

Higher Ink Exercise
Block 2 - Recurrence

Calculators should only be used when necessary

1. A job offers a starting salary of £30000 with an annual percentage increase of 3.5% plus an annual increment of £1500.

- a) Find the recurrence relation for the total annual salary. [1]
b) Calculate the expected salary 5 years after starting. [2]

2. A 60 litre water tank is leaking at a rate of 22% of its volume per day. In an attempt to make up for this, 5 litres of water are added to the tank each day.

- a) Find the recurrence relation to describe this. [1]
b) Calculate the volume of water in the tank after 1 week. [2]
c) If the water in the tank falls below 20 litres there will not be enough water. If this situation is to continue, will there be enough water in the tank? [3]

3. Two sequences are generated by the recurrence relations

$$u_{n+1} = 0.4u_n + 8.4$$

$$v_{n+1} = kv_n + 2$$

The two sequences approach the same limit as $n \rightarrow \infty$.

- a) Evaluate this limit. [2]
b) Hence determine the value of k . [2]

4. A recurrence relation is defined as $u_{n+1} = 0.6u_n + b$ $u_0 = 2$

- a) Given the limit of this relation is 10, find b . [2]
b) Calculate the value of u_4 . [2]

5. Find the equation of the straight line perpendicular to the line with equation $2x + 4y = 20$ which has an x-intercept of 6. [3]

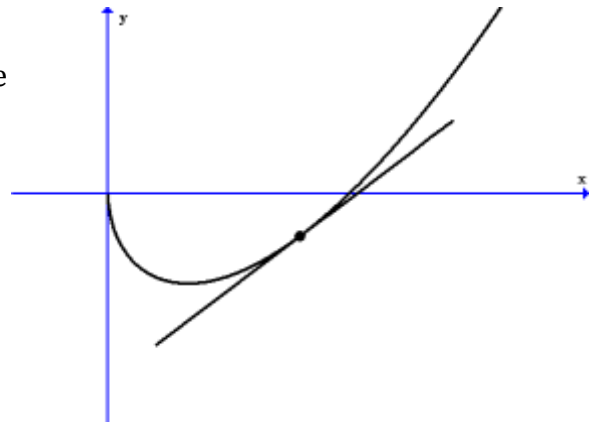
6. One of the roots of $x^3 - 2x^2 - px + 12 = 0$ is 4.

Calculate p and hence find the other roots of this polynomial. [5]

7. Find the equation of the tangent to the curve

$$y = \frac{2x^2 - 10x}{\sqrt{x}} \quad \text{where } x \in \mathbf{R}$$

at the point where $x = 4$



[5]

Total [30]

Higher Ink Exercise
Block 2 – Recurrence

Marking Scheme

Question 1

a)

- $u_{n+1} = 1.035u_n + £1500$

b)

- $u_0 = £30000$

$$u_1 = 1.035 \times £30000 + £1500 = £32550$$

- $u_2 = £35189.25$

$$u_3 = £37920.87$$

$$u_4 = £40748.10$$

$$u_5 = £43674.29$$

Question 2

a)

- $u_{n+1} = 0.78u_n + 5$

b)

- $u_0 = 60$

$$u_1 = 0.78 \times 60 + 5 = 51.8$$

- ...

$$u_7 = 29.27$$

c)

- Limit exists since $-1 < 0.78 < 1$

- $L = \frac{b}{1-a} = \frac{5}{1-0.78} = 22.73L$

- \therefore There will be enough water in the tank

Question 3

a)

- $L = \frac{b}{1-a} = \frac{8.4}{1-0.4} = 14$

b)

- $\frac{2}{1-k} = 14$

- $14(1-k) = 2$

$$14 - 14k = 2$$

$$-14k = -12$$

$$k = \frac{-12}{-14} = \frac{6}{7} \approx 0.86$$

Question 4

a)

- $\frac{b}{1-0.6} = 10$

- $\frac{b}{0.4} = 10$
 $b = 10 \times 0.4 = 4$

b)

- $u_0 = 2$
 $u_1 = 0.6 \times 2 + 4 = 5.2$
- ... $u_4 = 8.9632$

Question 5

- $4y = -2x + 20$
 $y = -\frac{2}{4}x + 5$
 $y = -\frac{1}{2}x + 5$
- $m = -\frac{1}{2} \therefore m_{\text{perp}} = 2$
- $y - 0 = 2(x - 6)$
 $y - 0 = 2x - 12$
 $2x - y - 12 = 0$

Question 6

-
- $p = 11$ (by valid strategy)
- $(x - 4)(x^2 + 2x - 3) = 0$
- $(x - 4)((x - 3)(x + 1)) = 0$
- \therefore roots at 4, 3 and 1

Question 7

- $y = (2x^2 - 10x)/\sqrt{x}$
 $= (2(4)^2 - 10(4))/\sqrt{4}$
 $= -4$
 \therefore point of tangent = (4, -4)
- $y = (2x^2 - 10x)x^{-1/2}$
 $= 2x^{3/2} - 10x^{1/2}$
- $\frac{dy}{dx} = 3x^{1/2} - 5x^{-1/2}$
 $= 3\sqrt{x} - 5/\sqrt{x}$
- $m = 3\sqrt{4} - 5/\sqrt{4}$
 $= 6 - 5/2$
 $= 3/5$
- $y + 4 = 3/5(x - 4)$
 $5y + 20 = 3(x - 4)$
 $5y + 20 = 3x - 12$
 $3x - 5y - 32 = 0$