

A-B Questions

Paper 1 Section A

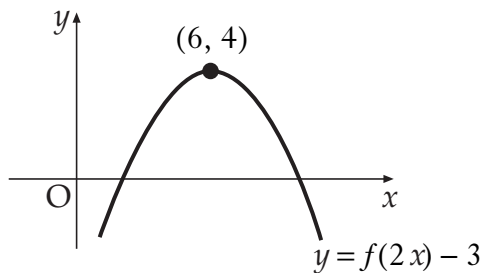
Each correct answer in this section is worth two marks.

1. On a suitable domain, D , a function g is defined by $g(x) = \sin^2 \sqrt{x-2}$.

Which of the following gives the real values of x in D and the corresponding values of $g(x)$?

- A. $x \geq 0$ and $-1 \leq g(x) \leq 1$
 B. $x \geq 0$ and $0 \leq g(x) \leq 1$
 C. $x \geq 2$ and $-1 \leq g(x) \leq 1$
 D. $x \geq 2$ and $0 \leq g(x) \leq 1$

2. The diagram shows the graph of $y = f(2x) - 3$.



What are the coordinates of the turning point on the graph of $y = f(x)$?

- A. (12, 7)
 B. (12, 1)
 C. (3, 7)
 D. (3, 1)

3. If $f(x) = (x-3)(x+5)$, for what values of x is the graph of $y = f(x)$ above the x -axis?

- A. $-5 < x < 3$
 B. $-3 < x < 5$
 C. $x < -5, x > 3$
 D. $x < -3, x > 5$

4. The discriminant of a quadratic equation is 23.

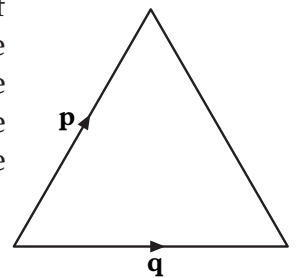
Here are two statements about this quadratic equation:

- I. the roots are real;
 II. the roots are rational.

Which of the following is true?

- A. neither statement is correct
 B. only statement I is correct
 C. only statement II is correct
 D. both statements are correct

5. An equilateral triangle of side 3 units is shown. The vectors p and q are as represented in the diagram. What is the value of $p \cdot q$?

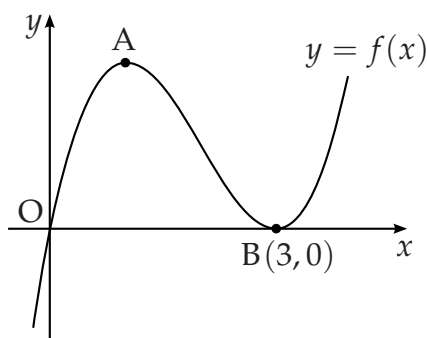


- A. 9
 B. $\frac{9}{2}$
 C. $\frac{9}{\sqrt{2}}$
 D. 0

[END OF PAPER 1 SECTION A]

Paper 1 Section B

- [SQA] 6. A sketch of the graph of $y = f(x)$ where $f(x) = x^3 - 6x^2 + 9x$ is shown below.
The graph has a maximum at A and a minimum at B(3,0).



- (a) Find the coordinates of the turning point at A. 4
- (b) Hence sketch the graph of $y = g(x)$ where $g(x) = f(x + 2) + 4$.
Indicate the coordinates of the turning points. There is no need to calculate the coordinates of the points of intersection with the axes. 2
- (c) Write down the range of values of k for which $g(x) = k$ has 3 real roots. 1
- [SQA] 7. Two sequences are generated by the recurrence relations $u_{n+1} = au_n + 10$ and $v_{n+1} = a^2v_n + 16$.
The two sequences approach the same limit as $n \rightarrow \infty$.
Determine the value of a and evaluate the limit. 5
- [SQA] 8. 5
- (a) Solve $\cos 2x^\circ - 3 \cos x^\circ + 2 = 0$ for $0 \leq x < 360$.
- (b) Hence solve $\cos 4x^\circ - 3 \cos 2x^\circ + 2 = 0$ for $0 \leq x < 360$

9. (a) Diagram 1 shows a right angled triangle, where the line OA has equation $3x - 2y = 0$.

- (i) Show that $\tan a = \frac{3}{2}$.
 (ii) Find the value of $\sin a$.

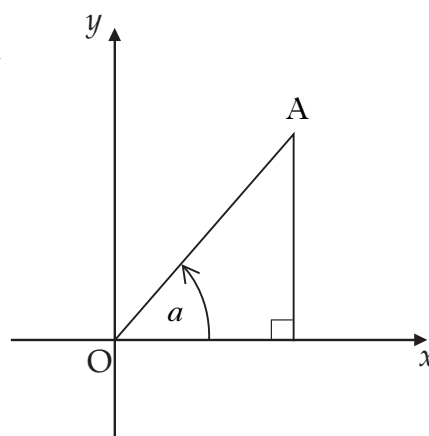


Diagram 1

- (b) A second right angled triangle is added as shown in Diagram 2.

The line OB has equation $3x - 4y = 0$.

Find the values of $\sin b$ and $\cos b$.

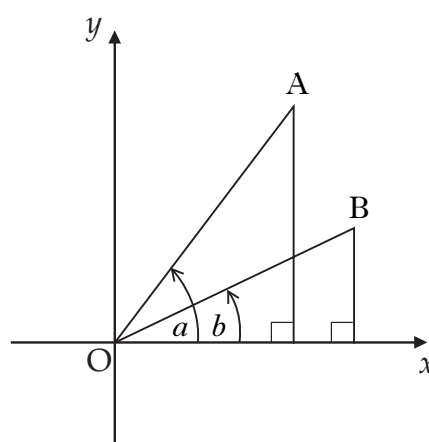


Diagram 2

- (c) (i) Find the value of $\sin(a - b)$.
 (ii) State the value of $\sin(b - a)$.

- [SQA] 10. Circle P has equation $x^2 + y^2 - 8x - 10y + 9 = 0$. Circle Q has centre $(-2, -1)$ and radius $2\sqrt{2}$.

- (a) (i) Show that the radius of circle P is $4\sqrt{2}$.
 (ii) Hence show that circles P and Q touch.

- (b) Find the equation of the tangent to the circle Q at the point $(-4, 1)$.

- (c) The tangent in (b) intersects circle P in two points. Find the x -coordinates of the points of intersection, expressing your answers in the form $a \pm b\sqrt{3}$.

- [SQA] 11. For what range of values of k does the equation $x^2 + y^2 + 4kx - 2ky - k - 2 = 0$ represent a circle?

- [SQA] 12. The graph of $y = f(x)$ passes through the point $(\frac{\pi}{9}, 1)$.
If $f'(x) = \sin(3x)$ express y in terms of x . 4
- [SQA] 13. (a) Find the derivative of the function $f(x) = (8 - x^3)^{\frac{1}{2}}$, $x < 2$. 2
(b) Hence write down $\int \frac{x^2}{(8 - x^3)^{\frac{1}{2}}} dx$. 1
- [SQA] 14. Find the maximum value of $\cos x - \sin x$ and the value of x for which it occurs in the interval $0 \leq x \leq 2\pi$. 6

[END OF PAPER 1 SECTION B]

Paper 2

[SQA] 1. $f(x) = 3 - x$ and $g(x) = \frac{3}{x}, x \neq 0$.

(a) Find $p(x)$ where $p(x) = f(g(x))$. 2

(b) If $q(x) = \frac{3}{3-x}, x \neq 3$, find $p(q(x))$ in its simplest form. 3

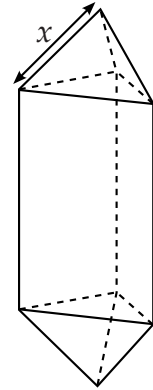
[SQA] 2. A goldsmith has built up a solid which consists of a triangular prism of fixed volume with a regular tetrahedron at each end.

The surface area, A , of the solid is given by

$$A(x) = \frac{3\sqrt{3}}{2} \left(x^2 + \frac{16}{x} \right)$$

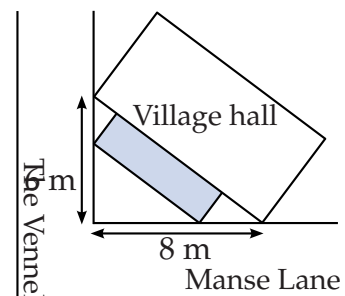
where x is the length of each edge of the tetrahedron.

Find the value of x which the goldsmith should use to minimise the amount of gold plating required to cover the solid.

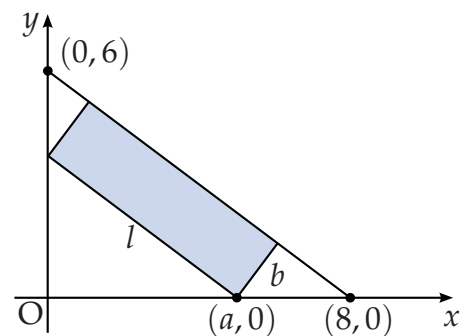


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[SQA] 3. The shaded rectangle on this map represents the planned extension to the village hall. It is hoped to provide the largest possible area for the extension.



The coordinate diagram represents the right angled triangle of ground behind the hall. The extension has length l metres and breadth b metres, as shown. One corner of the extension is at the point $(a, 0)$.



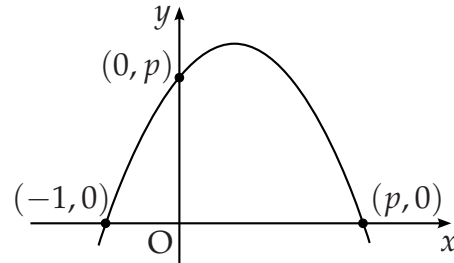
(a) (i) Show that $l = \frac{5}{4}a$.

(ii) Express b in terms of a and hence deduce that the area, $A \text{ m}^2$, of the extension is given by $A = \frac{3}{4}a(8 - a)$. 3

(b) Find the value of a which produces the largest area of the extension. 4

[SQA] 4. Show that the equation $(1 - 2k)x^2 - 5kx - 2k = 0$ has real roots for all integer values of k . 5

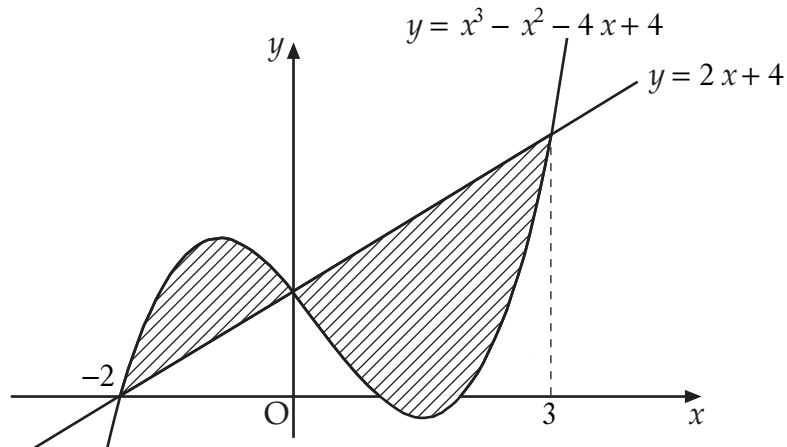
[SQA] 5. The diagram shows a sketch of a parabola passing through $(-1, 0)$, $(0, p)$ and $(p, 0)$.



(a) Show that the equation of the parabola is $y = p + (p - 1)x - x^2$. 3

(b) For what value of p will the line $y = x + p$ be a tangent to this curve? 3

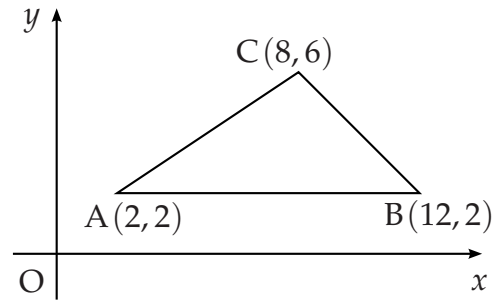
6. The diagram shows the curve with equation $y = x^3 - x^2 - 4x + 4$ and the line with equation $y = 2x + 4$. The curve and the line intersect at the points $(-2, 0)$, $(0, 4)$ and $(3, 10)$.



Calculate the total shaded area. 10

[SQA] 7. Solve the equation $3 \cos 2x^\circ + \cos x^\circ = -1$ in the interval $0 \leq x \leq 360$. 5

- [SQA] 8. Triangle ABC has vertices A(2,2), B(12,2) and C(8,6).
- Write down the equation of l_1 , the perpendicular bisector of AB.
 - Find the equation of l_2 , the perpendicular bisector of AC.
 - Find the point of intersection of lines l_1 and l_2 .
 - Hence find the equation of the circle passing through A, B and C.



1
4
1
2

9. Circle C_1 has equation $(x + 1)^2 + (y - 1)^2 = 121$.

A circle C_2 with equation $x^2 + y^2 - 4x + 6y + p = 0$ is drawn inside C_1 .

The circles have no points of contact.

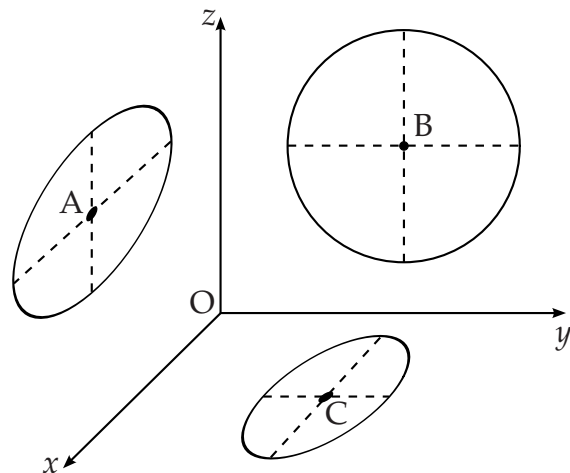
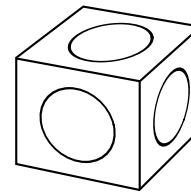
What is the range of values of p ?

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- [SQA] 10. A box in the shape of a cuboid is designed with **circles** of different sizes on each face.

The diagram shows three of the circles, where the origin represents one of the corners of the cuboid. The centres of the circles are A(6,0,7), B(0,5,6) and C(4,5,0).

Find the size of angle ABC.



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- [SQA] 11. A curve for which $\frac{dy}{dx} = 3 \sin(2x)$ passes through the point $(\frac{5\pi}{12}, \sqrt{3})$.

Find y in terms of x .

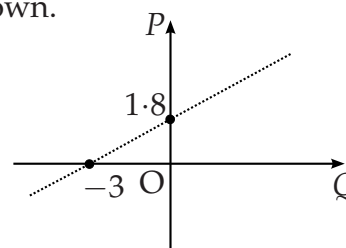
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[SQA] 12. Given that $f(x) = (5x - 4)^{\frac{1}{2}}$, evaluate $f'(4)$. 3

[SQA] 13. Find $\int \frac{1}{(7 - 3x)^2} dx$. 2

[SQA] 14. The results of an experiment give rise to the graph shown.

(a) Write down the equation of the line in terms of P and Q .



It is given that $P = \log_e p$ and $Q = \log_e q$.

(b) Show that p and q satisfy a relationship of the form $p = aq^b$, stating the values of a and b . 4

15. (a) The expression $3 \sin x - 5 \cos x$ can be written in the form $R \sin(x + a)$ where $R > 0$ and $0 \leq a < 2\pi$.

Calculate the values of R and a . 4

(b) Hence find the value of t , where $0 \leq t \leq 2$, for which

$$\int_0^t (3 \cos x + 5 \sin x) dx = 3.$$

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[END OF PAPER 2]