Calculus Calculator AB Grade

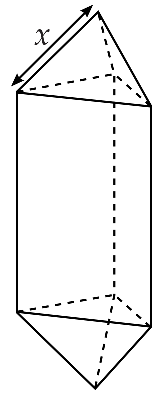
[SQA] 1. A goldsmith has built up a solid which consists of a triangular prism of fixed volume with a regular tetrahedron at each end.

The surface area, A, of the solid is given by

$$A(x) = \frac{3\sqrt{3}}{2}\left(x^2 + \frac{16}{x}\right)$$

where x is the length of each edge of the tetrahedron.

Find the value of x which the goldsmith should use to minimise the amount of gold plating required to cover the solid.

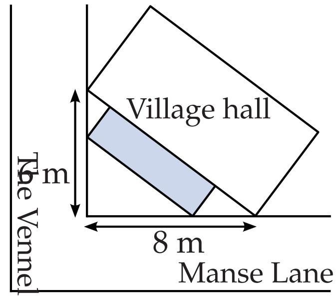


6

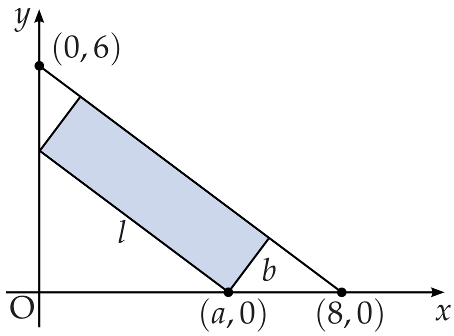
Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	6	A/B	CN	C11	$x = 2$	2000 P2 Q6

•1 ss: know to differentiate •2 pd: process •3 ss: know to set $f'(x) = 0$ •4 pd: deal with x^{-2} •5 pd: process •6 ic: check for minimum	•1 $A'(x) = \ldots$ •2 $\frac{3\sqrt{3}}{2}(2x - 16x^{-2})$ **or** $3\sqrt{3}x - 24\sqrt{3}x^{-2}$ •3 $A'(x) = 0$ •4 $-\frac{16}{x^2}$ **or** $-\frac{24\sqrt{3}}{x^2}$ •5 $x = 2$ •6 x | 2^- 2 2^+ ; $A'(x)$ | $-ve$ 0 $+ve$ so $x = 2$ is min.

[SQA] 2. The shaded rectangle on this map represents the planned extension to the village hall. It is hoped to provide the largest possible area for the extension.



The coordinate diagram represents the right angled triangle of ground behind the hall. The extension has length l metres and breadth b metres, as shown. One corner of the extension is at the point $(a, 0)$.



(*a*) (i) Show that $l = \frac{5}{4}a$.

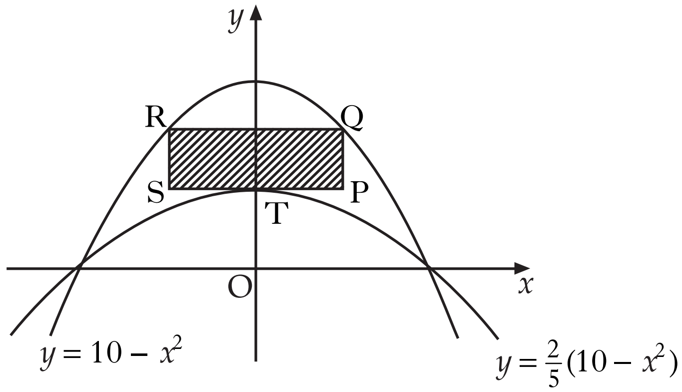
(ii) Express b in terms of a and hence deduce that the area, A m^2, of the extension is given by $A = \frac{3}{4}a(8 - a)$. **3**

(*b*) Find the value of a which produces the largest area of the extension. **4**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
(*a*)	3	A/B	CN	CGD	proof	2002 P2 Q10
(*b*)	4	A/B	CN	C11	$a = 4$	

•[1] ss: select strategy and carry through •[2] ss: select strategy and carry through •[3] ic: complete proof •[4] ss: know to set derivative to zero •[5] pd: differentiate •[6] pd: solve equation •[7] ic: justify maximum, e.g. nature table	•[1] proof of $l = \frac{5}{4}a$ •[2] $b = \frac{3}{5}(8 - a)$ •[3] complete proof leading to $A = \ldots$ •[4] $\frac{dA}{da} = \ldots = 0$ •[5] $6 - \frac{3}{2}a$ •[6] $a = 4$ •[7] e.g. nature table, comp. the square

3. The parabolas with equations $y = 10 - x^2$ and $y = \frac{2}{5}(10 - x^2)$ are shown in the diagram below.



A rectangle PQRS is placed between the two parabolas as shown, so that:

- Q and R lie on the upper parabola.
- RQ and SP are parallel to the x-axis.
- T , the turning point of the lower parabola, lies on SP.

(*a*) (i) If TP $= x$ units, find an expression for the length of PQ.

(ii) Hence show that the area, A, of rectangle PQRS is given by

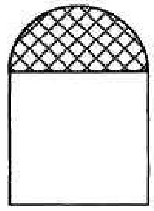
$$A(x) = 12x - 2x^3.$$ **3**

(*b*) Find the maximum area of this rectangle. **6**

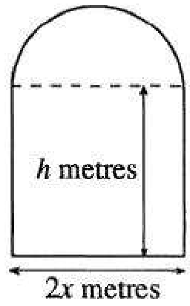
Part	Marks	Level	Calc.	Content	Answer	U1 OC3
(*ai*)	2	B	CN	C11	$6 - x^2$	2010 P2 Q5
(*aii*)	1	B	CN	C11	$2x \times (6 - x^2) = A(x)$	
(*b*)	6	C	CN	C11	max is $8\sqrt{2}$	

•1 ss: know to and find OT	•1 4
•2 ic: obtain an expression for PQ	•2 $10 - x^2 - 4$
•3 ic: complete area evaluation	•3 $2x(6 - x^2) = 12x - 2x^3$
•4 ss: know to and start to differentiate	•4 $A'(x) = 12\cdots$
•5 pd: complete differentiation	•5 $12 - 6x^2$
•6 ic: set derivative to zero	•6 $12 - 6x^2 = 0$
•7 pd: obtain	•7 $\sqrt{2}$
•8 ss: justify nature of stationary point	•8 x: $\cdots$, $\sqrt{2}$, $\cdots$; $A'(x)$: $+$, 0, $-$
•9 ic: interpret result and evaluate area	•9 Max and $8\sqrt{2}$

[SQA] 4. A window in the shape of a rectangle surmounted by a semicircle is being designed to let in the maximum amount of light.
The glass to be used for the semicircular part is stained glass which lets in one unit of light per square metre; the rectangular part uses clear glass which lets in 2 units of light per square metre.



The rectangle measures $2x$ metres by h metres.



(*a*) (i) If the perimeter of the whole window is 10 metres, express h in terms of x. **(2)**

(ii) Hence show that the amount of light, L, let in by the window is given by $L = 20x - 4x^2 - \frac{3}{2}\pi x^2$. **(2)**

(*b*) Find the values of x and h that must be used to allow this design to let in the maximum amount of light. **(5)**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
(*a*)	1	C	CN	CGD		1996 P2 Q11
(*a*)	3	A/B	CN	CGD		
(*b*)	2	C	CN	C11		
(*b*)	3	A/B	CN	C11		

(a) •1 eg $2h + 2x + \text{semicircle} = 10$

•2 $h = \frac{1}{2}(10 - \pi x - 2x)$

•3 $L = 2 \times 2xh + \frac{1}{2}\pi x^2$

•4 $L = 4x \times \frac{1}{2}(10 - \pi x - 2x) + \frac{1}{2}\pi x^2$

$L = 20x - 4x^2 - \frac{3}{2}\pi x^2$

(b) •5 $L' = 20 - 8x - 3\pi x$

•6 $L' = 0$

•7 $x = \dfrac{20}{3\pi + 8} = x_0$ $(= 1.148)$

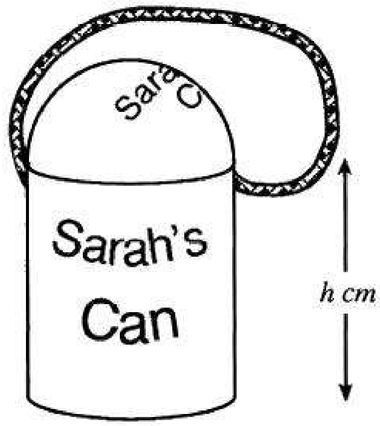
•8

x	x_0^-	x_0	x_0^+
L'	$+$	0	$-$

maximum at x_0

•9 $h = \dfrac{5\pi + 20}{3\pi + 8}$ $(= 2.049)$

[SQA] 5. A child's drinking beaker is in the shape of a cylinder with a hemispherical lid and a circular flat base. The radius of the cylinder is r cm and the height is h cm. The volume of the cylinder is 400 cm^3.



(*a*) Show that the surface area of plastic, $A(r)$, needed to make the beaker is given by $A(r) = 3\pi r^2 + \frac{800}{r}$. **(3)**

Note: The curved surface area of a hemisphere of radius r is $2\pi r^2$.

(*b*) Find the value of r which ensures that the surface area of plastic is minimised. **(6)**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
(*a*)	3	A/B	CR	CGD		1998 P2 Q10
(*b*)	3	C	CR	C11		
(*b*)	3	A/B	CR	C11		

(*a*)
•¹ $\pi r^2 + 2\pi rh + 2\pi r^2$
•² $h = \frac{400}{\pi r^2}$ or equivalent ($e.g.$ $\pi rh = \frac{400}{r}$)
•³ $2\pi r \frac{400}{\pi r^2} + 3\pi r^2$ and completes proof

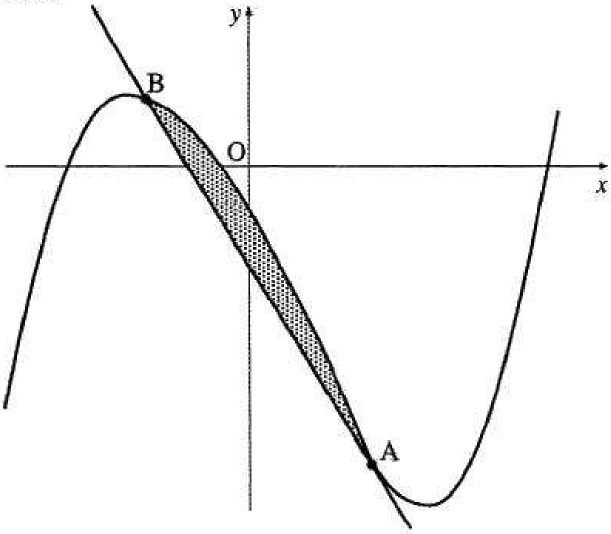
(*b*)
•⁴ $\frac{dA}{dr} = \ldots$
•⁵ $800r^{-1}$
•⁶ $6\pi r - 800r^{-2}$
•⁷ $e.g.$ $6\pi r - \frac{800}{r^2} = 0$
•⁸ 3.5
•⁹

r	3.5^-	3.5	3.5^+
$\frac{dA}{dr}$	$-ve$	0	$+ve$

[SQA] 6. In the diagram below a winding river has been modelled by the curve $y = x^3 - x^2 - 6x - 2$ and a road has been modelled by the straight line AB. The road is a tangent to the river at the point A(1, − 8).

(*a*) Find the equation of the tangent at A and hence find the coordinates of B. **(8)**

(*b*) Find the area of the shaded part which represents the land bounded by the river and the road. **(3)**



Part	Marks	Level	Calc.	Content	Answer	U2 OC2
(*a*)	5	C	CN	C4, G3, A23		1996 P2 Q8
(*a*)	3	A/B	CN	C4, G3, A23		
(*b*)	3	A/B	CN	C17		

(a)

•1 *strat*: $\frac{dy}{dx} = \ldots$

•2 $\frac{dy}{dx} = 3x^2 - 2x - 6$

•3 $m_{tgt} = -5$

•4 $y + 8 = -5(x - 1)$

•5 *strat*: attempt to simplify and equate *y*'s

•6 $x^3 - x^2 - x + 1 = 0$

•7 *strat*: *e.g.* try to factorise

•8 $B = (-1, 2)$

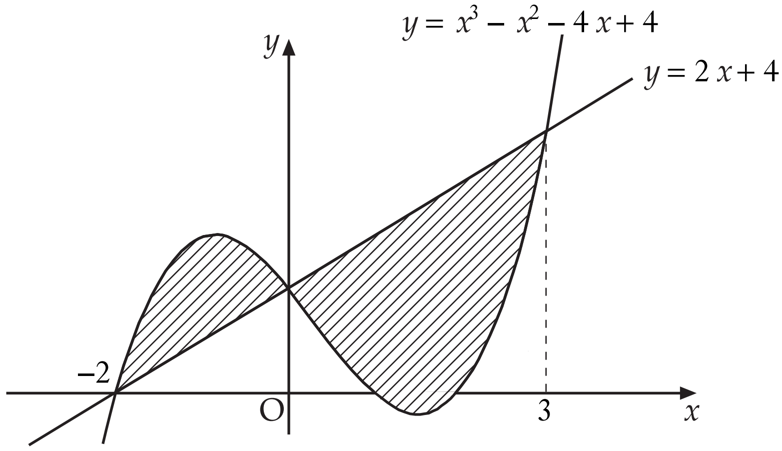
(b)

•9 $\int \left(x^3 - x^2 - 6x - 2\right) - (-5x - 3)\ dx$

•10 $\left[\frac{1}{4}x^4 - \frac{1}{3}x^3 - \frac{1}{2}x^2 + x\right]$

•11 $1\frac{1}{3}$

7. The diagram shows the curve with equation $y = x^3 - x^2 - 4x + 4$ and the line with equation $y = 2x + 4$. The curve and the line intersect at the points $(-2, 0)$, $(0, 4)$ and $(3, 10)$.



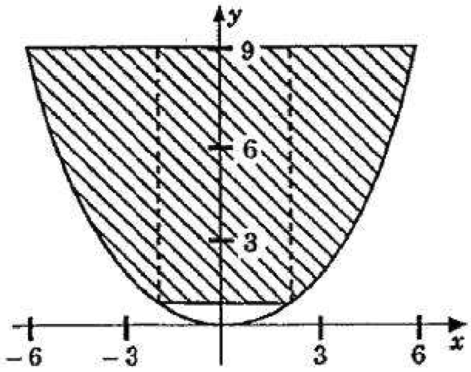
Calculate the total shaded area. **10**

Part	Marks	Level	Calc.	Content	Answer	U2 OC2
	10	B	CN	C17	$21\frac{1}{12}$	2011 P2 Q4

•1 ss: know to integrate	•1 $\int \cdots$ or attempt integration
•2 ic: know to deal with areas on each side of y-axis	•2 evidence of treating areas separately
•3 ic: interpret limits of one area	•3 e.g. $\int_0^3$
•4 ic: use "upper − lower"	•4 $(2x+4) - (x^3 - x^2 - 4x + 4)$
•5 pd: integrate	•5 $3x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4$
•6 ic: substitute in limits	•6 $\left(3(3)^2 + \frac{1}{3}(3)^3 - \frac{1}{4}(3)^4\right)$
•7 pd: evaluate the area on one side	•7 $\frac{63}{4}$
•8 ss: interpret integrand with limits of the other area	•8 $\int_{-2}^{0} (x^3 - x^2 - 4x + 4) - (2x + 4)\,dx$
•9 pd: evaluate the area on the other side	•9 $\frac{16}{3}$
•10 ic: state total area	•10 $21\frac{1}{12}$ or $\frac{253}{12}$ or $21{\cdot}1$

[SQA] 8. The cargo space of a small bulk carrier is 60m long.

The shaded part of the diagram represents the uniform cross-section of this space. It is shaped like the parabola with equation $y = \frac{1}{4}x^2$, $-6 \le x \le 6$, between the lines $y = 1$ and $y = 9$. Find the area of this cross-section and hence find the volume of cargo that this ship can carry.



(9)

Part	Marks	Level	Calc.	Content	Answer	U2 OC2
	3	C	CN	C17, CGD		1994 P2 Q10
	6	A/B	CN	C17, CGD		

- •[1] strategy: split into approp. parts
- •[2] $y = 1 \Rightarrow x = \pm 2$
- •[3] first rectangular area
- •[4] $9 - \frac{1}{4}x^2$ for integrand of shaped area
- •[5] $\int_2^5 dx$ for limits of shaped area
- •[6] for integrating...... $\left(9x - \frac{1}{12}x^3\right)$
- •[7] for evaluating...... $\left(\frac{56}{3}\right)$
- •[8] total cross-sectional area $= \frac{208}{3} (m^2)$
- •[9] volume $= 4160\ (m^3)$

9. (*a*) A curve has equation $y = (2x-9)^{\frac{1}{2}}$.

Show that the equation of the tangent to this curve at the point where $x = 9$ is $y = \frac{1}{3}x$. **5**

(*b*) Diagram 1 shows part of the curve and the tangent.

The curve cuts the x-axis at the point A.

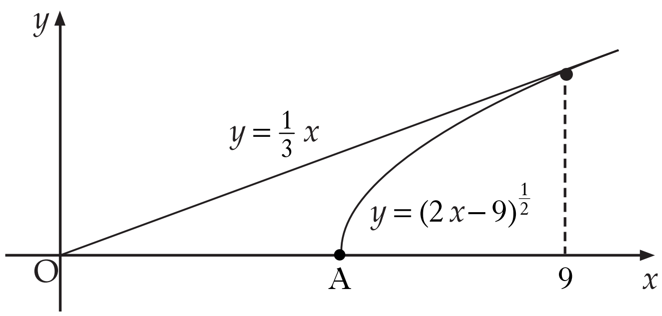


Diagram 1

Find the coordinates of point A. **1**

(*c*) Calculate the shaded area shown in diagram 2. **7**

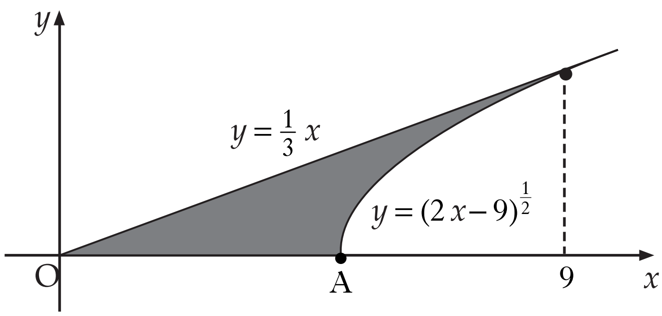


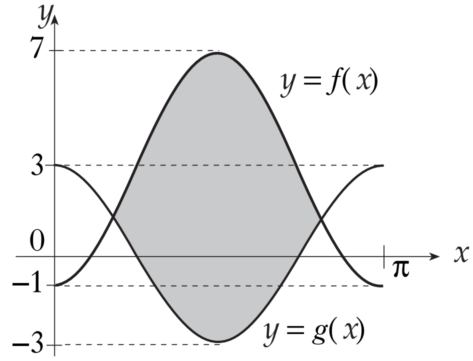
Diagram 2

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
(*a*)	5	B	CN	C21, C24	proof	2010 P2 Q6
(*b*)	1	C	CN	A6	$(\frac{9}{2}, 0)$	
(*c*)	7	A	CN	C17, C22	$\frac{9}{2} = 4\frac{1}{2} = 4{\cdot}5$	

•1 ss: know to and start to differentiate	•1 $\frac{1}{2}(2x-9)^{-\frac{1}{2}}\cdots$
•2 pd: complete chain rule derivative	•2 $\cdots \times 2$
•3 pd: gradient via differentiation	•3 $\frac{1}{3}$
•4 pd: obtain y_{curve} at $x = 9$	•4 3
•5 ic: state equation and complete	•5 $y - 3 = \frac{1}{3}(x-9)$ and complete
•6 ic: obtain coordinates of A	•6 $(\frac{9}{2}, 0)$
•7 ss: strategy for finding shaded area	•7 Shaded area = area of large △ − area under curve
•8 ss: know to integrate $(2x-9)^{\frac{1}{2}}$	•8 $\int (2x-9)^{\frac{1}{2}}\,dx$
•9 pd: start integration	•9 $\dfrac{(2x-9)^{\frac{3}{2}}}{\frac{3}{2}}\cdots$
•10 pd: complete integration	•10 $\cdots \times \frac{1}{2}$
•11 ic: limits x_A and 9	•11 $\frac{9}{2}$ and 9
•12 pd: substitute limits	•12 $\frac{1}{3}(18-9)^{\frac{3}{2}} - 0$
•13 pd: evaluate area and complete strategy	

[SQA] 10. The graphs of $y = f(x)$ and $y = g(x)$ are shown in the diagram.

$f(x) = -4\cos(2x) + 3$ and $g(x)$ is of the form $g(x) = m\cos(nx)$.



(*a*) Write down the values of m and n. **1**

(*b*) Find, correct to one decimal place, the coordinates of the points of intersection of the two graphs in the interval $0 \le x \le \pi$. **5**

(*c*) Calculate the shaded area. **6**

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
(*a*)	1	C	CN	T4	$m = 3$ and $n = 2$	2009 P2 Q5
(*b*)	5	C	CR	T6	$(0{\cdot}6, 1{\cdot}3), (2{\cdot}6, 1{\cdot}3)$	
(*c*)	6	B	CR	C17, C23	$12{\cdot}4$	

•1 ic: interprets graph	•1 $m = 3$ and $n = 2$
•2 ss: knows how to find intersection	•2 $3\cos 2x = -4\cos 2x + 3$
•3 pd: starts to solve	•3 $\cos 2x = \frac{3}{7}$
•4 pd: finds x-coord in 1st quadrant	•4 $x = 0{\cdot}6$
•5 pd: finds x-coord in 2nd quadrant	•5 $x = 2{\cdot}6$
•6 pd: finds y-coordinates	•6 $y = 1{\cdot}3, 1{\cdot}3$
•7 ss: knows how to find area	•7 $\int \left(-4\cos 2x + 3 - 3\cos 2x\right)\,dx$
•8 ic: states limits	•8 $\int_{0\cdot6}^{2\cdot6} \ldots$
•9 pd: integrate	•9 "$-7\sin 2x$"
•10 pd: integrate	•10 $3x - \frac{7}{2}\sin 2x$
•11 ic: substitute limits	•11 $(3 \times 2{\cdot}6 - \frac{7}{2}\sin 5{\cdot}2) - (3 \times 0{\cdot}6 - \frac{7}{2}\sin 1{\cdot}2)$
•12 pd: evaluate area	•12 $12{\cdot}4$

[SQA] 11. A curve for which $\dfrac{dy}{dx} = 3\sin(2x)$ passes through the point $\left(\frac{5\pi}{12}, \sqrt{3}\right)$.

Find y in terms of x. **4**

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	4	A/B	CN	C18, C23	$y = -\frac{3}{2}\cos(2x) + \frac{1}{4}\sqrt{3}$	2001 P2 Q10

•1 pd: integrate trig function	•1 $\int 3\sin(2x)\,dx$ *stated or implied by* •2
•2 pd: integrate composite function	•2 $-\frac{3}{2}\cos(2x)$
•3 ss: use given point to find "c"	•3 $\sqrt{3} = -\frac{3}{2}\cos(2 \times \frac{5}{12}\pi) + c$
•4 pd: evaluate "c"	•4 $c = \frac{1}{4}\sqrt{3}$ ($\approx 0{\cdot}4$)

[SQA] 12. Find $\int \frac{1}{(7-3x)^2}\,dx$. **2**

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	2	A/B	CN	C22, C14	$\frac{1}{3(7-3x)}+c$	2000 P2 Q10

•[1] pd: integrate function •[2] pd: deal with function of function	•[1] $\frac{1}{-1}(7-3x)^{-1}$ •[2] $\times\frac{1}{-3}$

[SQA] 13. Given that $f(x)=(5x-4)^{\frac{1}{2}}$, evaluate $f'(4)$. **3**

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	1	C	CN	C21	$\frac{5}{8}$	2000 P2 Q8
	2	A/B	CN	C21		

•[1] pd: differentiate power •[2] pd: differentiate 2nd function •[3] pd: evaluate $f'(x)$	•[1] $\frac{1}{2}(5x-4)^{-\frac{1}{2}}$ •[2] $\times 5$ •[3] $f'(4)=\frac{5}{8}$

[END OF QUESTIONS]