A-B Questions

Paper 1 Section A

Each correct answer in this section is worth two marks.

1. On a suitable domain, D, a function g is defined by $g(x) = \sin^2 \sqrt{x-2}$.

   Which of the following gives the real values of x in D and the corresponding values of $g(x)$?

   A. $x \geq 0$ and $-1 \leq g(x) \leq 1$

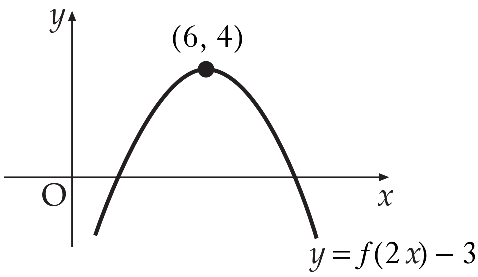
   B. $x \geq 0$ and $0 \leq g(x) \leq 1$

   C. $x \geq 2$ and $-1 \leq g(x) \leq 1$

   D. $x \geq 2$ and $0 \leq g(x) \leq 1$

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
D	1.2	A/B	0	0	CN	A1, T1	2011 P1 Q20

2. The diagram shows the graph of $y = f(2x) - 3$.



What are the coordinates of the turning point on the graph of $y = f(x)$?

A. $(12, 7)$

B. $(12, 1)$

C. $(3, 7)$

D. $(3, 1)$

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
A	1.2	A/B	0	0	CN	A3	2010 P1 Q20

3. If $f(x) = (x-3)(x+5)$, for what values of x is the graph of $y = f(x)$ above the x-axis?

A. $-5 < x < 3$

B. $-3 < x < 5$

C. $x < -5,\ x > 3$

D. $x < -3,\ x > 5$

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
C	2.1	A/B	0	0	CN	A16	2011 P1 Q18

4. The discriminant of a quadratic equation is 23.

   Here are two statements about this quadratic equation:

   I. the roots are real;

   II. the roots are rational.

   Which of the following is true?

   A. neither statement is correct

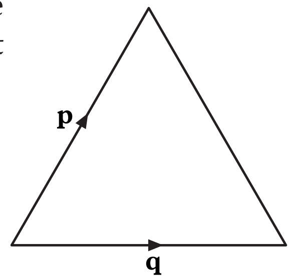
   B. only statement I is correct

   C. only statement II is correct

   D. both statements are correct

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
B	2.1	A/B	0	0	NC	A17	2011 P1 Q9

5. An equilateral triangle of side 3 units is shown.The vectors $\boldsymbol{p}$ and $\boldsymbol{q}$ are as represented in the diagram.What is the value of $\boldsymbol{p}.\boldsymbol{q}$?



A. 9

B. $\frac{9}{2}$

C. $\frac{9}{\sqrt{2}}$

D. 0

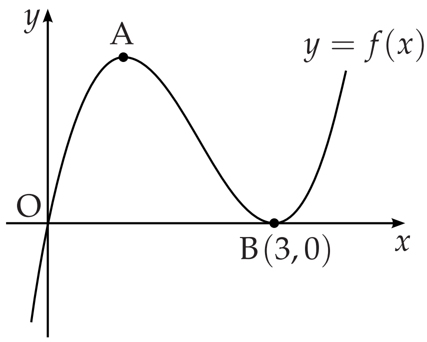
Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
B	3.1	A/B	0	0	CN	G26, G28	2011 P1 Q14

[END OF PAPER 1 SECTION A]

Paper 1 Section B

[SQA] 6. A sketch of the graph of $y = f(x)$ where $f(x) = x^3 - 6x^2 + 9x$ is shown below.

The graph has a maximum at A and a minimum at $B(3, 0)$.



(*a*) Find the coordinates of the turning point at A. **4**

(*b*) Hence sketch the graph of $y = g(x)$ where $g(x) = f(x+2) + 4$.

Indicate the coordinates of the turning points. There is no need to calculate the coordinates of the points of intersection with the axes. **2**

(*c*) Write down the range of values of k for which $g(x) = k$ has 3 real roots. **1**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
(*a*)	4	C	NC	C8	$A(1, 4)$	2000 P1 Q2
(*b*)	2	C	NC	A3	sketch (translate 4 up, 2 left)	
(*c*)	1	A/B	NC	A2	$4 < k < 8$	

•1 ss: know to differentiate •2 pd: differentiate correctly •3 ss: know gradient = 0 •4 pd: process •5 ic: interpret transformation •6 ic: interpret transformation •7 ic: interpret sketch	•1 $\frac{dy}{dx} = \dots$ •2 $\frac{dy}{dx} = 3x^2 - 12x + 9$ •3 $3x^2 - 12x + 9 = 0$ •4 $A = (1, 4)$ translate $f(x)$ 4 units up, 2 units left •5 sketch with coord. of $A'(-1, 8)$ •6 sketch with coord. of $B'(1, 4)$ •7 $4 < k < 8$ (accept $4 \le k \le 8$)

[SQA] 7. Two sequences are generated by the recurrence relations $u_{n+1} = au_n + 10$ and $v_{n+1} = a^2 v_n + 16$.

The two sequences approach the same limit as $n \to \infty$.

Determine the value of a and evaluate the limit. **5**

Part	Marks	Level	Calc.	Content	Answer	U1 OC4
	4	C	NC	A13	$a = \frac{3}{5}, L = 25$	2000 P1 Q5
	1	A/B	NC	A12		

•1 ss: know how to find limit •2 pd: process •3 pd: process •4 ic: interpret coeff. of u_n •5 pd: process	•1 $L = aL + 10$ **or** $L = a^2L + 16$ **or** $L = \frac{b}{1-a}$ •2 $L = \frac{10}{1-a}$ **or** $L = \frac{16}{1-a^2}$ •3 $\frac{10}{1-a}$ **or** $\frac{16}{1-a^2}$ •4 $10a^2 - 16a + 6 = 0$ •5 $a = \frac{3}{5}$ **and** $L = 25$

[SQA] 8.

(*a*) Solve $\cos 2x^\circ - 3\cos x^\circ + 2 = 0$ for $0 \leq x < 360$. **5**

(*b*) Hence solve $\cos 4x^\circ - 3\cos 2x^\circ + 2 = 0$ for $0 \leq x < 360$

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
(*a*)	5	C	NC	T7	$x = 0, 60, 300$	2011 P1 Q23
(*b*)	2	A	NC	T11	$x = 0, 30, 150, 180, 210, 330$	

•1 ss: know to use double angle formula •2 ic: express as a quadratic in $\cos x^\circ$ •3 ss: start to solve •4 pd: reduce to equations in cos only •5 ic: process solutions in given domain •6 ic: interpret relationship with (a) •7 ic: interpret periodicity	•1 $2\cos^2 x^\circ - 1\ldots$ •2 $2\cos^2 x^\circ - 3\cos x^\circ + 1 = 0$ •3 $(2\cos x^\circ - 1)(\cos x^\circ - 1)$ •4 $\cos x^\circ = \frac{1}{2}, 1$ •5 $0, 60, 300$ •6 $2x = 0$ and 60 and 300 •7 0, 30, 150, 180, 210 and 330

9. (*a*) Diagram 1 shows a right angled triangle, where the line OA has equation $3x - 2y = 0$.

   (i) Show that $\tan a = \frac{3}{2}$.

   (ii) Find the value of $\sin a$. **4**

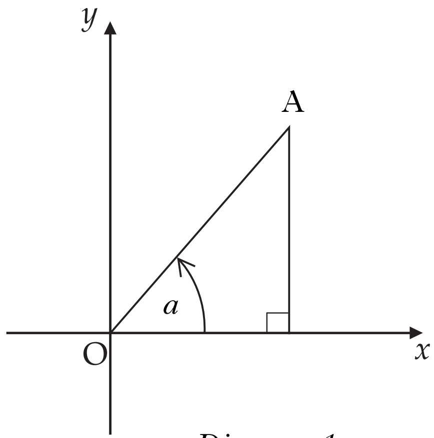


Diagram 1

(*b*) A second right angled triangle is added as shown in Diagram 2.

The line OB has equation $3x - 4y = 0$.

Find the values of $\sin b$ and $\cos b$. **4**

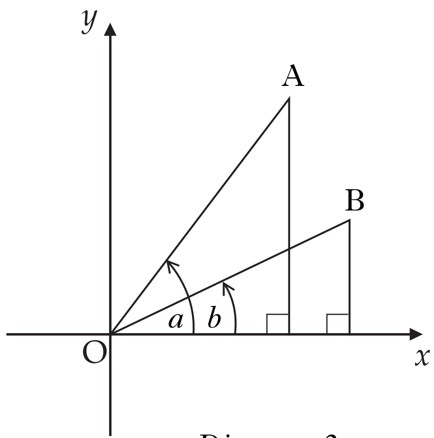


Diagram 2

(*c*) (i) Find the value of $\sin(a - b)$.

   (ii) State the value of $\sin(b - a)$. **4**

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
(*a*)	4	C	CN	G2, T5	proof, $\frac{3}{\sqrt{13}}$	2010 P1 Q23
(*b*)	4	C	CN	G2, T5	$\sin b = \frac{3}{5}, \cos b = \frac{4}{5}$	
(*c*)	4	B	CN	T8, T1	$\frac{6}{5\sqrt{13}}, -\frac{6}{5\sqrt{13}}$	

•[1] ss: write in slope/intercept form	•[1] $y = \frac{3}{2}x$
•[2] ic: connect gradient and $\tan a$	•[2] $m = \frac{3}{2}$ and $\tan a = \frac{3}{2}$
•[3] pd: calculate hypotenuse	•[3] $\sqrt{13}$
•[4] ic: state value of sine ratio	•[4] $\frac{3}{\sqrt{13}}$ or $\frac{3\sqrt{13}}{13}$
•[5] ss: determine $\tan b$	•[5] $\tan b = \frac{3}{4}$
•[6] ss: know to complete triangle	•[6] right-angled triangle with 3, 4
•[7] pd: determine hypotenuse	•[7] 5
•[8] ic: state values of sine and cosine ratios	•[8] $\sin b = \frac{3}{5}$ and $\cos b = \frac{4}{5}$
•[9] ss: know to use addition formula	•[9] $\sin a \cos b - \cos a \sin b$
•[10] ic: substitute into expansion	•[10] $\frac{3}{\sqrt{13}} \times \frac{4}{5} - \frac{2}{\sqrt{13}} \times \frac{3}{5}$
•[11] pd: evaluate sine of compound angle	•[11] $\frac{6}{5\sqrt{13}}$
•[12] ss: use $\sin(-x) = -\sin x$	•[12] $-\frac{6}{5\sqrt{13}}$

[SQA] 10. Circle P has equation $x^2 + y^2 - 8x - 10y + 9 = 0$. Circle Q has centre $(-2, -1)$ and radius $2\sqrt{2}$.

(*a*) (i) Show that the radius of circle P is $4\sqrt{2}$.

(ii) Hence show that circles P and Q touch. **4**

(*b*) Find the equation of the tangent to the circle Q at the point $(-4, 1)$. **3**

(*c*) The tangent in (*b*) intersects circle P in two points. Find the x-coordinates of the points of intersection, expressing you answers in the form $a \pm b\sqrt{3}$. **3**

Part	Marks	Level	Calc.	Content	Answer	U2 OC4
(*a*)	2	C	CN	G9	proof	2001 P1 Q11
(*a*)	2	A/B	CN	G14		
(*b*)	3	C	CN	G11	$y = x + 5$	
(*c*)	3	C	CN	G12	$x = 2 \pm 2\sqrt{3}$	

•1 ic: interpret centre of circle (P)	•1 $C_P = (4, 5)$
•2 ss: find radius of circle (P)	•2 $r_P = \sqrt{16 + 25 - 9} = \sqrt{32} = 4\sqrt{2}$
•3 ss: find sum of radii	•3 $r_P + r_Q = 4\sqrt{2} + 2\sqrt{2} = 6\sqrt{2}$
•4 pd: compare with distance between centres	•4 $C_PC_Q = \sqrt{6^2 + 6^2} = 6\sqrt{2}$ **and** "so touch"
•5 ss: find gradient of radius	•5 $m_r = -1$
•6 ss: use $m_1m_2 = -1$	•6 $m_{tgt} = +1$
•7 ic: state equation of tangent	•7 $y - 1 = 1(x + 4)$
•8 ss: substitute linear into circle	•8 $x^2 + (x+5)^2 - 8x - 10(x+5) + 9 = 0$
•9 pd: express in standard form	•9 $2x^2 - 8x - 16 = 0$
•10 pd: solve (quadratic) equation	•10 $x = 2 \pm 2\sqrt{3}$

[SQA] 11. For what range of values of k does the equation $x^2 + y^2 + 4kx - 2ky - k - 2 = 0$ represent a circle? **5**

Part	Marks	Level	Calc.	Content	Answer	U2 OC4
	5	A	NC	G9, A17	for all k	2000 P1 Q6

•1 ss: know to examine radius	•1 $g = 2k$, $f = -k$, $c = -k - 2$ *stated or implied by* •2
•2 pd: process	•2 $r^2 = 5k^2 + k + 2$
•3 pd: process	•3 (real $r \Rightarrow$) $5k^2 + k + 2 > 0$ (*accept* $\geq$)
•4 ic: interpret quadratic inequation	•4 use discr. **or** complete sq. **or** diff.
•5 ic: interpret quadratic inequation	•5 true for all k

[SQA] 12. The graph of $y = f(x)$ passes through the point $\left(\frac{\pi}{9}, 1\right)$.

If $f'(x) = \sin(3x)$ express y in terms of x. **4**

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	4	A/B	NC	C18, C23	$y = -\frac{1}{3}\cos(3x) + \frac{7}{6}$	2000 P1 Q8

•1 ss: know to integrate	•1 $y = \int \sin(3x)\,dx$ *stated or implied by* •2
•2 pd: integrate	•2 $-\frac{1}{3}\cos(3x)$
•3 ic: interpret $(\frac{\pi}{9}, 1)$	•3 $1 = -\frac{1}{3}\cos(\frac{3\pi}{9}) + c$ or equiv.
•4 pd: process	•4 $c = \frac{7}{6}$

[SQA] 13. (*a*) Find the derivative of the function $f(x) = (8 - x^3)^{\frac{1}{2}}$, $x < 2$. **2**

(*b*) Hence write down $\displaystyle\int \frac{x^2}{(8 - x^3)^{\frac{1}{2}}}\,dx$. **1**

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
(*a*)	2	A/B	CN	C21	$-\frac{3}{2}x^2(8 - x^3)^{-\frac{1}{2}}$	2002 P1 Q10
(*b*)	1	A/B	CN	C24	$-\frac{2}{3}(8 - x^3)^{\frac{1}{2}} + c$	

•1 pd: process differentiation	•1 $\frac{1}{2}(8 - x^3)^{-\frac{1}{2}}$
•2 pd: use the chain rule	•2 $\ldots \times -3x^2$
•3 ic: interpret answer from (a)	•3 $-\frac{2}{3}f(x)$ **or** $-\frac{2}{3}(8 - x^3)^{\frac{1}{2}}$

[SQA] 14. Find the maximum value of $\cos x - \sin x$ and the value of x for which it occurs in the interval $0 \leq x \leq 2\pi$. **6**

Part	Marks	Level	Calc.	Content	Answer	U3 OC4
	6	A/B	CN	T14	max value $\sqrt{2}$ when $x = \frac{7\pi}{4}$	2000 P1 Q10

•1 ss: use e.g. $k\cos(x + a)$	•1 e.g. use $k\cos(x + a)$
•2 ic: expand chosen rule	•2 $k\cos x\cos a - k\sin x\sin a$
•3 pd: compare coefficients	•3 $k\cos a = 1$ and $k\sin a = 1$
•4 pd: process	•4 $k = \sqrt{2}$
•5 pd: process	•5 $\tan a = 1$, $a = \frac{\pi}{4}$ (45° *is bad form*)
•6 ic: interpret trig expression	•6 max. value $= \sqrt{2}$ when $x = \frac{7\pi}{4}$ (*do not accept* 45°)

[END OF PAPER 1 SECTION B]

Paper 2

[SQA] 1. $f(x) = 3 - x$ and $g(x) = \dfrac{3}{x}, x \neq 0$.

(*a*) Find $p(x)$ where $p(x) = f(g(x))$. **2**

(*b*) If $q(x) = \dfrac{3}{3-x}, x \neq 3$, find $p(q(x))$ in its simplest form. **3**

Part	Marks	Level	Calc.	Content	Answer	U1 OC2
(*a*)	2	C	CN	A4	$3 - \frac{3}{x}$	2000 P2 Q3
(*b*)	2	C	CN	A4	x	
(*b*)	1	A/B	CN	A4		

•1 ic: interpret composite func.	•1 $f\left(\frac{3}{x}\right)$ *stated or implied by* •2
•2 pd: process	•2 $3 - \frac{3}{x}$
•3 ic: interpret composite func.	•3 $p\left(\frac{3}{3-x}\right)$ *stated or implied by* •4
•4 pd: process	•4 $3 - \frac{3}{\frac{3}{3-x}}$
•5 pd: process	•5 x

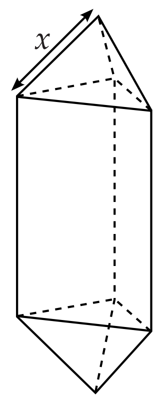
[SQA] 2. A goldsmith has built up a solid which consists of a triangular prism of fixed volume with a regular tetrahedron at each end.

The surface area, A, of the solid is given by

$$A(x) = \frac{3\sqrt{3}}{2}\left(x^2 + \frac{16}{x}\right)$$

where x is the length of each edge of the tetrahedron.

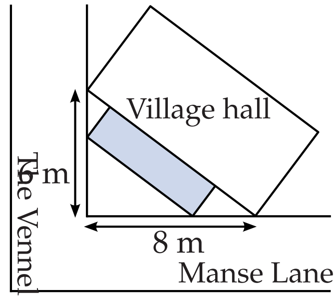
Find the value of x which the goldsmith should use to minimise the amount of gold plating required to cover the solid. **6**



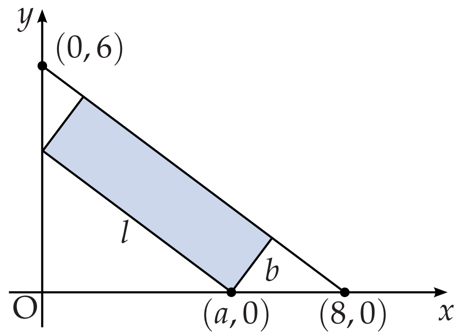
Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	6	A/B	CN	C11	$x = 2$	2000 P2 Q6

•1 ss: know to differentiate	•1 $A'(x) = \ldots$
•2 pd: process	•2 $\frac{3\sqrt{3}}{2}(2x - 16x^{-2})$ **or** $3\sqrt{3}x - 24\sqrt{3}x^{-2}$
•3 ss: know to set $f'(x) = 0$	•3 $A'(x) = 0$
•4 pd: deal with x^{-2}	•4 $-\frac{16}{x^2}$ **or** $-\frac{24\sqrt{3}}{x^2}$
•5 pd: process	•5 $x = 2$
•6 ic: check for minimum	•6 x: 2^-, 2, 2^+; $A'(x)$: $-ve$, 0, $+ve$; so $x = 2$ is min.

[SQA] 3. The shaded rectangle on this map represents the planned extension to the village hall. It is hoped to provide the largest possible area for the extension.



The coordinate diagram represents the right angled triangle of ground behind the hall. The extension has length l metres and breadth b metres, as shown. One corner of the extension is at the point $(a, 0)$.



(*a*) (i) Show that $l = \frac{5}{4}a$.

(ii) Express b in terms of a and hence deduce that the area, A m^2, of the extension is given by $A = \frac{3}{4}a(8 - a)$. **3**

(*b*) Find the value of a which produces the largest area of the extension. **4**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
(*a*)	3	A/B	CN	CGD	proof	2002 P2 Q10
(*b*)	4	A/B	CN	C11	$a = 4$	

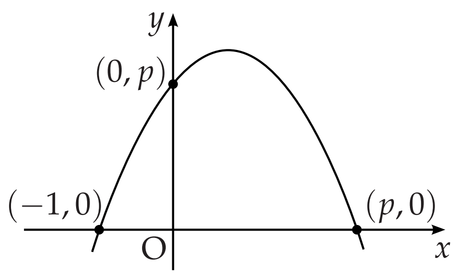
•1 ss: select strategy and carry through •2 ss: select strategy and carry through •3 ic: complete proof •4 ss: know to set derivative to zero •5 pd: differentiate •6 pd: solve equation •7 ic: justify maximum, e.g. nature table	•1 proof of $l = \frac{5}{4}a$ •2 $b = \frac{3}{5}(8 - a)$ •3 complete proof leading to $A = \ldots$ •4 $\frac{dA}{da} = \ldots = 0$ •5 $6 - \frac{3}{2}a$ •6 $a = 4$ •7 e.g. nature table, comp. the square

[SQA] 4. Show that the equation $(1-2k)x^2 - 5kx - 2k = 0$ has real roots for all integer values of k. **5**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
	5	A/B	CN	A18, A16, CGD	proof	2002 P2 Q9

•[1] ss: know to use discriminant •[2] ic: pick out discriminant •[3] pd: simplify to quadratic •[4] ss: choose to draw table or graph •[5] pd: complete proof using disc.≥ 0	•[1] discriminant = ... •[2] disc $= (-5k)^2 - 4(1-2k)(-2k)$ •[3] $9k^2 + 8k$ •[4] e.g. draw a table, graph, complete the square •[5] complete proof and conclusion relating to disc.≥ 0

[SQA] 5. The diagram shows a sketch of a parabola passing through $(-1,0)$, $(0,p)$ and $(p,0)$.



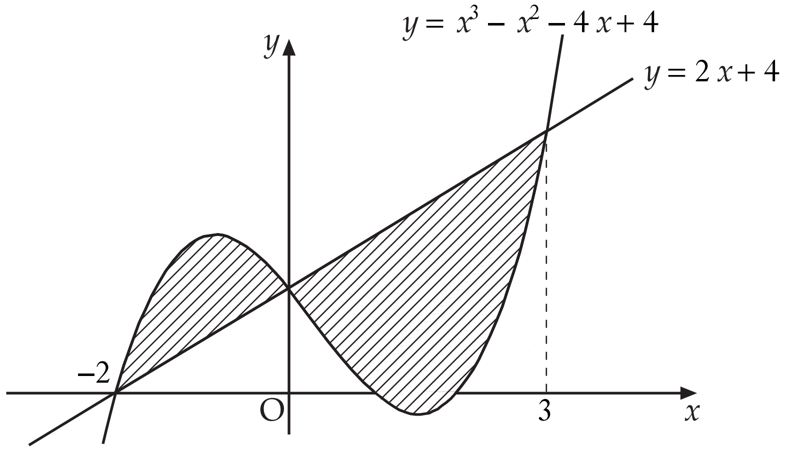
(*a*) Show that the equation of the parabola is $y = p + (p-1)x - x^2$. **3**

(*b*) For what value of p will the line $y = x + p$ be a tangent to this curve? **3**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
(*a*)	3	A/B	CN	A19	proof	2001 P2 Q11
(*b*)	3	A/B	CN	A24	2	

•[1] ss: use a standard form of parabola •[2] ss: use 3rd point to determine k •[3] pd: complete proof •[4] ss: equate and simplify to zero •[5] ss: use discriminant for tangency •[6] pd: process	•[1] $y = k(x+1)(x-p)$ •[2] $k = -1$ with justification (i.e. substitute $(0,p)$) •[3] $y = -1(x+1)(x-p)$ **and complete** •[4] $x^2 + 2x - px = 0$ •[5] $b^2 - 4ac = (2-p)^2 = 0$ **or** $(2-p)^2 - 4 \times 0 = 0$ •[6] $p = 2$

6. The diagram shows the curve with equation $y = x^3 - x^2 - 4x + 4$ and the line with equation $y = 2x + 4$. The curve and the line intersect at the points $(-2, 0)$, $(0, 4)$ and $(3, 10)$.



Calculate the total shaded area. **10**

Part	Marks	Level	Calc.	Content	Answer	U2 OC2
	10	B	CN	C17	$21\frac{1}{12}$	2011 P2 Q4

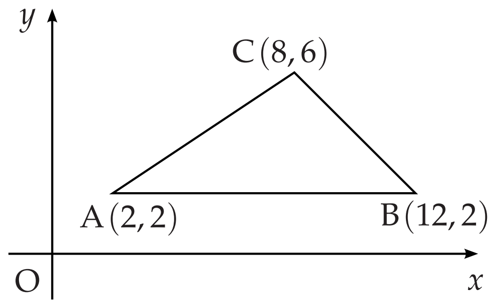
•[1] ss: know to integrate	•[1] $\int \cdots$ or attempt integration
•[2] ic: know to deal with areas on each side of y-axis	•[2] evidence of treating areas separately
•[3] ic: interpret limits of one area	•[3] e.g. $\int_0^3$
•[4] ic: use "upper − lower"	•[4] $(2x+4) - (x^3 - x^2 - 4x + 4)$
•[5] pd: integrate	•[5] $3x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4$
•[6] ic: substitute in limits	•[6] $\left(3(3)^2 + \frac{1}{3}(3)^3 - \frac{1}{4}(3)^4\right)$
•[7] pd: evaluate the area on one side	•[7] $\frac{63}{4}$
•[8] ss: interpret integrand with limits of the other area	•[8] $\int_{-2}^{0}(x^3 - x^2 - 4x + 4) - (2x + 4)\,dx$
•[9] pd: evaluate the area on the other side	•[9] $\frac{16}{3}$
•[10] ic: state total area	•[10] $21\frac{1}{12}$ or $\frac{253}{12}$ or $21{\cdot}1$

[SQA] 7. Solve the equation $3\cos 2x^\circ + \cos x^\circ = -1$ in the interval $0 \leq x \leq 360$. **5**

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
	5	A/B	CR	T10	$60, 131{\cdot}8, 228{\cdot}2, 300$	2000 P2 Q5

•[1] ss: know to use $\cos 2x = 2\cos^2 x - 1$	•[1] $3(2\cos^2 x^\circ - 1)$
•[2] pd: process	•[2] $6\cos^2 x^\circ + \cos x^\circ - 2 = 0$
•[3] ss: know to/and factorise quadratic	•[3] $(2\cos x^\circ - 1)(3\cos x^\circ + 2)$
•[4] pd: process	•[4] $\cos x^\circ = \frac{1}{2}, x = 60, 30$
•[5] pd: process	•[5] $\cos x^\circ = -\frac{2}{3}, x = 132, 228$

[SQA] 8. Triangle ABC has vertices A$(2,2)$, B$(12,2)$ and C$(8,6)$.



(*a*) Write down the equation of l_1, the perpendicular bisector of AB. **1**

(*b*) Find the equation of l_2, the perpendicular bisector of AC. **4**

(*c*) Find the point of intersection of lines l_1 and l_2. **1**

(*d*) Hence find the equation of the circle passing through A, B and C. **2**

Part	Marks	Level	Calc.	Content	Answer	U2 OC4
(*a*)	1	C	CN	G3, G7	$x = 7$	2001 P2 Q7
(*b*)	4	C	CN	G7	$3x + 2y = 23$	
(*c*)	1	C	CN	G8	$(7,1)$	
(*d*)	2	A/B	CN	G8, G9, G10	$(x-7)^2 + (y-1)^2 = 26$	

•1 ic: state equation of a vertical line	•1 $x = 7$
•2 pd: process coord. of a midpoint	•2 midpoint $= (5,4)$
•3 ss: find gradient of AC	•3 $m_{AC} = \frac{2}{3}$
•4 ic: state gradient of perpendicular	•4 $m_{\perp} = -\frac{3}{2}$
•5 ic: state equation of straight line	•5 $y - 4 = -\frac{3}{2}(x-5)$
•6 pd: find pt of intersection	•6 $x = 7, y = 1$
•7 ss: use standard form of circle equ.	•7 $(x-7)^2 + (y-1)^2$
•8 ic: find radius and complete	•8 $(x-7)^2 + (y-1)^2 = 26$
	or
	•7 $x^2 + y^2 - 14x - 2y + c = 0$
	•8 $c = 24$

9. Circle C_1 has equation $(x+1)^2 + (y-1)^2 = 121$.

A circle C_2 with equation $x^2 + y^2 - 4x + 6y + p = 0$ is drawn inside C_1.

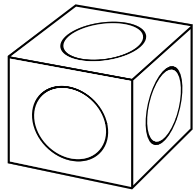
The circles have no points of contact.

What is the range of values of p? **9**

Part	Marks	Level	Calc.	Content	Answer	U2 OC4
	9	A	CN	G9, G15	$-23 < p < 13$	2011 P2 Q7

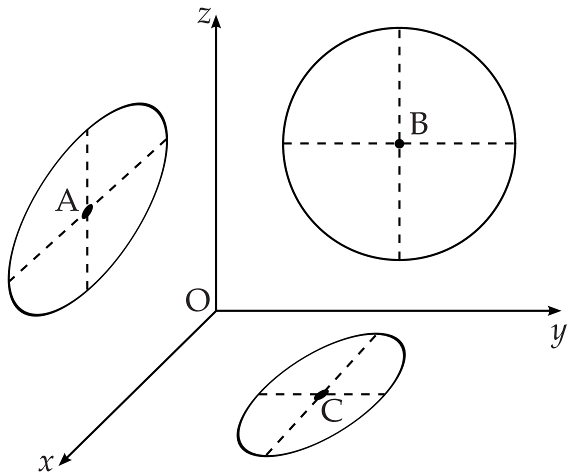
•[1] ic: state centre of C_1 •[2] ic: state radius of C_1 •[3] ic: state centre of C_2 •[4] pd: find radius of C_2 in terms of p •[5] ic: interpret upper bound for p •[6] ic: find distance between centres, d •[7] ss: identify relevant relationship •[8] ic: develop relationship by squaring •[9] pd: find lower bound for p	•[1] $(-1, 1)$ •[2] 11 ($\sqrt{121}$ not accepted) •[3] $(2, -3)$ •[4] $\sqrt{13-p}$ •[5] $p < 13$ •[6] 5 •[7] $\sqrt{13-p} < 6$ or $r_2 + d < 11$ or $r_2 < 6$ •[8] $13 - p < 36$ •[9] $p > -23$

[SQA] 10. A box in the shape of a cuboid is designed with **circles** of different sizes on each face.



The diagram shows three of the circles, where the origin represents one of the corners of the cuboid. The centres of the circles are $A(6,0,7)$, $B(0,5,6)$ and $C(4,5,0)$.

Find the size of angle ABC. **7**



Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	5	C	CR	G17, G16, G22		2001 P2 Q4
	2	A/B	CR	G26, G28	71·5°	

•1 ss: use $\frac{\overrightarrow{BA}.\overrightarrow{BC}}{\lvert\overrightarrow{BA}\rvert\lvert\overrightarrow{BC}\rvert}$	•1 use $\frac{\overrightarrow{BA}.\overrightarrow{BC}}{\lvert\overrightarrow{BA}\rvert\lvert\overrightarrow{BC}\rvert}$ *stated or implied by* •7
•2 ic: state vector e.g. $\overrightarrow{BA}$	•2 $\overrightarrow{BA} = \begin{pmatrix} 6 \\ -5 \\ 1 \end{pmatrix}$
•3 ic: state a consistent vector e.g. $\overrightarrow{BC}$	•3 $\overrightarrow{BC} = \begin{pmatrix} 4 \\ 0 \\ -6 \end{pmatrix}$
•4 pd: process $\lvert\overrightarrow{BA}\rvert$	•4 $\lvert\overrightarrow{BA}\rvert = \sqrt{62}$
•5 pd: process $\lvert\overrightarrow{BC}\rvert$	•5 $\lvert\overrightarrow{BC}\rvert = \sqrt{52}$
•6 pd: process scalar product	•6 $\overrightarrow{BA}.\overrightarrow{BC} = 18$
•7 pd: find angle	•7 $A\hat{B}C = 71{\cdot}5°$

[SQA] 11. A curve for which $\frac{dy}{dx} = 3\sin(2x)$ passes through the point $\left(\frac{5\pi}{12}, \sqrt{3}\right)$.

Find y in terms of x. **4**

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	4	A/B	CN	C18, C23	$y = -\frac{3}{2}\cos(2x) + \frac{1}{4}\sqrt{3}$	2001 P2 Q10

•1 pd: integrate trig function •2 pd: integrate composite function •3 ss: use given point to find "c" •4 pd: evaluate "c"	•1 $\int 3\sin(2x)\,dx$ *stated or implied by* •2 •2 $-\frac{3}{2}\cos(2x)$ •3 $\sqrt{3} = -\frac{3}{2}\cos(2 \times \frac{5}{12}\pi) + c$ •4 $c = \frac{1}{4}\sqrt{3}$ ($\approx 0{\cdot}4$)

[SQA] 12. Given that $f(x) = (5x-4)^{\frac{1}{2}}$, evaluate $f'(4)$. **3**

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	1	C	CN	C21	$\frac{5}{8}$	2000 P2 Q8
	2	A/B	CN	C21		

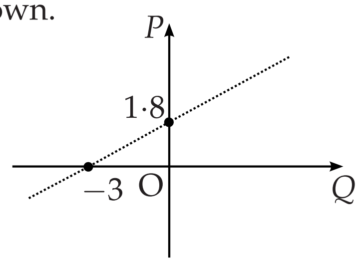
•1 pd: differentiate power •2 pd: differentiate 2nd function •3 pd: evaluate $f'(x)$	•1 $\frac{1}{2}(5x-4)^{-\frac{1}{2}}$ •2 $\times 5$ •3 $f'(4) = \frac{5}{8}$

[SQA] 13. Find $\int \frac{1}{(7-3x)^2}\,dx$. **2**

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	2	A/B	CN	C22, C14	$\frac{1}{3(7-3x)} + c$	2000 P2 Q10

•1 pd: integrate function •2 pd: deal with function of function	•1 $\frac{1}{-1}(7-3x)^{-1}$ •2 $\times \frac{1}{-3}$

[SQA] 14. The results of an experiment give rise to the graph shown.



(*a*) Write down the equation of the line in terms of P and Q. **2**

It is given that $P = \log_e p$ and $Q = \log_e q$.

(*b*) Show that p and q satisfy a relationship of the form $p = aq^b$, stating the values of a and b. **4**

Part	Marks	Level	Calc.	Content	Answer	U3 OC3
(*a*)	2	A/B	CR	G3	$P = 0{\cdot}6Q + 1{\cdot}8$	2000 P2 Q11
(*b*)	4	A/B	CR	A33	$a = 6{\cdot}05$, $b = 0{\cdot}6$	

•[1] ic: interpret gradient •[2] ic: state equ. of line •[3] ic: interpret straight line •[4] ss: know how to deal with x of $x \log y$ •[5] ss: know how to express number as log •[6] ic: interpret sum of two logs	•[1] $m = \frac{1{\cdot}8}{3} = 0{\cdot}6$ •[2] $P = 0{\cdot}6Q + 1{\cdot}8$ Method 1 •[3] $\log_e p = 0{\cdot}6 \log_e q + 1{\cdot}8$ •[4] $\log_e q^{0{\cdot}6}$ •[5] $\log_e 6{\cdot}05$ •[6] $p = 6{\cdot}05q^{0{\cdot}6}$ Method 2 $\ln p = \ln aq^b$ •[3] $\ln p = \ln a + b \ln q$ •[4] $\ln p = 0{\cdot}6 \ln q + 1{\cdot}8$ *stated or implied by •[5] or •[6]* •[5] $\ln a = 1{\cdot}8$ •[6] $a = 6{\cdot}05$, $b = 0{\cdot}6$

15. (*a*) The expression $3\sin x - 5\cos x$ can be written in the form $R\sin(x+a)$ where $R > 0$ and $0 \leq a < 2\pi$.

Calculate the values of R and a. **4**

(*b*) Hence find the value of t, where $0 \leq t \leq 2$, for which

$$\int_0^t (3\cos x + 5\sin x)\ dx = 3.$$

7

Part	Marks	Level	Calc.	Content	Answer	U3 OC4
(*a*)	4	C	CN	T13	$R = \sqrt{34}, a = 5{\cdot}253$	2011 P2 Q6
(*b*)	7	B	CN	C23, T3, T16	$t = 0{\cdot}6$	

•1 ss: use compound angle formula	•1 $R\sin x\cos a + R\cos x\sin a$
•2 ic: compare coefficients	•2 $R\cos a = 3$ and $R\sin a = -5$
•3 pd: process R	•3 $\sqrt{34}$ (accept 5·8)
•4 pd: process a	•4 5·253 (accept 5·3)
•5 pd: integrate given expression	•5 $3\sin x - 5\cos x$
•6 ic: substitute limits	•6 $(3\sin t - 5\cos t) - (3\sin 0 - 5\cos 0)$
•7 pd: process limits	•7 $3\sin t - 5\cos t + 5$
•8 ss: know to use wave equation	•8 $\sqrt{34}\sin(t + 5{\cdot}3) + 5$
•9 ic: write in standard format	•9 $\sin(t + 5{\cdot}3) = -\frac{2}{\sqrt{34}}$
•10 ss: start to solve equation	•10 $t + 5{\cdot}3 = 3{\cdot}5, 5{\cdot}9$
•11 pd: complete and state solution	•11 $t = 0{\cdot}6$

[END OF PAPER 2]