) can be solved in exactly the same way as for equations. The only difference is that you do not have a '=' sign in the middle.

Example

Solve algebraically the inequation $2y - 2 < 7$

Solution

Step one – move the '–2' over to the other side to become +2'

$$2y - 2 < 7$$
$$2y < 7 + 2$$

Step two – divide by the 2

$$2y < 9$$
$$y < \tfrac{9}{2}$$
$$y < 4.5$$

Step three – write down the answer.

Example

Solve algebraically the inequation $\frac{1}{2}a - 3 \geq 5$

Solution

Step one – move the '–3' over to the other side to become +3'

Step two – the opposite of ½ is doubling, so ×2

Step three – write down the answer.

$$\frac{1}{2}a - 3 \geq 5$$
$$\frac{1}{2}a \geq 5 + 3$$
$$\frac{1}{2}a \geq 8$$
$$a \geq 8 \times 2$$
$$a \geq 16$$

Equations with letters on both sides

In your exam, you will have to solve an equation that has letters on both sides.

*[**Before you start** (optional) – write the "invisible plus signs" in, in front of anything that does not have a sign in front of it already, two remind you it is positive.]*
Step one – move everything with a letter in to the left-hand side, and all the numbers to the right-hand side, remembering to "change side and do the opposite".
Step two – simplify each side
Step three – solve the resulting equation
Final step – check your answer works by substituting it back in to both sides of the original equation and checking both sides give the same answer.

Example 1

Solve algebraically the equation $5y - 5 = 3y + 9$

Solution

Optional first step *– write in "invisible plus signs" in front of anything that does not already have a sign*

Step one – move the +3y over to the left-hand side where it becomes –3y. Move –5 to the right-hand side where it becomes +5.

Step two – simplify both sides
Step three – divide to get the final answer

$$5y - 5 = 3y + 9$$
$$+5y - 5 = +3y + 9$$
$$+5y - 3y = +9 + 5$$
$$2y = 14$$
$$y = \frac{14}{2}$$
$$y = 7$$

Final step: check, by substituting y=7 into the original equation
The left-hand side is 5y–5. If we replace y with 7, we get 5×7–5, which equals 30.
The left-hand side is 3y+9. If we replace y with 7, we get 3×7+9, which equals 30.

These are the same, so our answer has to be correct.

Example 2

Solve algebraically the equation $2a + 5 = 15 - 2a$

Solution

Optional first step *– write in "invisible plus signs" in front of anything that does not already have a sign*

$$2a + 5 = 15 - 2a$$
$$+2a + 5 = +15 - 2a$$

Step one – move the –2a over to the left-hand side where it becomes +2a. Move +5 to the right-hand side where it becomes –5.

$$+2a + 2a = +15 - 5$$

Step two – simplify both sides

$$4a = 10$$

Step three – divide to get the final answer

$$a = \tfrac{10}{4}$$
$$a = 10 \div 4 = \underline{2.5}$$

Final step: check, by substituting a=2.5 into the original equation
The left-hand side is 2a+5. If we substitute a=2.5, we get 2×2.5+5, which equals 10.
The left-hand side is 15–2a. If we substitute a=2.5, we get 15–2×2.5, which equals 10.

These are the same, so our answer has to be correct.

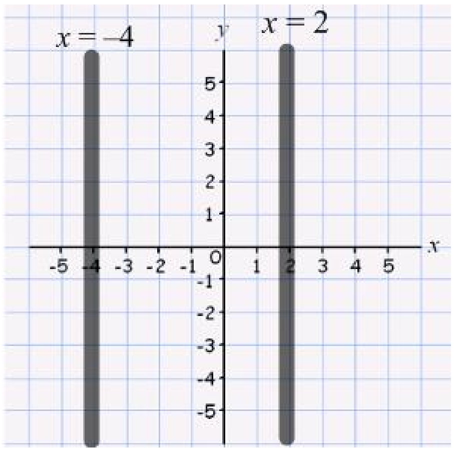
Unit 3 Outcome 2 – Straight Line Graphs

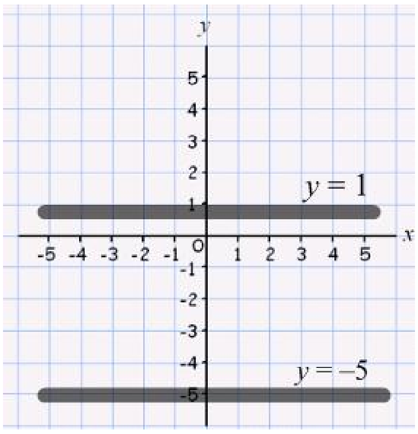
Given an equation such as $y = 3x - 5$ or $y = 10 - \frac{1}{2}x$, we can use algebra to come up with points to plot on a coordinate grid. These points will always lie on a straight line. If we join the points together, we say we have drawn "the graph of the equation".

Vertical and Horizontal Lines

The simplest lines are vertical and horizontal ones. They are very easy to draw. However because we spend most of the time talking about the more complicated lines, people tend to forget these ones.

Vertical lines have equations such as $x = 2$ or $x = -4$ (we say that these lines have equations "of the form $x = a$" where *a* can be any number). The line $x = 2$ is a line going vertically through 2 on the *x* axis. The line $x = -4$ is a line going vertically through –4 on the *x* axis. (see diagram on the right)





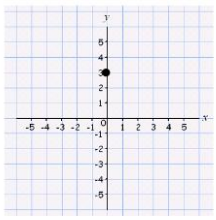
Horizontal lines have equations such as $y = 1$ or $y = -5$ (we say that these lines have equations "of the form $y = b$" where *b* can be any number). The line $y = 1$ is a line going horizontally through 1 on the *y* axis. The line $y = -5$ is a line going horizontally through –5 on the *y* axis. (see diagram on the left)

Example

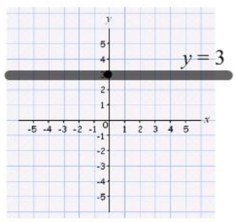
Draw the line with equation $y = 3$

Solution

$y = 3$ has to go through number 3 on the *y*-axis, so we can mark that point straight away.



To go through that point, the line has to be horizontal, so we can draw the line and complete the graph.



We must remember to label the graph $y = 3$.

Drawing a Straight line from its equation

Other straight line equations that have x in are a little more complicated to do. We have to use a table of values to work out what y is for different values of x.

When doing a table of values, we can choose any values of x that we want to. However it makes sense to choose simple numbers: 0, 1, 2, 3 is a common choice. However in an exam they will tell you which numbers to use, and they will usually make it harder for you by including a negative value.

Example

Draw the straight line with equation $y = 3x - 4$

Solution

Step one – draw up a table of values

x	0	1	2	3
y				

Step two – use the equation to work out what y is for each value of x. You should *always* write your working down for more difficult examples (e.g. exam questions).

a) when x=0,
$$\begin{aligned} y &= 3x - 4 \\ &= 3 \times 0 - 4 \\ &= -4 \end{aligned}$$

x	0	1	2	3
y	–4			

b) when x=1,
$$\begin{aligned} y &= 3x - 4 \\ &= 3 \times 1 - 4 \\ &= -1 \end{aligned}$$

x	0	1	2	3
y	–4	–1		

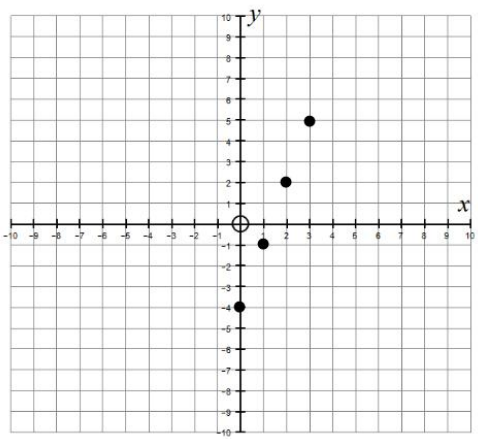
c) In a similar way, when x=2, y=2 and when x=3, y=5:

x	0	1	2	3
y	–4	–1	3	5

Step three – every column in this table of values now gives you a coordinate. Write these down

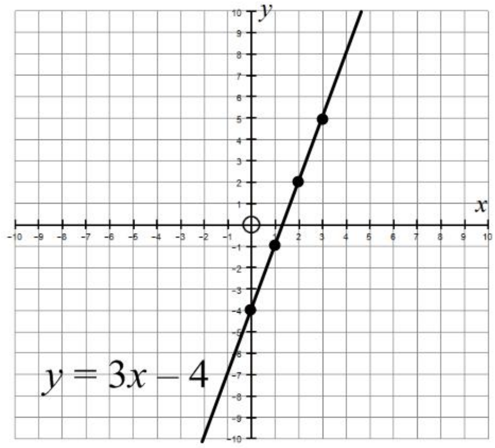
x	0	1	2	3
y	–4	–1	3	5
	(0,–4)	(1, –1)	(2,3)	(3,5)

Step four – plot these points on a coordinate grid. They should lie in a straight line. ***If they don't, you have made a mistake – go back and check step two carefully***.



Step five – draw a line through the points, making sure the line goes all the way from the top to the bottom of the grid.

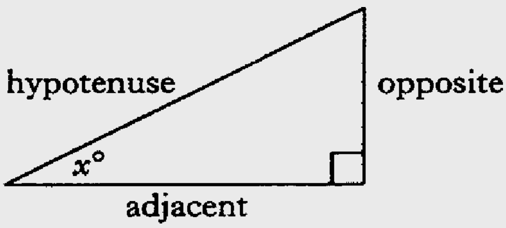
Label your line with its equation.



Unit 3 Outcome 3 – Trigonometry (SOH CAH TOA)

These formulae are given on the exam paper

Trigonometric ratios in a right angled triangle:



$$\tan x° = \frac{\text{opposite}}{\text{adjacent}}$$

$$\sin x° = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos x° = \frac{\text{adjacent}}{\text{hypotenuse}}$$

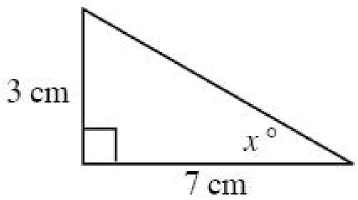
There is always a question on trigonometry in right angled triangles on the calculator paper. You will be asked to work out the length of a side or the size of an angle.

Calculating an Angle

Example

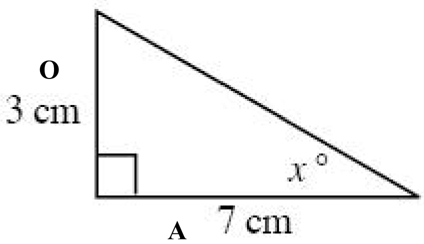
Find the size of angle x in this right-angled triangle.

Do not use a scale drawing.



Solution

Step one – label the sides O, H or A. Do not bother labelling the side that we do not know the length of (in this case it is H), as we don't need it.



Step two – here we have the *opposite* and the *adjacent*. Now use SOH CAH TOA to tick off the sides you have:

√ √ √√

SOH CAH TOA

TOA has two ticks above it, telling us to use *tan* in this question.

Step three – copy out the formula carefully $\tan x = \frac{\text{opposite}}{\text{adjacent}}$

Step four – substitute the numbers 3 and 7 in the correct places $\tan x = \frac{3}{7}$

Step five – use $\tan^{-1}$ (or $\sin^{-1}$ or $\cos^{-1}$) to find the angle

$$x = \tan^{-1}(3 \div 7)$$

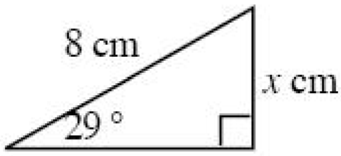
$$x = 23.198... = \underline{23.2°} \text{ (1d.p.)}$$

Working out a length

Example

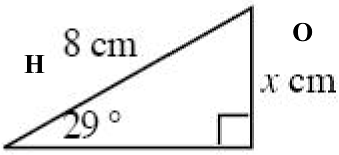
Find the size of angle *x* in this right-angled triangle.

Do not use a scale drawing.



Solution

Step one – label the sides O, H or A. One side will have nothing written on it – do not bother labelling that side (in this case it is A) as we do not need it.



Step two – we have the *opposite* and the *hypotenuse*. Now use SOH CAH TOA to tick off the sides you have:

√√ √ √

SOH CAH TOA

SOH has two ticks above it, telling us to use *sin* in this question.

Step three – copy out the formula carefully

Step four – substitute the numbers 29 and 8 in the correct places. ***The angle has to go straight after sin (or cos or tan)***

Step five – multiply to get the answer

$$\sin x = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 29 = \frac{x}{8}$$

$$x = 8 \times \sin 29$$

$$x = 3.878... = \underline{3.88\,\text{cm}} \text{ (2d.p.)}$$

Exam Questions

During the exam the Trigonometry question is almost always hidden, look hard because it will definitely always be in the exam – and always in paper 2.

One way you can spot a SOH CAH TOA question is by watching out for the phrase **"Do not use a scale drawing"**. This phrase usually indicates that the question is either Pythagoras or SOH CAH TOA:

- If there is <u>any</u> angle (other than the right angle) in the question, then use SOH CAH TOA
- If there are <u>no angles</u> involved in the question (only lengths), then use Pythagoras

Unit 3 Outcome 4 – Standard Form

Definition: Standard form (also known as Scientific notation) is a quicker way of showing really large or really small numbers.

Definition: a number written normally is said to be "in **normal form**".

A really small number (one that begins 'zero point…' e.g. 0.02 or 0.00015) will have a *negative* number in the power when written in standard form

Examples

Normal Form	Standard Form
92 000 000	9.2×10^7
0.000456	4.56×10^{-4}
305 000	3.05×10^5

Changing Numbers from Standard Form into Normal Form

The basic rule for changing numbers back into normal form (or "writing a number in full") is to move the decimal point the number of places shown by the power.

If the power is positive, move the point to the right, if it negative, move to the left.

Example

Change 6.3×10^5 into normal form

Solution

Start with the number 6.3. Move the decimal point 5 places to the right.

Answer: 630 000

Example

Write 2.15×10^{-3} in full

Solution

Start with the number 2.15. Move the decimal point 3 places to the left.

Answer: 0.00215

Example

Change 4×10^8 into normal form

Solution

Start with the number 4. This has no decimal point, so we use 4.0 instead.
Move the decimal point 8 places to the right.

Answer: 400 000 000

Changing Numbers from Normal Form into Standard Form

The number before the times sign *has to be* between 1 and 10. An easy way of making this number is to put a point after the first digit**:**

- e.g. for 235000, we use 2.35 (and not 235)
- e.g. for 70543, we use 7.0543
- e.g. for 0.00027, we use 2.7

Example

Change 85000 into standard form

Solution

Step one – putting a decimal point after the first digit gives us 8.5. So our number is $8.5\times10^?$

Step two – count how many times you have to move the decimal point to go from 8.5 to 85000. The answer is 4.

Answer: **8.5×10^4**

Example

Write 0.0027 in standard form

Solution

Step one – putting a decimal point after the first digit gives us 2.7. So our number is $2.7\times10^?$

Step two – count how many times you have to move the decimal point to go from 2.7 to 0.0027. The answer is 3. However we are going backwards to do this as the number is less than zero, so the power has to be negative (i.e. it is –3)

Answer: **2.7×10^{-3}**

Example

Write 4008500 in standard form

Solution

Step one – putting a decimal point after the first digit gives us 4.0085. So our number is $4.0085\times10^?$

Step two – count how many times you have to move the decimal point to go from 4.0085 to 4008500. The answer is 6.

Answer: **4.0085×10^6**

Index of Key Words