

## Calculus Non Calculator AB Grade

- [SQA] 1. A curve has equation  $y = 2x^3 + 3x^2 + 4x - 5$ .

Prove that this curve has no stationary points.

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Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	2	C	NC	C8, C7		1999 P1 Q16
	3	A/B	NC	C8, C7		

<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\frac{dy}{dx} = \dots\dots</math></li> <li>•<sup>2</sup> <math>6x^2 + 6x + 4</math></li> <li>•<sup>3</sup> e.g. "<math>b^2 - 4ac</math>" = .....</li> <li>•<sup>4</sup> <math>-60</math> or <math>-15</math> (from <math>3x^2 + 3x + 2</math>)</li> <li>•<sup>5</sup> <math>\Delta</math> negative so no st. points</li> </ul>	OR	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\frac{dy}{dx} = \dots\dots</math></li> <li>•<sup>2</sup> <math>6x^2 + 6x + 4</math></li> <li>•<sup>3</sup> e.g. complete square.....</li> <li>•<sup>4</sup> <math>S = 6\left(x + \frac{1}{2}\right)^2 + 2\frac{1}{2}</math></li> <li>•<sup>5</sup> <math>S \geq 2\frac{1}{2}</math> so no st. points</li> </ul>
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- [SQA] 2. Find the values of  $x$  for which the function  $f(x) = 2x^3 - 3x^2 - 36x$  is increasing.

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Part	Marks	Level	Calc.	Content	Answer	U2 OC1
	2	C	NC	C7, A16		1996 P1 Q16
	2	A/B	NC	C7, A16		

- <sup>1</sup> know to consider  $f'(x) > 0$       stated or implied by the evidence for •<sup>4</sup>.
- <sup>2</sup>  $\frac{dy}{dx} = 6x^2 - 6x - 36$
- <sup>3</sup>  $6(x - 3)(x + 2) > 0$       or by formula or completing the square
- <sup>4</sup>  $x < -2, x > 3$

3. If  $f(x) = (x - 3)(x + 5)$ , for what values of  $x$  is the graph of  $y = f(x)$  above the  $x$ -axis?

- A.  $-5 < x < 3$
- B.  $-3 < x < 5$
- C.  $x < -5, x > 3$
- D.  $x < -3, x > 5$

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Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
C	2.1	A/B	0	0	CN	A16	2011 P1 Q18

4. The discriminant of a quadratic equation is 23.

Here are two statements about this quadratic equation:

- I. the roots are real;
- II. the roots are rational.

Which of the following is true?

- A. neither statement is correct
- B. only statement I is correct
- C. only statement II is correct
- D. both statements are correct

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Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
B	2.1	A/B	0	0	NC	A17	2011 P1 Q9

[SQA] 5. Find  $\int \frac{x^2 - 5}{x\sqrt{x}} dx$ .

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Part	Marks	Level	Calc.	Content	Answer	U2 OC2
	2	C	NC	C14		1999 P1 Q20
	2	A/B	NC	C13		

$\bullet^1 \left( \frac{x^2}{x\sqrt{x}} = \right) x^{\frac{1}{2}}$	$\bullet^3 \frac{x^{\frac{3}{2}}}{\frac{3}{2}}$
$\bullet^2 \left( \frac{-5}{x\sqrt{x}} = \right) -5x^{-\frac{3}{2}}$	$\bullet^4 \frac{-5}{-\frac{1}{2}} x^{-\frac{1}{2}}$

[SQA] 6. Find the value of  $\int_1^2 \frac{u^2 + 2}{2u^2} du$ .

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Part	Marks	Level	Calc.	Content	Answer	U2 OC2
	4	C	NC	C15		1989 P1 Q16
	1	A/B	NC	C15		

- <sup>1</sup> strat: know to divide
- <sup>2</sup>  $\frac{1}{2} + u^{-2}$
- <sup>3</sup>  $\frac{1}{2}u$
- <sup>4</sup>  $-u^{-1}$
- <sup>5</sup> 1

[SQA] 7. Differentiate  $\sin 2x + \frac{2}{\sqrt{x}}$  with respect to  $x$ .

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Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	2	C	NC	C3		1989 P1 Q10
	2	A/B	NC	C20		

- <sup>1</sup>  $2x^{-\frac{1}{2}}$
- <sup>2</sup>  $\cos 2x$
- <sup>3</sup>  $\times 2$
- <sup>4</sup>  $-x^{-\frac{3}{2}}$

[SQA] 8. (a) Find the derivative of the function  $f(x) = (8 - x^3)^{\frac{1}{2}}$ ,  $x < 2$ .

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(b) Hence write down  $\int \frac{x^2}{(8 - x^3)^{\frac{1}{2}}} dx$ .

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Part	Marks	Level	Calc.	Content	Answer	U3 OC2
(a)	2	A/B	CN	C21	$-\frac{3}{2}x^2(8 - x^3)^{-\frac{1}{2}}$	2002 P1 Q10
(b)	1	A/B	CN	C24	$-\frac{2}{3}(8 - x^3)^{\frac{1}{2}} + c$	

<ul style="list-style-type: none"> <li>•<sup>1</sup> pd: process differentiation</li> <li>•<sup>2</sup> pd: use the chain rule</li> <li>•<sup>3</sup> ic: interpret answer from (a)</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\frac{1}{2}(8 - x^3)^{-\frac{1}{2}}</math></li> <li>•<sup>2</sup> <math>\dots \times -3x^2</math></li> <li>•<sup>3</sup> <math>-\frac{2}{3}f(x)</math> or <math>-\frac{2}{3}(8 - x^3)^{\frac{1}{2}}</math></li> </ul>
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[SQA] 9. Given that  $f(x) = 5(7 - 2x)^3$ , find the value of  $f'(4)$ .

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Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	4	A/B	NC	C21		1991 P1 Q13

- <sup>1</sup>  $(7 - 2x)^2$
- <sup>2</sup>  $\times 15$
- <sup>3</sup>  $\times -2$
- <sup>4</sup>  $-30$

[SQA] 10. Differentiate  $2x^{\frac{3}{2}} + \sin^2 x$  with respect to  $x$ .

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Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	1	C	NC	C21		1992 P1 Q11
	3	A/B	NC	C21		

- <sup>1</sup>  $3x^{\frac{1}{2}}$
- <sup>2</sup>  $(\sin x)^2$  stated or implied by •<sup>3</sup>
- <sup>3</sup>  $2 \sin x$
- <sup>4</sup>  $\times \cos x$

[SQA] 11. Find the derivative, with respect to  $x$ , of  $\frac{1}{x^3} + \cos 3x$ .

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Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	4	A/B	NC	C21		1994 P1 Q10

- <sup>1</sup>  $x^{-3}$  stated or implied by •<sup>2</sup>
- <sup>2</sup>  $-3x^{-4}$
- <sup>3</sup>  $-\sin 3x$
- <sup>4</sup>  $\times 3$

[SQA] 12. Differentiate  $\sin^3 x$  with respect to  $x$ .

Hence find  $\int \sin^2 x \cos x dx$ .

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Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	1	C	NC	C21, C19		1994 P1 Q17
	3	A/B	NC	C21, C19		

- <sup>1</sup> using  $(\sin x)^3$  stated or implied by •<sup>2</sup>
- <sup>2</sup>  $3 \sin^2 x$
- <sup>3</sup>  $\times \cos x$
- <sup>4</sup>  $\frac{1}{3} \sin^3 x$

[SQA] 13. Given  $f(x) = \cos^2 x - \sin^2 x$ , find  $f'(x)$ .

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Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	1	C	NC	C21		1999 P1 Q19
	2	A/B	NC	C21, C20		

<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>f(x) = \cos 2x</math></li> <li>•<sup>2</sup> <math>-\sin 2x</math></li> <li>•<sup>3</sup> <math>\times 2</math></li> </ul>		<p style="text-align: center;">For <math>\frac{d}{dx}(\cos^2 x)</math>    OR    For <math>\frac{d}{dx}(-\sin^2 x)</math></p> <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>2 \cos x</math></li> <li>•<sup>2</sup> <math>\times -\sin x</math></li> </ul> <p style="text-align: center;">For <math>\frac{d}{dx}(-\sin^2 x)</math></p> <ul style="list-style-type: none"> <li>•<sup>3</sup> <math>-2 \sin x \times \cos x</math></li> </ul>		<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>-2 \sin x</math></li> <li>•<sup>2</sup> <math>\times \cos x</math></li> </ul> <p style="text-align: center;">For <math>\frac{d}{dx}(\cos^2 x)</math></p> <ul style="list-style-type: none"> <li>•<sup>3</sup> <math>2 \cos x \times -\sin x</math></li> </ul>
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[SQA] 14. Find  $\frac{dy}{dx}$  given that  $y = \sqrt{1 + \cos x}$ .

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Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	3	A/B	NC	C21, C20		1996 P1 Q13

- <sup>1</sup>  $(1 + \cos x)^{\frac{1}{2}}$  stated or implied by •<sup>2</sup>
- <sup>2</sup>  $\frac{1}{2}(1 + \cos x)^{-\frac{1}{2}}$
- <sup>3</sup>  $\times -\sin x$

[SQA] 15. Given  $f(x) = (\sin x + 1)^2$ , find the exact value of  $f'(\frac{\pi}{6})$ .

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Part	Marks	Level	Calc.	Content	Answer	U3 OC2			
	3	A/B	NC	C21, C20, T2		1998 P1 Q16			
<table style="width:100%; border:none;"> <tr> <td style="width:50%; border:none;"> <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>2(\sin x + 1)</math></li> <li>•<sup>2</sup> <math>\times \cos x</math></li> <li>•<sup>3</sup> <math>\frac{3\sqrt{3}}{2}</math></li> </ul> </td> <td style="width:10%; border:none; text-align: center;">Alternative</td> <td style="width:40%; border:none;"> <ul style="list-style-type: none"> <li>•<sup>1</sup> expand and differentiate <math>2 \sin x + 1</math></li> <li>•<sup>2</sup> differentiate <math>\sin^2 x</math></li> <li>•<sup>3</sup> <math>\frac{3\sqrt{3}}{2}</math></li> </ul> </td> </tr> </table>							<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>2(\sin x + 1)</math></li> <li>•<sup>2</sup> <math>\times \cos x</math></li> <li>•<sup>3</sup> <math>\frac{3\sqrt{3}}{2}</math></li> </ul>	Alternative	<ul style="list-style-type: none"> <li>•<sup>1</sup> expand and differentiate <math>2 \sin x + 1</math></li> <li>•<sup>2</sup> differentiate <math>\sin^2 x</math></li> <li>•<sup>3</sup> <math>\frac{3\sqrt{3}}{2}</math></li> </ul>
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[SQA] 16. Find  $\int \sqrt{1 + 3x} dx$  and hence find the exact value of  $\int_0^1 \sqrt{1 + 3x} dx$ .

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Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	4	A/B	NC	C22		1993 P1 Q16
<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>(1+3x)^{\frac{1}{2}}</math></li> <li>•<sup>2</sup> <math>\frac{1}{\frac{3}{2}}(1+3x)^{-\frac{1}{2}}</math></li> <li>•<sup>3</sup> <math>+ 3</math></li> <li>•<sup>4</sup> <math>\frac{14}{9}</math></li> </ul>						

[SQA] 17.

(a) Evaluate  $\int_0^{\frac{\pi}{2}} \cos 2x dx$ .

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(b) Draw a sketch and explain your answer.

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Part	Marks	Level	Calc.	Content	Answer	U3 OC2		
(a)	3	A/B	NC	C23		1992 P1 Q14		
(b)	1	C	NC	T1, C16				
(b)	1	A/B	NC	T1, C16				
<table style="width:100%; border:none;"> <tr> <td style="width:40%; border:none;"> <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\frac{1}{2}</math></li> <li>•<sup>2</sup> <math>\sin 2x</math></li> <li>•<sup>3</sup> 0</li> <li>•<sup>4</sup> diagram</li> <li>•<sup>5</sup> +ve and -ve cancel out</li> </ul> </td> <td style="width:60%; border:none; text-align: center;"> </td> </tr> </table>							<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\frac{1}{2}</math></li> <li>•<sup>2</sup> <math>\sin 2x</math></li> <li>•<sup>3</sup> 0</li> <li>•<sup>4</sup> diagram</li> <li>•<sup>5</sup> +ve and -ve cancel out</li> </ul>	
<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\frac{1}{2}</math></li> <li>•<sup>2</sup> <math>\sin 2x</math></li> <li>•<sup>3</sup> 0</li> <li>•<sup>4</sup> diagram</li> <li>•<sup>5</sup> +ve and -ve cancel out</li> </ul>								

- [SQA] 18. The curve  $y = f(x)$  passes through the point  $(\frac{\pi}{12}, 1)$  and  $f'(x) = \cos 2x$ .  
Find  $f(x)$ .

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Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	3	A/B	NC	C23		1997 P1 Q15

- <sup>1</sup>  $\frac{1}{2} \sin 2x$
- <sup>2</sup>  $1 = \frac{1}{2} \sin \frac{\pi}{6} + c$
- <sup>3</sup>  $c = \frac{3}{4}$

[END OF QUESTIONS]