

# Springburn Academy : Mathematics Department

## Higher Mathematics : Lesson Starters

Block 1 Without using a calculator :

### Task 6

1 Find the resultant vector  $\frac{1}{2}(\mathbf{u} + \mathbf{v})$  when  $\mathbf{u} = \begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix}$  and  $\mathbf{v} = \begin{pmatrix} 1 \\ 0 \\ -6 \end{pmatrix}$

2 Make a sketch of a parabola with equation  $y = ax^2 + bx + c$ , where

- $a > 0$
- $b^2 - 4ac > 0$

3 The vectors  $\mathbf{p} = \begin{pmatrix} k \\ -1 \\ 1 \end{pmatrix}$  and  $\mathbf{q} = \begin{pmatrix} 0 \\ 4 \\ k \end{pmatrix}$  are perpendicular. What is the value of  $k$ ?

4 Triangle PQR has vertices at  $P(-3, -2)$ ,  $Q(-1, 4)$  and  $R(5, 3)$ .  
PS is a median. What is the gradient of PS?

### Task 7

1 The roots of  $f(x) = 2x^2 - x - 9$  are equal. (True or False?)

2 If  $\mathbf{f} = 4\mathbf{j} - \mathbf{k}$  and  $\mathbf{g} = 5\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ , find  $|\mathbf{f} + \mathbf{g}|$ .

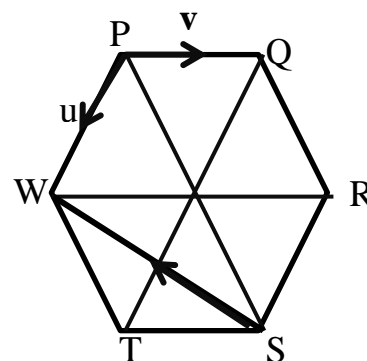
3 Find the range of values of  $k$  such that the equation  $kx^2 - x - 1 = 0$  has no real roots.

4 The unit vectors  $\mathbf{a}$  and  $\mathbf{b}$  are such that  $\mathbf{a} \cdot \mathbf{b} = \frac{2}{3}$ . Determine the value of  $\mathbf{a} \cdot (\mathbf{a} + 2\mathbf{b})$ .

### Task 8

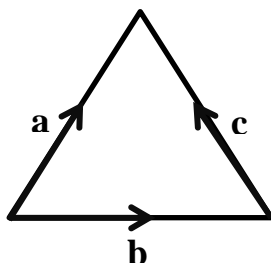
- 1 Vectors  $\mathbf{p}$  and  $\mathbf{q}$  are such that  $|\mathbf{p}| = 3$ ,  $|\mathbf{q}| = 4$  and  $\mathbf{p} \cdot \mathbf{q} = 10$ .  
Find the value of  $\mathbf{q} \cdot (\mathbf{p} + \mathbf{q})$ .
- 2 Calculate the gradient of the line perpendicular to the line  $6y + 9x = 1$ .
- 3 Here are two statements about the roots of the equation  $x^2 + x + 1 = 0$ :  
(1) the roots are equal;  
(2) the roots are real.  
Which, if any, of the 2 statements above are true?

- 4 The diagram shows a regular hexagon PQRSTW.  
 $\overrightarrow{PW}$  and  $\overrightarrow{PQ}$  represent the vectors  $\mathbf{u}$  and  $\mathbf{v}$  respectively.  
What is  $\overrightarrow{SW}$  in terms of  $\mathbf{u}$  and  $\mathbf{v}$ ?



### Task 9

1. Show that the points  $A(-7, -8, 1)$ ,  $T(3, 2, 5)$  and  $B(8, 7, 7)$  are collinear.  
Find the ratio in which T divides AB.
2. If,  $2x^2 + 4x + 7$  is expressed in the form  $2(x + p)^2 + q$ , what is the value of  $q$ ?
3. Find the range of values of  $p$  such that the equation  $px^2 - 2x + 3 = 0$ ,  $p \neq 0$ , has no real roots.
4. The triangle below is equilateral with sides of length = 2cm. Calculate the value of  $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c})$ .



### Task 10

The diagram shows a cuboid OPQRSTUV relative to the coordinate axes.

P is the point  $(4, 0, 0)$ , Q is  $(4, 2, 0)$  and U is  $(4, 2, 3)$ .

M is the midpoint of OR.

N is the point on UQ such that  $UN = \frac{1}{3} UQ$ .

(a) State the coordinates of M and N.

(b) Express VM and VN in component form.

(c) Calculate the size of angle MVN.

**(Calculator may be used here.)**

