Higher Ink Exercise

Block 2 - Integration 1

<u>Calculators should only be used when necessary</u>

1. Find the anti-derivative of each:

a) $f'(x) = 12x^3$ b) $f'(x) = 6x^4$ c) $\frac{dy}{dz} = z^{-31}$ d) $\frac{dy}{dh} = 4h^{3/5}$

[5]

2. Integrate, expressing all answers with positive powers:

a) $\int x^9\, dx$ b) $\int t^{-5}\, dt$ c) $\int x^{7/8}\, dx$ d) $\int \frac{-\frac{5}{6}\, du}{u^3}$

[5]

3. Evaluate

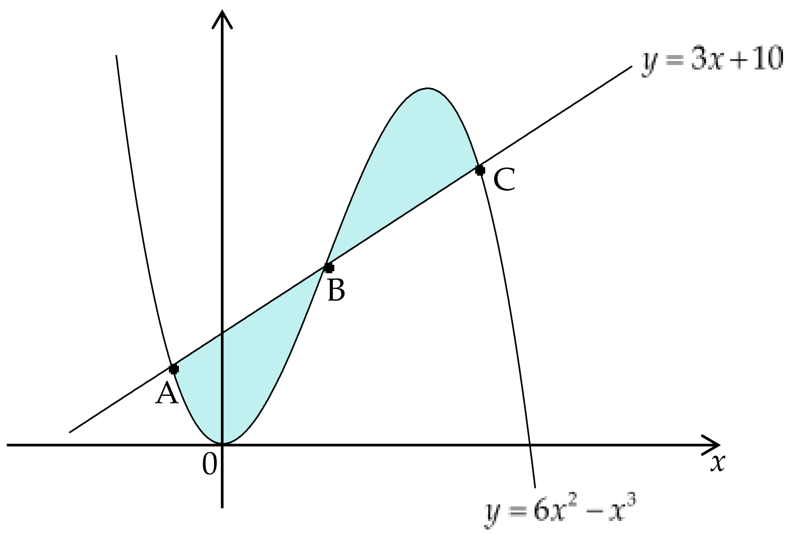
a) $\int_1^4 \frac{3x+1}{\sqrt{x}} dx$ b) $\int_1^2 \frac{u+2}{x^3}\, du$

[7]

4a) i. Show that $(x - 2)$ is a factor of $x^3 - 6x^2 + 3x + 10$ [1]

ii. Hence factorise $x^3 - 6x^2 + 3x + 10$

[2]



The line with equation $y = 3x + 10$ intersects the curve with equation $y = 6x^2 - x^3$ at the points A, B and C.

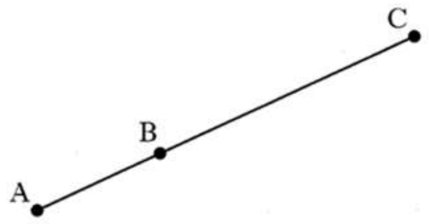
b) Find the x-coordinates of the points A, B and C. [3]

c) Calculate the total shaded area shown in the diagram. [10]

5. Relative to a suitable co-ordinate system A and B are the points (-2, 1, -1) and (1, 3, 2) respectively.

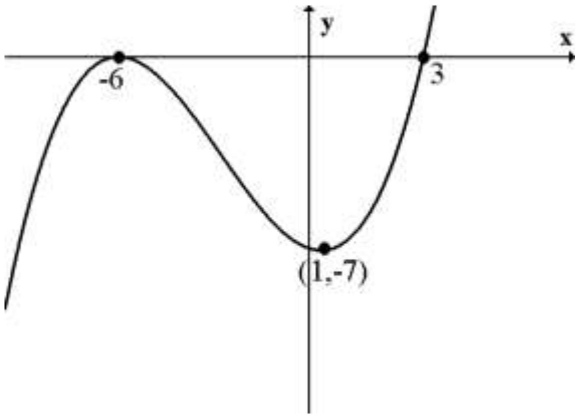
A, B and C are collinear points and C is positioned such that BC = 2AB.

Find the coordinates of C.



[4]

6. Part of the graph of y = g(x) is shown.



On separate diagrams sketch the graphs of

(i) $y = -3g(x)$
(ii) $y = g(x - 6)$
(iii) $y = g'(x)$

[4]

Total [41]

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Marking Scheme

Question 1
a)
- $f(x) = \frac{12x^4}{4} = 3x^4$

b)
- $f(x) = \frac{6x^5}{5}$

c)
- $y = \frac{z^{-30}}{-30}$

d)
- $y = \frac{4h^{8/5}}{8/5}$
- $= \frac{20h^{8/5}}{8} = \frac{5h^{8/5}}{4}$

Question 2
a)
- $\frac{x^{10}}{10} + C$

b)
- $\frac{t^{-4}}{-4} + C$

c)
- $\frac{x^{15/8}}{15/8} + C = \frac{8x^{15/8}}{15} + C$

d)
- $-\frac{5}{6}u^{-3}\,du$
- $\frac{-\frac{5}{6}u^{-2}}{-2} + C = \frac{5u^{-2}}{12} + C$

Question 3
a)
- $\int_1^4 (3x+1)x^{-½}\,dx$
  $= \int_1^4 3x^{-½} + x^{-½}\,dx$
  $= \int_1^4 4x^{-½}\,dx$
- $= [4x^{½}/½]_1^4$
  $= [8x^{½}]_1^4$
- $= 8(4)^{½} - 8(1)^{½}$
  $= 8$

b)
- $\int_1^2 (u+2)u^{-3}\,du$
  $= \int_1^2 u^{-2} + 2u^{-3}\,du$
- $= [u^{-1}/-1 + 2u^{-2}/-2]_1^2$

$= [-u^{-1} - u^{-2}]_1^2$

- $= [-(2)^{-1} - (2)^{-2}] - [-(1)^{-1} - (1)^{-2}]$
  $= [-\frac{1}{2} - \frac{1}{4}] - [-1-1]$
- $\frac{5}{4}$

<u>Question 4</u>

a)

- i. (shows using valid strategy)
- ii. $(x - 2)(x^2 - 4x - 5)$
- $= (x - 2)(x - 5)(x + 1)$

b)

- $3x + 10 = 6x^2 - x^3$
- $x^3 - 6x^2 + 3x + 10 = 0$
- $\therefore A = -1, B = 2, C = 5$

c)

- $\int_{-1}^{2} x^3 - 6x^2 + 3x + 10 \, dx$
- $\int_{2}^{5} 6x^2 - x^3 - 3x - 10 \, dx$