

Calculus Calculator C Grade

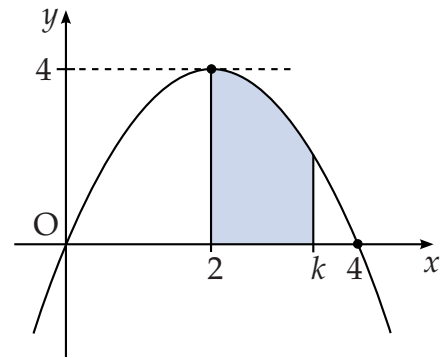
[SQA] 1. Find $\int \frac{(x^2 - 2)(x^2 + 2)}{x^2} dx, x \neq 0$.

4

Part	Marks	Level	Calc.	Content	Answer	U2 OC2
	4	C	CN	C14, C12, C13	$\frac{1}{3}x^3 + 4x^{-1} + c$	2001 P2 Q6
<ul style="list-style-type: none"> •¹ ss: start to write in standard form •² pd: complete process •³ pd: integrate •⁴ pd: integrate a -ve index 				<ul style="list-style-type: none"> •¹ $\frac{x^4 - 4}{x^2}$ •² $x^2 - 4x^{-2}$ •³ $\frac{1}{3}x^3 + c$ •⁴ $\frac{-4x^{-1}}{-1}$ 		

[SQA] 2. The parabola shown crosses the x -axis at $(0,0)$ and $(4,0)$, and has a maximum at $(2,4)$.

The shaded area is bounded by the parabola, the x -axis and the lines $x = 2$ and $x = k$.



- (a) Find the equation of the parabola.
- (b) Hence show that the shaded area, A , is given by

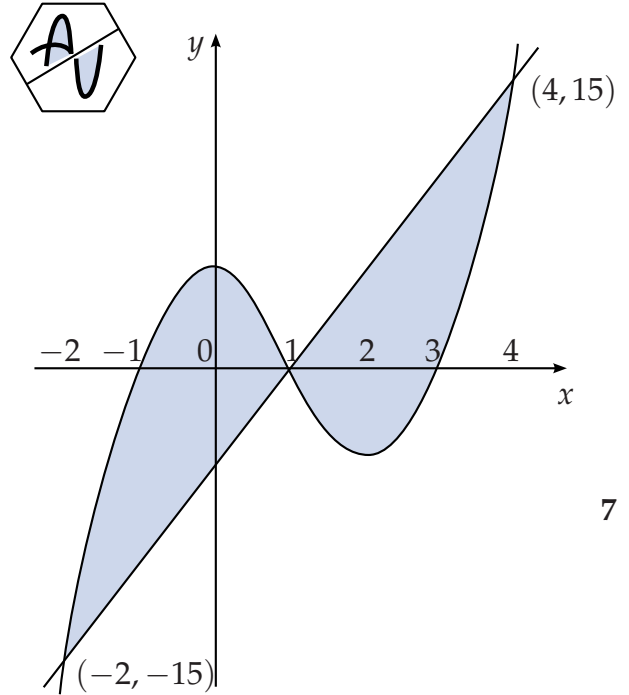
$$A = -\frac{1}{3}k^3 + 2k^2 - \frac{16}{3}.$$

2

3

Part	Marks	Level	Calc.	Content	Answer	U2 OC2
(a)	2	C	CN	A19	$y = 4x - x^2$	2000 P2 Q4
(b)	3	C	CN	C16	proof	
<ul style="list-style-type: none"> •¹ ic: state standard form •² pd: process for x^2 coeff. •³ ss: know to integrate •⁴ pd: integrate correctly •⁵ pd: process limits and complete proof 				<ul style="list-style-type: none"> •¹ $ax(x - 4)$ •² $a = -1$ •³ \int_2^k (function from (a)) •⁴ $-\frac{1}{3}x^3 + 2x^2$ •⁵ $-\frac{1}{3}k^3 + 2k^2 - (-\frac{8}{3} + 8)$ 		

- [SQA] 3. A firm asked for a logo to be designed involving the letters A and U. Their initial sketch is shown in the hexagon.



A mathematical representation of the final logo is shown in the coordinate diagram.

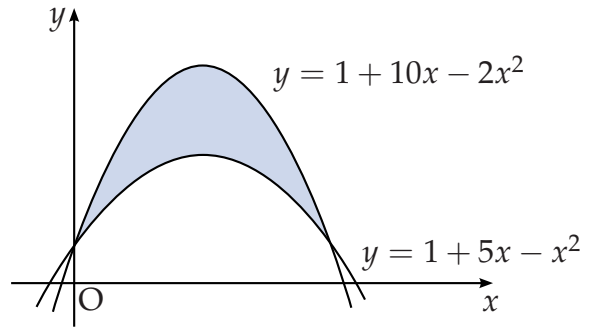
The curve has equation $y = (x + 1)(x - 1)(x - 3)$ and the straight line has equation $y = 5x - 5$. The point $(1, 0)$ is the centre of half-turn symmetry.

Calculate the total shaded area.

7

Part	Marks	Level	Calc.	Content	Answer	U2 OC2
	7	C	CN	C17	$40\frac{1}{2}$ units ²	2001 P2 Q8
<ul style="list-style-type: none"> •¹ ss: express in standard form •² ss: split area and integrate •³ ss: subtract functions •⁴ pd: process •⁵ pd: process •⁶ pd: process •⁷ ic: use symmetry or otherwise for total area 					<ul style="list-style-type: none"> •¹ $y = x^3 - 3x^2 - x + 3$ •² $\int_1^4 (\dots) dx$ or $\int_{-2}^1 (\dots) dx$ •³ $\int [(5x - 5) - (x^3 - 3x^2 - x + 3)] dx$ or $\int [(x^3 - 3x^2 - x + 3) - (5x - 5)] dx$ •⁴ $\int (-x^3 + 3x^2 + 6x - 8) dx$ •⁵ $[-\frac{1}{4}x^4 + x^3 + 3x^2 - 8x]$ •⁶ $20\frac{1}{4}$ or $-20\frac{1}{4}$ depending on chosen integrals •⁷ $40\frac{1}{2}$ 	

[SQA] 4. Calculate the shaded area enclosed between the parabolas with equations $y = 1 + 10x - 2x^2$ and $y = 1 + 5x - x^2$.



6

Part	Marks	Level	Calc.	Content	Answer	U2 OC2
	6	C	CN	C17	$20\frac{5}{6}$	2002 P2 Q5
<ul style="list-style-type: none"> •¹ ss: find intersections •² ss: know to find limits •³ ss: know to integrate (upper – lower) •⁴ pd: simplify •⁵ pd: integrate •⁶ pd: process limits 				<ul style="list-style-type: none"> •¹ $1 + 10x - 2x^2 = 1 + 5x - x^2$ •² $x = 0,5$ and $\int_0^5 ()$ •³ $\int ((1 + 10x - 2x^2) - (1 + 5x - x^2)) dx$ •⁴ $\int (5x - x^2) dx$ •⁵ $\frac{5}{2}x^2 - \frac{1}{3}x^3$ •⁶ $20\frac{5}{6}$ 		

[SQA] 5. Find the equation of the tangent to the curve $y = 2 \sin(x - \frac{\pi}{6})$ at the point where $x = \frac{\pi}{3}$.

4

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	4	C	CN	C5, C20	$y = \sqrt{3}x + 1 - \frac{\pi}{\sqrt{3}}$	2002 P2 Q6
<ul style="list-style-type: none"> •¹ pd: find derivative •² ss: know derivative at $x = \dots$ represents grad. •³ pd: find corresponding y-coordinate •⁴ ic: state equation of tangent 				<ul style="list-style-type: none"> •¹ $\frac{dy}{dx} = 2 \cos(x - \frac{\pi}{6})$ •² $m = \sqrt{3}$ •³ $y_{x=\frac{\pi}{3}} = 1$ •⁴ $y - 1 = \sqrt{3}(x - \frac{\pi}{3})$ 		

[SQA] 6. Given that $f(x) = (5x - 4)^{\frac{1}{2}}$, evaluate $f'(4)$.

3

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	1	C	CN	C21	$\frac{5}{8}$	2000 P2 Q8
	2	A/B	CN	C21		

<ul style="list-style-type: none"> •¹ pd: differentiate power •² pd: differentiate 2nd function •³ pd: evaluate $f'(x)$ 	<ul style="list-style-type: none"> •¹ $\frac{1}{2}(5x - 4)^{-\frac{1}{2}}$ •² $\times 5$ •³ $f'(4) = \frac{5}{8}$
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[END OF QUESTIONS]