

# Springburn Academy : Mathematics Department

## Higher Mathematics : Lesson Starters

### Block 3 ( Wave Equation 2)

Without using a calculator :

#### Task 1

1 If  $f(x) = 2x^3 - x^2 - 5x + 4$ .

What is the remainder when  $f(x)$  is divided by  $(x+2)$ ?

2 For what value of  $k$  does the equation  $kx^2 - 6x + 1 = 0$  have equal roots?

3 If  $k \sin x^\circ = \sqrt{3}$  and  $k \cos x^\circ = 1$ , find the values of  $k$  and  $x$

4 Expand  $\cos\left(\theta + \frac{\pi}{4}\right)$  and simplify

#### Task 2

1 Find  $\int (2x+7)^4 dx$ .

2 What are the coordinates of the centre of the circle with equation

$$3x^2 + 3y^2 - 6x + 18y - 5 = 0?$$

3 Solve  $2 \sin x^\circ = \sqrt{3}$ , for  $0^\circ \leq x^\circ \leq 360^\circ$

4 Express  $\cos x^\circ + \sqrt{2} \sin x^\circ$  in the form  $k \cos(x-a)^\circ$ , where  $k > 0$  and  $0 \leq a < 360$ .

#### Task 3

1 A line joins the points P(-4, 3) and Q(2, -7).

Find the equation of the perpendicular bisector of PQ

2 Evaluate  $4\sqrt{2} \sin \frac{\pi}{4} \cos \frac{2\pi}{3}$ .

3 A circle with centre (-3, 4) passes through the point (-2, 2).

What is the equation of the circle?

4 Express  $2\sqrt{2} \cos x^\circ - 2\sqrt{2} \sin x^\circ$  in the form  $k \cos(x-a)^\circ$ , where  $k > 0$  and  $0 \leq a < 360$ .

#### Task 4

- 1 The population of hamsters in a breeding centre increases by 5% during each month.  
At the end of each month the breeder sells 30 hamsters.  
If  $u_n$  represents the hamster population at the beginning of a month, find an expression for  $u_{n+1}$
- 2 The equation  $x^2 + 2x + p = 0$  has no real roots.  
What is the range of values of  $p$ ?
- 3 Find  $\int \sin(1+2x) dx$ .
- 4 If  $\sqrt{3} \cos x + \sin x = k \cos x \cos p + k \sin x \sin p$ , where  $k > 0$ , what is the value of  $k$ ?

#### Task 5

- 1 Show that the line with equation  $y = 2x + 10$  is a tangent to the circle with equation  $x^2 + y^2 - 2x - 4y - 15 = 0$   
and find the coordinates of the point of contact of the tangent and circle.
- 2 Find the maximum value of  $\cos x - \sin x$   
and the value of  $x$  for which it occurs in the interval  $0 \leq x \leq 2\pi$ .