National 5 Revision Booklet

Expressions and Formula

This revision covers the following topics.

- 1.1 Surds
- 1.2 Indices
- 1.3 Significant Figures

1.1 Surds – This is a non calculator exercise.

1. Simplify:

a. $\sqrt{12}$ **b.** $\sqrt{20}$ **c.** $\sqrt{44}$

d. $\sqrt{50}$ **e.** $\sqrt{300}$ **f.** $\sqrt{125}$

g. $\sqrt{72}$ **h.** $\sqrt{450}$ **i.** $\sqrt{1500}$

2. Simplify as far as possible:

a. $\sqrt{3} \times \sqrt{8}$ **b.** $\sqrt{6} \times \sqrt{12}$ **c.** $\sqrt{7} \times \sqrt{8}$

d. $\sqrt{2} \times \sqrt{3} \times \sqrt{5}$ **e.** $\sqrt{8} \times \sqrt{2} \times \sqrt{5}$ **f.** $\sqrt{3} \times \sqrt{5} \times \sqrt{15}$

3. Add or subtract these surds and simplify as far as possible:

a. $4\sqrt{3} + 5\sqrt{3}$ **b.** $6\sqrt{7} - 2\sqrt{7}$

c. $5\sqrt{10} + 7\sqrt{10} - \sqrt{10}$ **d.** $\sqrt{2} + \sqrt{2}$

e. $2\sqrt{3} + 5\sqrt{5} + 6\sqrt{3}$ **f.** $4\sqrt{3} + 5\sqrt{3}$

g. $\sqrt{72} + 5\sqrt{2}$ **h.** $\sqrt{75} + \sqrt{108} - \sqrt{3}$

4. Expand and simplify:

a. $\sqrt{2}(1-\sqrt{2})$

b. $\sqrt{3}(\sqrt{3}+1$

c. $\sqrt{2}(3+\sqrt{6})$

d. $\sqrt{5}(\sqrt{5}+2)$

e. $\sqrt{5}(\sqrt{200}+\sqrt{50})$

f. $2\sqrt{12}(\sqrt{3}+\sqrt{6})$

5. Expand and simplify:

a. $(\sqrt{3}+1)(\sqrt{2}+6)$

b. $(\sqrt{5}+\ )(\sqrt{2}-3)$

c. $(\sqrt{7}-1)(\sqrt{7}-3)$

d. $(3\sqrt{3}+8)(5\sqrt{2}+10)$

e. $(\sqrt{2}+3)^2$

f. $(5-2\sqrt{3}\,)^2$

6. Rationalise the denominator in each fraction and simplify as far as possible:

a. $\frac{4}{\sqrt{3}}$

b. $\frac{6}{\sqrt{2}}$

c. $\frac{20}{\sqrt{8}}$

d. $\frac{8}{5\sqrt{8}}$

e. $\frac{\sqrt{5}}{\sqrt{8}}$

f. $\frac{20}{\sqrt{30}}$

7. Extensions Work - Use the conjugate to rationalise each denominator:

a. $\frac{3}{\sqrt{3}+2}$

b. $\frac{16}{4-\sqrt{6}}$

c. $\frac{\sqrt{10}}{\sqrt{7}-9}$

d. $\frac{4+\sqrt{5}}{\sqrt{3}+2}$

8. A rectangle has sides measuring $(2+\sqrt{5})cm\ and\ (3-\sqrt{3})cm.$

Calculate the area of the rectangle.

9. The exact area of a rectangle is $2(\sqrt{6}+\sqrt{3})square\ centimetres.$

Given that the breadth of the rectangle is $\sqrt{6}cm$, show that the length is equal to $(2+\sqrt{2})$cm.

End of Surds review.

1.2 Indices – This is a non calculator exercise

10. Simplify the following expressions, expressing your answers with positive indices:

a. $y \times y \times y$ **b.** $p \times p \times p \times p \times q$ **c.** $a \times a \times b \times b \times a$

d. $2^3 \times 2^4$ **e.** $p^3 \times p^7$ **f.** $a^3 \times a^{-4}$

g. $3 \times y^2 \times 2 \times y$ **h.** $2 \times a^2 \times b^3 \times a^{-1}$ **i.** $y^4 \div y^2$

j. $4r^8 \div 2r$ **k.** $d^4 \div d^{-3}$ **l.** $a^0 \times 6a^3 \div 2a^{-2}$

m. $\frac{h^5}{h^2}$ **n.** $\frac{10e^{-3}}{5e^3}$ **o.** $\frac{2p^{-2}}{p^{-6}}$

p. $\frac{a^2 \times 2a^5}{a^4}$ **q.** $\frac{w \times w \times w}{p^2 w^4}$ **r.** $\frac{3a^4b^3}{6a^2b^5}$

11. Simplify the following expressions, expressing your answers with positive indices:

a. $(3^2)^5$ **b.** $(4^{-3})^2$ **c.** $(a^3)^7$ **d.** $(p^4)^{-3}$ **e.** $(d^{-5})^{-2}$

f. $(a^{¼})^8$ **g.** $(y^{\frac{2}{3}})^{\frac{3}{4}}$ **h.** $2(p^{\frac{4}{5}})^{\frac{-5}{2}}$ **i.** $3(c^{-2})^0$ **j.** $(k^{\frac{1}{3}})^{\frac{2}{5}}$

k. $(ab)^3$ **l.** $(xy^2)^4$ **m.** $(2m^3)^3$ **n.** $(2xy^2)^4$ **o.** $3(a^2b^4)^{\frac{1}{2}}$

12. Express without root signs:

(a) $\sqrt[4]{a}$ (b) $\sqrt{p^3}$ (c) $\sqrt[3]{x^5}$ (d) $\sqrt[5]{r^2}$

(e) $\sqrt[q]{b^3}$

13. Express without root signs (Write with positive indices first where necessary)

(a) $p^{\frac{1}{5}}$ (b) $w^{\frac{3}{4}}$ (c) $x^{\frac{1}{2}}$ (d) $a^{-\frac{3}{4}}$

(e) $y^{-\frac{1}{5}}$

14. Evaluate each of the following without the use of a calculator

$(a)\ 25^{\frac{1}{2}}$ $(b)\ 8^{\frac{1}{3}}$ $(c)\ 4^{-\frac{1}{2}}$ $(d)\ 16^{-\frac{1}{4}}$ $(e)\ 13^{0}$ $(f)\ 7^{-1}$

$(g)\ 16^{\frac{3}{2}}$ $(h)\ 27^{\frac{2}{3}}$ $(i)\ 8^{-\frac{4}{3}}$ $(j)\ (-8)^{\frac{1}{3}}$ $(k)\ 64^{\frac{2}{3}}$ $(l)\ 100^{-\frac{3}{2}}$

$(m)\ (\frac{1}{2})^{-1}$ $(n)\ (\frac{1}{8})^{\frac{4}{3}}$ $(o)\ (\frac{2}{3})^{3}$ $(p)\ (\frac{1}{2})^{-5}$ $(q)\ (\frac{3}{4})^{-2}$ $(r)\ (\frac{8}{27})^{-\frac{4}{3}}$

15. Simplify each of the following by:
    - changing root signs to fractional powers
    - moving x's onto the numerators
    - expanding brackets where necessary

$(a)\ x^{\frac{1}{2}}(x^4+1)$ $(b)\ x^{-\frac{1}{2}}(x^{\frac{3}{2}}-x^2)$ $(c)\ \frac{1}{x^2}\left(x^{\frac{1}{2}}+x\right)$

$(d)\ \frac{2}{x^{-3}}\left(x^2+\frac{1}{x}\right)$ $(e)\ \frac{1}{\sqrt{x}}\left(x^2-\sqrt{x}\right)$ $(f)\ \left(x^2+\frac{1}{x}\right)^2$

$(g)\ \frac{1}{x}\left(\sqrt{x}+x\right)$ $(h)\ \left(x+\frac{1}{\sqrt{x}}\right)^2$ $(i)\ x^{-2}\left(\frac{1}{x}-\sqrt[3]{x}\right)$

$(j)\ \frac{x^2+3}{x}$ $(k)\ \frac{\sqrt{x}\ -\ x}{x^2}$ $(l)\ \frac{(2x+1)^2}{x^{\frac{3}{2}}}$

End of Indices Review

1.3 Significant Figures- This is a non calculator exercise

16. Round each number to the amount of significant figures asked:

a. 5068 (1) **b.** 38383 (2) **c.** 626817 (3) **d.** 0.0649 (1)

17. Calculate the following, giving your answers to 2 significant figures:

a. √14 **b.** √0.26 **c.** $\sqrt{6^2+4^2}$ **d.** $\sqrt{9^2-2^2}$

End of Significant Figures review.

END OF REVIEW

National 5 Revision Booklet

Expressions and Formula

This revision covers the following topics.

- 2.1 Expanding Brackets
- 2.2 Factorising
- 2.3 Completing the Square
- 2.4 Simplifying Algebraic Fractions
- 2.5 Algebraic Fractions – 4 Operations

2.1 Expanding brackets – This is a non calculator exercise

1. Multiply out the bracket(s) below:

(a) $2\,(3h + 1)$ **(b)** $-8(6 - 5x)$ **(c)** $3n\,(n^3 - 2n^2)$

(d) $7 + 3\,(n - 2)$ **(e)** $(5 - 2y) - 4(2 + 3y)$

2. Multiply out the bracket(s) below:

(a) $(y + 2)(y + 5)$ **(b)** $(t - 3)(t - 1)$ **(c)** $(4x + 3)(2x - 1)$

(d) $(y - 1)(y^2 + 2y + 1)$ **(e)** $(m + 3)(m^2 + 3m - 2)$ **(f)** $(x - 3)(2x^2 - 3x - 6)$

3. Multiply out the bracket(s) below:

(a) $(x + 5)^2$ **(b)** $(2c - 5)^2$ **(c)** $(4b - 3c)^2$

(d) $(a + \frac{}{a})^2$ **(e)** $(\frac{3}{p} - 4p)^2$ **(f)** $(x+4)^2 - (x+2)^2$

End of Expanding brackets review

2.2 Factorising – This is a non calculator exercise

4. Factorise:

(a) $4a + 8$ **(b)** $16 – 8x$ **(c)** $a^2 + a$

(d) $6z^2 - 8z$ **(e)** $2ab – 4ac$ **(f)** $15w^2 - 6wx$

5. Factorise:

(a) $c^2 - d^2$ **(b)** $w^2 - 16$ **(c)** $81 - e^2$

(d) $a^2 - 100$ **(e)** $49 - n^2$ **(f)** $4y^2 - 25$

(g) $9y^2 - 16$ **(h)** $81x^2 - a^2$ **(i)** $4w^2 - 9x^2$

(j) $1 – 121k^2$ **(k)** $64u^2 - 9v^2$ **(l)** $3a^2 - 12$

(m) $3a^2 - 3b^2$ **(n)** $3c^2 - 27d^2$ **(o)** $16y^2 - 36t^2$

(p) $ax^2 - ay^2$ **(q)** $72 – 2m^2$

6. Factorise:

(a) $a^2 + 3a + 2$ **(b)** $x^2 + 5x + 6$ **(c)** $m^2 + 4m - 5$

(d) $w^2 - 6w + 9$ **(e)** $2w^2 + 3w + 1$ **(f)** $3y^2 - 4y + 1$

(g) $2x^2 - 7x + 3$ **(h)** $13u^2 + 7u – 6$

7. Factorise:

(a) $2x^2 - 18$ **(b)** $4p^2 - 49$ **(c)** $5x^2 - 45$

(d) $x^2 - 4y^2$ **(e)** $x^2 - 5x – 24$ **(f)** $x^2 + x – 6$

(g) $a^2 + 2a + b^2$

End of Factorising review

2.3 Completing the Square – This is a non calculator exercise

8. In parts a – f add a number to make a perfect square:

(a) $x^2 + 2x$ **(b)** $y^2 + 12y$ **(c)** $m^2 - 6m$

(d) $w^2 - 20w$ **(e)** $r^2 - 9r$ **(f)** $v^2 - \frac{2}{3}v$

9. In parts g – j, write in the form $(x+p)^2 + q$:

(g) $x^2 + 6x + 10$ **(h)** $y^2 - 2y + 3$ **(i)** $z^2 + 8z - 10$

(j) $m^2 + \frac{1}{2}m + \frac{1}{4}$

10. Write in the form $p - (x+q)^2$: **(Extension Work)**

(a) $4 + 2x - x^2$ **(b)** $5 - 4x - x^2$ **(c)** $6 + 3x - x^2$

(d) $4x - 6 - x^2$

11. Write in the form $a(x+p)^2 + q$: **(Extension Work)**

(a) $2x^2 + 4x - 1$ **(b)** $5a^2 - 30a - 18$ **(c)** $2n^2 + 2n + 1$

(d) $2m^2 - 3m - 6$

End of Completing the Square review

2.4 Simplifying Algebraic Fractions

12. Simplify these expressions:

(a) $\frac{5}{25x}$ **(b)** $\frac{ax}{bx}$ **(c)** $\frac{3x^2}{6x}$ **(d)** $\frac{4pq}{12q}$

(e) $\frac{6abc}{12a^2c}$ **(f)** $\frac{a(b+1)}{3(b+1)}$ **(g)** $\frac{(x-1)(x+1)}{3(x-1)}$ **(h)** $\frac{(x-1)(x+2)}{(x-2)(x+3)}$

(i) $\frac{a^2}{a^2(a+1)}$

13. Factorise first then simplify these expressions

(a) $\frac{x^2+4x-5}{x^2+x-2}$ **(b)** $\frac{9x^2-1}{3x^2+11x-4}$ **(c)** $\frac{2x^2-x-1}{2x^2+x}$

End of Simplifying Algebraic Fractions Review

2.5 Applying the four operations to algebraic fractions

14. Simplify:

(a) $\frac{a}{2}\times\frac{b}{2}$ **(b)** $\frac{b^2}{ax}\times\frac{a^2}{by}$ **(c)** $\frac{3}{k}\times 4k^2$ **(d)** $\frac{x+1}{4}\times\frac{16x}{x+1}$

(e) $\frac{3}{x}\div\frac{5}{2x}$ **(f)** $\frac{3a}{xy}\div\frac{4ab}{x^2y}$ **(g)** $\frac{2x+4}{5}\div\frac{3x+9}{10}$

15. Simplify:

(a) $\frac{x}{5}+\frac{x}{7}$ **(b)** $\frac{g}{3}+\frac{5}{g}$ **(c)** $\frac{3}{m}+\frac{4}{n}$

(d) $\frac{2}{x+1}+\frac{3}{x-1}$ **(e)** $\frac{x+1}{x-1}+\frac{x}{x+1}$ **(f)** $7+\frac{3}{2d-1}$

(g) $\frac{a}{2}-\frac{a}{3}$ **(h)** $\frac{1}{x}-\frac{1}{y}$ **(i)** $\frac{3}{x}-4$

(j) $\frac{1}{x-1}-\frac{1}{x+1}$ **(k)** $\frac{x+1}{x+2}-\frac{x}{x+1}$

16. Simplify:

(a) $\frac{12e^2\ f^2}{2ef^3}$ **(b)** $\frac{(x-4)(3-x)}{4(x-4)(3-x)}$ **(c)** $\frac{x^4-16}{x^2+6x-16}$

(d) $\frac{s^2}{t}\times\frac{3t}{2s}$ **(e)** $\frac{3y-1}{2}\times\frac{y+1}{3y-1}$ **(f)** $\frac{5p}{8}\div\frac{p}{2}$

(h) $\frac{1}{p}+\frac{2}{(p+5)}$ **(i)** $\frac{a}{b}+\frac{b}{a}$ **(j)** $\frac{7}{x}-\frac{3}{x+1}$

(k) $\frac{3}{x}-\frac{4}{x+1}$ **(l)** $\frac{5}{x}-\frac{3}{(x-2)}$ **(m)** $\frac{3}{x}-\frac{5}{x+2}$

End of Applying four operations to Algebraic fractions review

END OF REVIEW

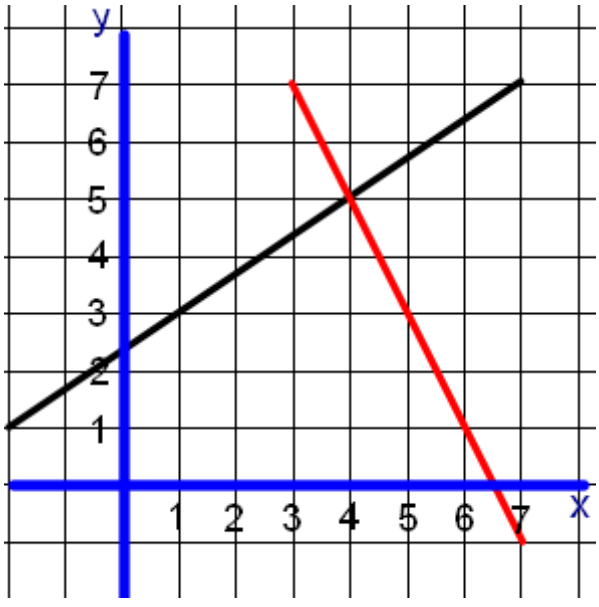
National 5 Revision Booklet

Expressions and Formula

MIXED EXERCISE 3
This homework covers the following topics.
• 3.1 Calculating Gradient • 3.2 Length of Arc/Area of Sector • 3.3 Volumes of Solids

3.1 Finding Gradient- This is a non calculator exercise

1. Calculate the gradient of the lines drawn on the following grid:

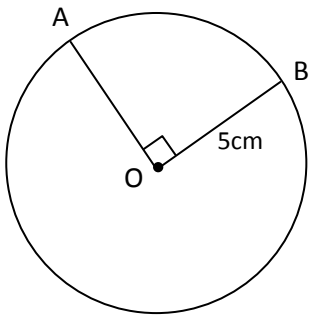


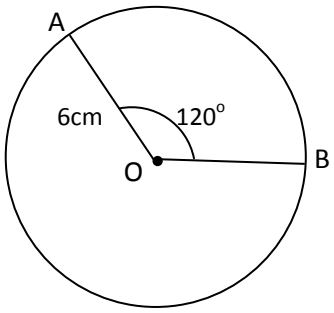
2. Calculate the gradient of a line passing through the points A(-2,-4) and B(8,1)

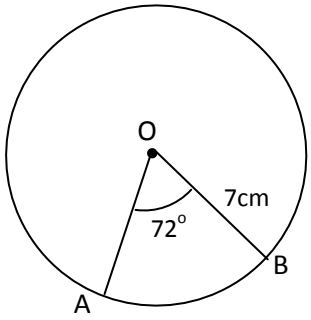
End of Gradient review.

3.2 Arcs and Sectors - This is a calculator exercise

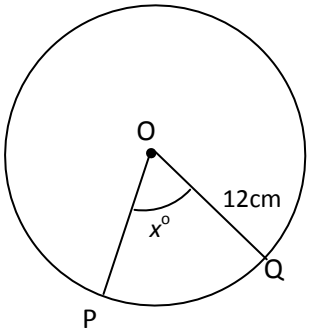
3. Calculate the length of the minor arc AB and the area of sector AOB in each of the following where O is the centre of the circle:



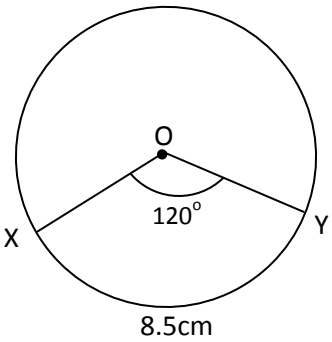




4. Calculate the angle marked x in the diagram if the area of the sector is 75cm². Give angle to nearest whole degree.

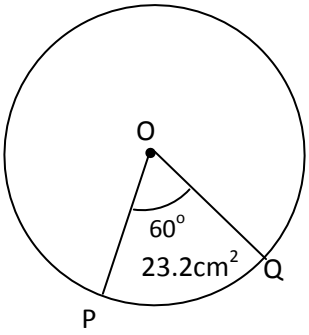


5. The length of arc XY is 8.5cm. Calculate the circumference of the circle:

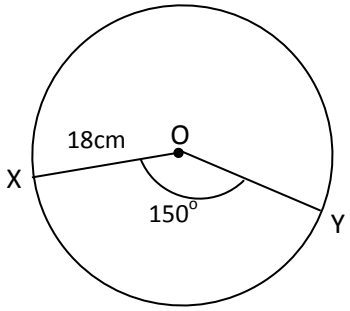


3.2 Arcs and Sectors - This is a calculator exercise

6. The area of sector PQ is 23.2cm^2. Calculate the radius of the circle:



7. From the diagram below
   a) Calculate the **perimeter** of the sector XOY.
   b) Calculate the area of the circle.



End of Arcs and Sectors Review

3.3 Volume - This is a calculator exercise

Important Formulae:

Volume of a cuboid – $V = l \times b \times h$

Volume of a prism – $V =$ Area of face $\times$ length (height)

Volume of a cylinder – $V = \pi r^2 h$

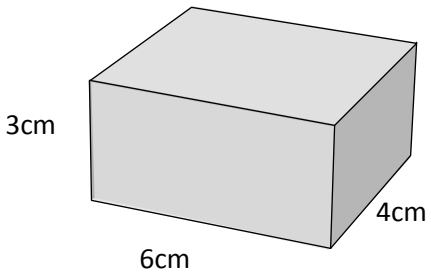
Volume of a sphere – $V = \frac{4}{3}\pi r^3$

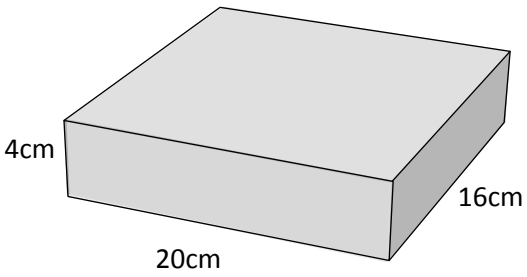
Volume of a cone – $V = \frac{1}{3}\pi r^2 h$

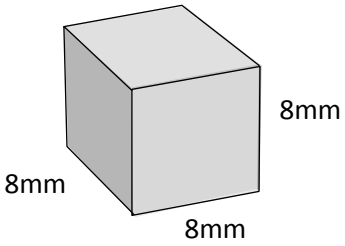
Volume of a pyramid – $V = \frac{1}{3}$(area of base) $\times$ height

$V = \frac{1}{3} \times A \times h$

8. Calculate the volume of each cuboid below:

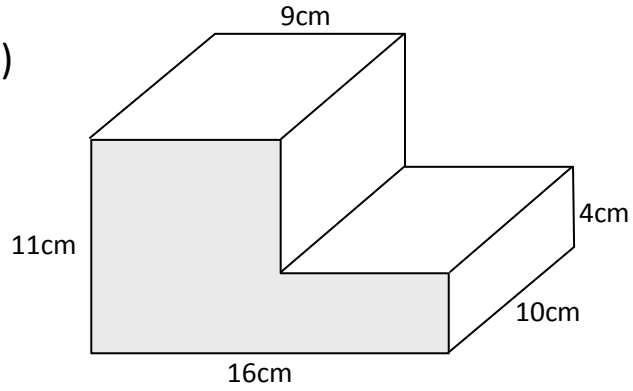




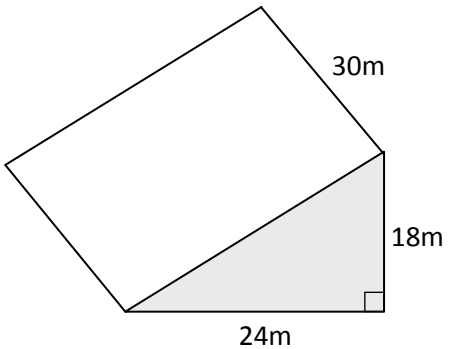


9. Calculate the volume of each prism below:

a)

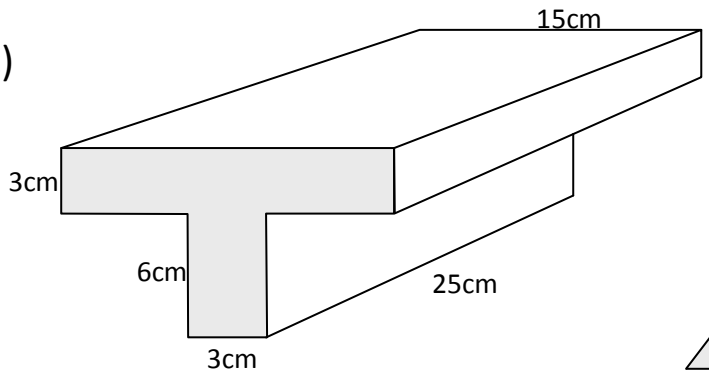


b)

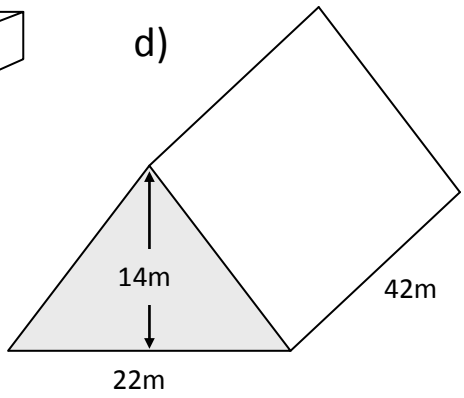


3.3 Volume - This is a calculator exercise

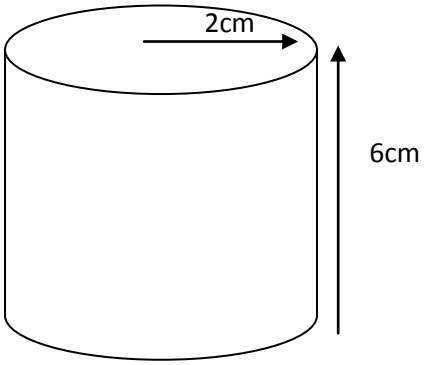
c)

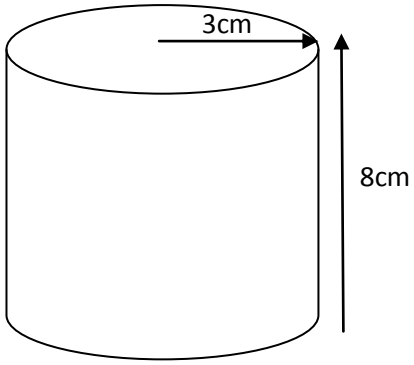


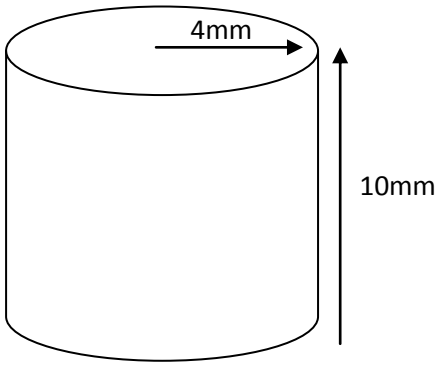
d)

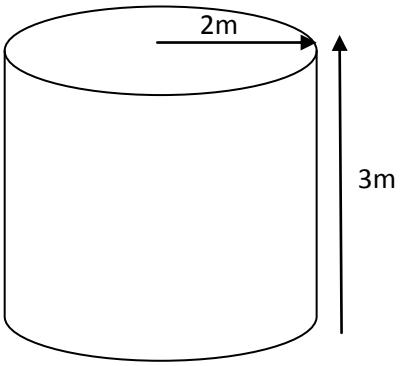


10. Calculate the volume of each cylinder:





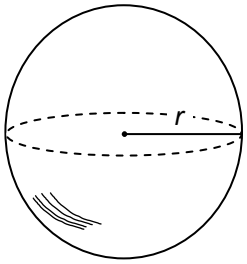




3.3 Volume- This is a calculator exercise

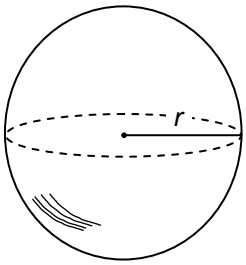
11. A cylinder has a diameter of 20 cm and a height of 16cm. Calculate its volume.
12. A cylinder has a diameter of 2.6m and a height of 80cm. Calculate its volume to the nearest cubic metre.
13. Calculate the volume of each sphere below rounding your answers to 1 decimal place:

(a)



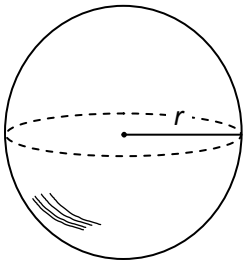
Radius = 6cm

(b)



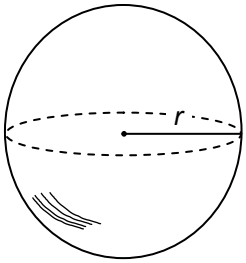
Radius = 2m

(c)



Radius = 9mm

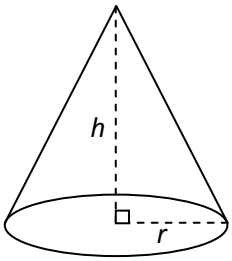
(d)



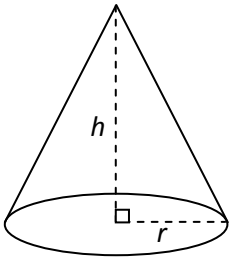
Radius = 3cm

3.3 Volume- This is a calculator exercise

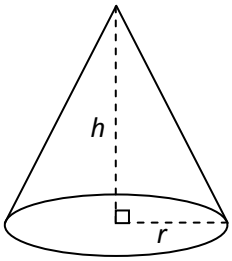
14. A sphere has a diameter of 8cm. Calculate its volume giving your answers to 3 significant figures.
15. Calculate the volume of each cone below rounding your answers to 1 decimal place:



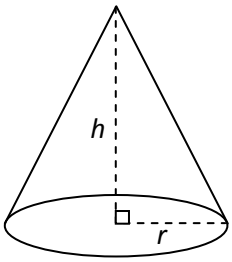
Height = 6cm
Radius = 3cm



Height = 12mm
Radius = 8mm



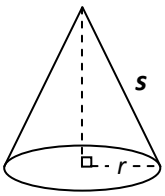
Height = 5cm
Radius = 3cm



Height = 6m
Radius = 2m

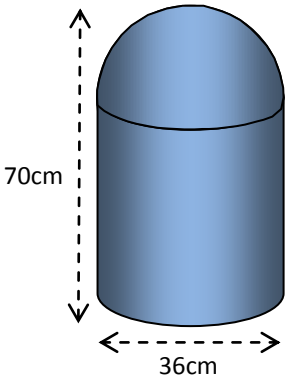
3.3 Volume- This is a calculator exercise

16. A cone has a base diameter of 8cm and a height of 5cm. Calculate the volume of the cone.
17. A cone has a base diameter of 10cm and a **slant height** of 13cm. Calculate the volume of the cone:



18. A cone has a base radius of 9cm and a slant height of 15cm. Calculate the volume of the cone.
19. A pyramid has a square base of side 4cm and a vertical height of 7cm. Calculate the volume of the pyramid to 2 significant figures.
20. A pyramid has a rectangular base measuring 16mm by 12mm and a vertical height of 10mm. Calculate its volume.
21. A "Binnit" waste bin is in the shape of a cylinder with a hemi-sphere on top.

    The diameter of the bin is 36cm and the total height is 70cm. Calculate the volume of the bin giving your answer to the nearest litre:



End of Volume Review