

Higher Ink Exercise
Block 2 - Integration 2

Calculators should only be used when necessary

1. Integrate, expressing all answers with positive powers:

a) $\int (5 + 3x)^2 dx$ b) $\int (x + 2)^3 dx$ c) $\int 8\cos x dx$ d) $\int (\sin 5t + \cos (-3t)) dt$
[5]

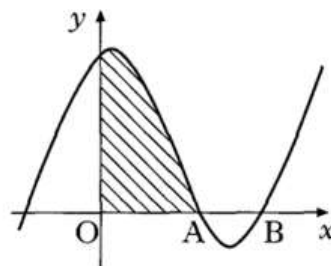
2. Evaluate

a) $\int_1^3 (x + 2)^3 dx$ b) $\int_{-1}^{5/6} (5 - 6t)^5 dt$ c) $\int_{3\pi/4}^{5\pi/4} \cos 2x dx$

[9]

3. The diagram shows a sketch of the graph of $y = x^3 - 4x^2 + x + 6$.

- a) Show that the graph cuts the x-axis at (3,0).
b) Hence or otherwise find the coordinates of A.
c) Find the shaded area.



[9]

4. Acceleration is defined as the rate of change of velocity.

An object is traveling in a straight line.

The velocity, v m/s, of this object, t seconds after the start of the motion, is given by:

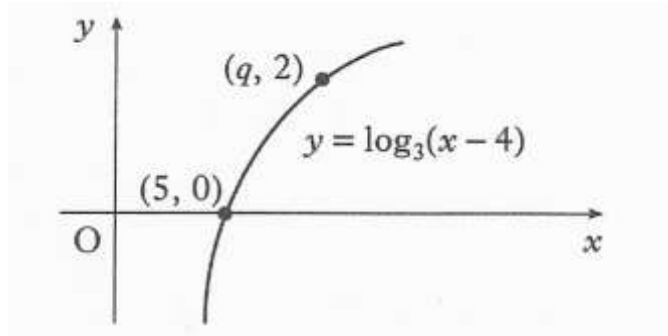
$$v(t) = 8\cos\left(2t - \frac{\pi}{2}\right)$$

- a) Find a formula for $a(t)$, the acceleration of this object, t seconds after the start of the motion.
b) Determine whether the velocity is increasing or decreasing when $t = 10$.
c) Velocity is defined as the rate of change of displacement. Determine a formula for $s(t)$, the displacement of the object, given that $s(t) = 4$ when $t = 0$.

[8]

5. The diagram shows part of the graph of $y = \log_3(x - 4)$.
The point $(q, 2)$ lies on the graph.

What is the value of q ?



[2]

6. Given that the lines $x + 4y = 7$, $3x + y = 10$ and $x - 5y + a = 0$ are concurrent, find the value of a .

[5]

Total [38]

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Marking Scheme

Question 1

a)

- $\frac{1}{9} (5+3x)^3 + C$

b)

- $\frac{1}{4} (x+2)^4 + C$

c)

- $8\sin x + C$

d)

- $-\frac{1}{5}\cos 5t$

- $\dots - \frac{1}{3}\sin(-3t) + C$

Question 2

a)

- $\left[\frac{1}{4}(x+2)^4\right]_1^3$

- $\frac{1}{4}((3)+2)^4 - \frac{1}{4}((1)+2)^4$

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b)

- $\left[\frac{1}{-36}(5-6t)^6\right]_{-1}^{5/6}$

- $\left[\frac{1}{-36}(5-6(5/6))^6\right] - \left[\frac{1}{-36}(5-6(-1))^6\right]$

- $\frac{1}{36}$

c)

- $\left[\frac{1}{2}\sin 2x\right]_{\frac{3\pi}{4}}^{\frac{5\pi}{4}}$

- $\frac{1}{2}\sin 2\left(\frac{5\pi}{4}\right) - \frac{1}{2}\sin 2\left(\frac{3\pi}{4}\right)$

- 1

Question 3

a)

- no remainder $\therefore (x-3)$ is a factor and 3 is a root

b)

- $(x-3)(x^2-x-2)$

- $(x-3)(x-2)(x+1)$

- $\therefore A = (2,0)$

c)

- $\int_0^2 x^3 - 4x^2 + x + 6 dx$

- $\left[\frac{x^4}{4} - \frac{4x^3}{3} + \frac{x^2}{2} + 6x\right]_0^2$

- $\frac{(2)^4}{4} - \frac{4(2)^3}{3} + \frac{2^2}{2} + 6(2)$
- $4 - \frac{32}{3} + 2 + 12$
- $9\frac{1}{3}$

Question 4

a)

- $a(t) = v'(t)$
 $= -8\sin(2t - \frac{\pi}{2})2$
- $= -16(2t - \frac{\pi}{2})$

b)

- $a(10) = -16\sin(2(10) - \frac{\pi}{2})$
 $= 6.5$
- $a(10) > 0 \Rightarrow$ velocity is increasing

c)

- $s(t) = \int v(t)$
 $= 4\sin(2t - \frac{\pi}{2}) + C$
- $s(0) = 4$
 $4\sin(2(0) - \frac{\pi}{2}) + C$
- $C = 8$
- $s(t) = 4\sin(2t - \frac{\pi}{2}) + 8$

Question 5

- $2 = \log_3(q - 4)$
 $3^2 = (q - 4)$
- $q - 4 = 9$
 $q = 13$

Question 6

- $x + 4y = 7$
 $3x + y = 10$
- $3x + 12y = 21$
- $11y = 11$
 $y = 1$
- $3x + 1 = 10$
 $3x = 9$
 $x = 3$
- $(3) - 5(1) + a = 0$
- $a = 12$