

# Springburn Academy : Mathematics Department

## Higher Mathematics : Lesson Starters

Block 2

### Integration

Only use a calculator if necessary.

#### Task 1

- 1 Show that  $(x - 1)$  is a factor of  $f(x) = x^3 - 6x^2 + 9x - 4$  and find the other factors.
- 2 ABCD is a parallelogram. A, B and C have coordinates (2, 3), (4, 7) and (8, 11). Find the equation of DC.
- 3 The roots of the equation  $(x - 1)(x + K) = -4$  are equal. Find the values of  $k$ .
- 4 Evaluate  $\log_2 2 + \log_2 50 - \log_2 4$ .

#### Task 2

- 1 The point  $P(-1, 7)$  lies on the curve with equation  $y = 5x^2 + 2$ . Find the equation of the tangent to the curve at  $P$ .
- 2 Given that  $f(x) = 5(7 - 2x)^3$ , find the value of  $f'(4)$ .
- 3 Find  $k$  if  $(x - 2)$  is a factor of  $x^3 + kx^2 - 4x - 12$ .
- 4 (a)  $\int 2x \, dx$     (b)  $\int 3x^2 \, dx$     (c)  $\int 4x^3 \, dx$     (d)  $\int x - 5 \, dx$     (e)  $\int 6x^2 - 1 \, dx$     (f)  $\int 9 \, dx$

#### Task 3

- 1 Show that the vectors  $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$  and  $\mathbf{b} = 3\mathbf{i} - \mathbf{j} + 3\mathbf{k}$  are perpendicular.
- 2 Show that the points  $L(-5, 6, -5)$ ,  $M(7, -2, -1)$  and  $N(10, -4, 0)$  are collinear and find the ratio in which  $M$  divides  $LN$ .
- 3 Find the equation of the tangent to the curve  $y = 3x^2 + 7$  at the point where  $x = 1$ .
- 4 (a)  $\int 5x^4 \, dx$     (b)  $\int 4x + 3 \, dx$     (c)  $\int -2 - 3x^2 \, dx$     (d)  $\int 5t^2 \, dt$     (e)  $\int \sqrt{x} \, dx$

#### Task 4

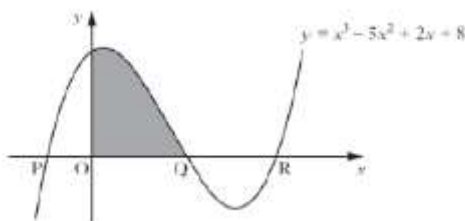
- 1 Differentiate  $3x^{\frac{2}{3}} + \sin^2 x$  with respect to  $x$ .
- 2 Find the equation of the line through the point  $(5, -2)$  which is parallel to the line with equation  $3x + 2y - 4 = 0$ .
- 3 Find  $f'(2)$  where  $f(x) = \frac{4}{x^2}$ .
- 4 (a)  $\int_0^3 2x \, dx$     (b)  $\int_1^2 4x^3 \, dx$     (c)  $\int_{-1}^1 (x^2 + 3) \, dx$     (d)  $\int_1^4 x^{\frac{1}{2}} \, dx$     (e)  $\int_0^2 x(x-1) \, dx$

### Task 5

- 1 Differentiate  $(2x - 5)^3$  with respect to  $x$ .
- 2 Solve the equation  $2(1 + \cos 3x) = 1$  in the interval  $0 \leq x < \pi$ .
- 3 (a)  $\int_{-1}^0 x^{-2} dx$  (b)  $\int_1^3 \frac{1}{x^3} dx$  (c)  $\int_1^2 (x-1)(x+1) dx$  (d)  $\int_{-1}^1 2x(3-x) dx$  (e)  $\int_0^t 2x dx = 16$ , find  $t$

### Task 6

- 1 For what value of  $k$  does the equation  $x^2 - 5x + (k + 6) = 0$  have equal roots.
- 2 Find  $x$  if  $4\log_x 6 - 2\log_x 4 = 1$ .
- 3 Find the equation of  $l_1$ , the perpendicular bisector of the line joining  $A(3, -3)$  to  $B(-1, 9)$ .
- 4 The diagram below shows the graph of  $y = x^3 - 5x^2 + 2x + 8$ . The points  $P$ ,  $Q$  and  $R$  have coordinates  $P(-1, 0)$ ,  $Q(2, 0)$  and  $R(4, 0)$ . Determine the shaded area.



### Day 7

- 1 For what value of  $k$  does the equation  $x^2 - 5x + (k + 6) = 0$  have equal roots.
- 2 Given that  $(x - 1)$  is a factor of  $x^3 + 3x^2 + x - 5$ , factorise this cubic fully.
- 3 (a)  $\int 3 \sin x dx$  (b)  $\int 7 \cos x dx$  (c)  $\int (\cos m - 3 \sin m) dm$  (d)  $\int (4x^3 + 3 \cos x) dx$

### Day 8

- 1 Given that  $f(x) = \cos^2 x - \sin^2 x$ , find  $f'(x)$ .
- 2 (a)  $\int_0^{\frac{\pi}{2}} 4 \sin x dx$  (b)  $\int_0^{\frac{\pi}{4}} (1 + \cos x) dx$  (c)  $\int \sin(2x + 3) dx$  (d)  $\int \cos 2x dx$

### Task 9

- 1 Functions  $f$  and  $g$  are defined on the set of real numbers by  $f(x) = x - 1$  and  $g(x) = x^2$ .

Find formulae for  $f(g(x))$  and  $g(f(x))$ .

2 (a)  $\int (x-4)^2 dx$       (b)  $\int \frac{dt}{(t-5)^3}$       (c)  $\int_1^3 (2x+3)^4 dx$       (d)  $\int_0^{\frac{\pi}{2}} \sin(3x + \frac{\pi}{4}) dx$

3  $\int_0^a 5 \sin 3x dx = \frac{10}{3}$ ,  $0 \leq a < \pi$ , calculate the value of  $a$ .