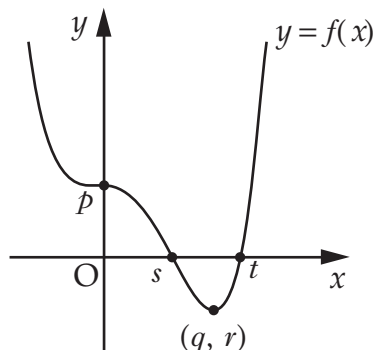


Calculus Non Calculator C Grade

1. The graph of $y = f(x)$ shown has stationary points at $(0, p)$ and (q, r) .



Here are two statements about $f(x)$:

- I. $f(x) < 0$ for $s < x < t$;
- II. $f'(x) < 0$ for $x < q$.

Which of the following is true?

- A. neither statement is correct
- B. only statement I is correct
- C. only statement II is correct
- D. both statements are correct

2

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
B	1.3	C	0	0	NC	A2, C7	2012 P1 Q18

2. If $y = 3x^{-2} + 2x^{\frac{3}{2}}$, $x > 0$, determine $\frac{dy}{dx}$.

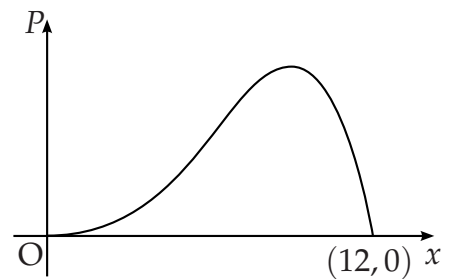
- A. $-6x^{-3} + \frac{4}{5}x^{\frac{5}{2}}$
- B. $-3x^{-1} + 3x^{\frac{1}{2}}$
- C. $-6x^{-3} + 3x^{\frac{1}{2}}$
- D. $-3x^{-1} + \frac{4}{5}x^{\frac{5}{2}}$

2

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
C	1.3	C	0	0	NC	C1, C2	2012 P1 Q6



[SQA] 3. A company spends x thousand pounds a year on advertising and this results in a profit of P thousand pounds. A mathematical model, illustrated in the diagram, suggests that P and x are related by $P = 12x^3 - x^4$ for $0 \leq x \leq 12$.



Find the value of x which gives the maximum profit.

5

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	5	C	NC	C11	$x = 9$	2001 P1 Q6

<ul style="list-style-type: none"> •¹ ss: start diff. process •² pd: process •³ ss: set derivative to zero •⁴ pd: process •⁵ ic: interpret solutions 	<ul style="list-style-type: none"> •¹ $\frac{dP}{dx} = 36x^2 \dots$ or $\frac{dP}{dx} = \dots - 4x^3$ •² $\frac{dP}{dx} = 36x^2 - 4x^3$ •³ $\frac{dP}{dx} = 0$ •⁴ $x = 0$ and $x = 9$ •⁵ nature table about $x = 0$ and $x = 9$
--	---

4. Differentiate $2\sqrt[3]{x}$ with respect to x .

- A. $6\sqrt{x}$
 B. $\frac{3}{2}\sqrt[3]{x^4}$
 C. $-\frac{4}{3\sqrt[3]{x^2}}$
 D. $\frac{2}{3\sqrt[3]{x^2}}$

2

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
D	1.3	C	0.83	0.38	NC	C2, C3	HSN 091

$$2\sqrt[3]{x} = 2x^{1/3}$$

$$\frac{d}{dx}(2x^{1/3}) = \frac{1}{3} \times 2x^{-2/3} = \frac{2}{3\sqrt[3]{x^2}} \quad \text{Option D}$$

5. If $f(x) = \frac{1}{\sqrt[5]{x}}$, $x \neq 0$, what is $f'(x)$?

- A. $-\frac{1}{5}x^{-6/5}$
 B. $-\frac{1}{5}x^{-4/5}$
 C. $-\frac{5}{2}x^{-7/2}$
 D. $-\frac{5}{2}x^{-3/2}$

2

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
A	1.3	C	0	0	CN	C3, C2	2010 P1 Q12

6. What is the derivative of $\frac{1}{4x^3}$, $x \neq 0$?

A. $\frac{1}{12x^2}$

B. $-\frac{1}{12x^2}$

C. $\frac{4}{x^4}$

D. $-\frac{3}{4x^4}$

2

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
D	1.3	C	0	0	NC	C3, C2	2009 P1 Q8

7. A tangent to the curve with equation $y = x^3 - 2x$ is drawn at the point $(2, 4)$.

What is the gradient of this tangent?

A. 2

B. 3

C. 4

D. 10

2

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
D	1.3	C	0	0	NC	C4	2011 P1 Q4

8. What is the gradient of the tangent to the curve with equation $y = x^3 - 6x + 1$ at the point where $x = -2$?
- A. -24
B. 3
C. 5
D. 6

2

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
D	1.3	C	0	0	NC	C4	2012 P1 Q2

9. A curve has equation $y = 5x^3 - 12x$.
What is the gradient of the tangent at the point $(1, -7)$?
- A. -7
B. -5
C. 3
D. 5

2

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
C	1.3	C	0	0	NC	C4	2009 P1 Q4

- [SQA] 10. Find the x -coordinate of each of the points on the curve $y = 2x^3 - 3x^2 - 12x + 20$ at which the tangent is parallel to the x -axis. 4

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	4	C	NC	C4		1993 P1 Q4

<ul style="list-style-type: none"> •¹ $\frac{dy}{dx} = \dots\dots$ •² $6x^2 - 6x - 12$ •³ $\dots\dots = 0$ •⁴ $x = -1, 2$

- [SQA] 11. The point $P(-1, 7)$ lies on the curve with equation $y = 5x^2 + 2$. Find the equation of the tangent to the curve at P . 4

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	4	C	NC	C4, G3		1999 P1 Q9

<ul style="list-style-type: none"> •¹ $\frac{dy}{dx} = \dots\dots$ •² $10x$ •³ -10 •⁴ $y - 7 = -10(x - (-1))$

- [SQA] 12. Find the equation of the tangent to the curve $y = 4x^3 - 2$ at the point where $x = -1$. 4

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	4	C	NC	C4, G3		1990 P1 Q2

<ul style="list-style-type: none"> •¹ strat: $\frac{dy}{dx} = \dots\dots$ •² $\frac{dy}{dx} = 12x^2$ •³ $m = 12$ •⁴ $y - (-6) = 12(x - (-1))$

- [SQA] 13. Find the equation of the tangent to the curve $y = 3x^2 + 2$ at the point where $x = 1$. 4

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	4	C	NC	C4, G3		1991 P1 Q5

<ul style="list-style-type: none"> •¹ strat: $\frac{dy}{dx} = \dots\dots$ •² $f'(1) = 6$ •³ $f(1) = 5$ •⁴ $y - 5 = 6(x - 1)$
--

- [SQA] 14. Find the equation of the tangent to the curve with equation $y = 5x^3 - 6x^2$ at the point where $x = 1$. 4

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	4	C	NC	C4, G3		1992 P1 Q1

<ul style="list-style-type: none"> •¹ $y' = 15x^2 - 12x$ •² $y'(1) = 3$ •³ $y(1) = -1$ •⁴ $y - (-1) = 3(x - 1)$

15. The derivative of a function f is given by $f'(x) = x^2 - 9$.

Here are two statements about f :

- (1) f is increasing at $x = 1$;
- (2) f is stationary at $x = -3$.

Which of the following is true?

- A. Neither statement is correct.
- B. Only statement (1) is correct.
- C. Only statement (2) is correct.
- D. Both statements are correct.

2

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
C	1.3	C	0	0	CN	C4, C7	2010 P1 Q15

16. If $s(t) = t^2 - 5t + 8$, what is the rate of change of s with respect to t when $t = 3$?

- A. -5
- B. 1
- C. 2
- D. 9

2

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
B	1.3	C	0	0	NC	C6	2010 P1 Q17

17. The volume of a sphere is given by the formula $V = \frac{4}{3}\pi r^3$. What is the rate of change of V with respect to r , at $r = 2$?

- A. $\frac{16\pi}{3}$
- B. $\frac{32\pi}{3}$
- C. 16π
- D. 32π

2

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
C	1.3	C	0	0	NC	C6	2012 P1 Q8

18. $A = 2\pi r^2 + 6\pi r$

What is the rate of change of A with respect to r when $r = 2$?

- A. 10π
- B. 12π
- C. 14π
- D. 20π

2

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
C	1.3	C	0	0	NC	C6	2009 P1 Q20

[SQA] 19. For what values of x is the function $f(x) = \frac{1}{3}x^3 - 2x^2 - 5x - 4$ increasing?

5

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	2	C	NC	C7		1990 P1 Q16
	3	A/B	NC	C7		

<ul style="list-style-type: none"> •¹ $f'(x) = x^2 - 4x - 5$ •² use $f'(x) > 0$ •³ zeros at $x = 5$ and $x = -1$ •⁴ strat. e.g. for $-1 < x < 5$ test $x = 0$ •⁵ $x < -1, x > 5$ _____
--

[SQA] 20. The point $P(-2, b)$ lies on the graph of the function $f(x) = 3x^3 - x^2 - 7x + 4$.

(a) Find the value of b .

1

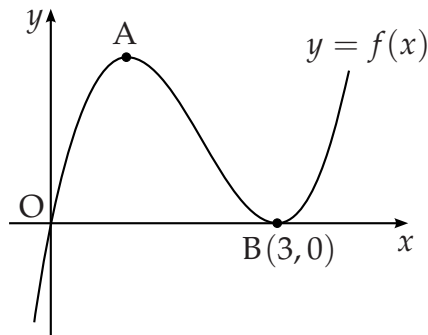
(b) Prove that this function is increasing at P .

3

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
(a)	1	C	NC	A6		1995 P1 Q10
(b)	3	C	NC	C7		

<ul style="list-style-type: none"> •¹ $b = -10$ •² know to differentiate and know to show $\frac{dy}{dx} \Big _{x=-2} > 0$ •³ $9x^2 - 2x - 7$ •⁴ show that $\frac{dy}{dx} \Big _{x=-2} > 0$

- [SQA] 21. A sketch of the graph of $y = f(x)$ where $f(x) = x^3 - 6x^2 + 9x$ is shown below.
The graph has a maximum at A and a minimum at B(3,0).



- (a) Find the coordinates of the turning point at A. 4
- (b) Hence sketch the graph of $y = g(x)$ where $g(x) = f(x + 2) + 4$.
Indicate the coordinates of the turning points. There is no need to calculate the coordinates of the points of intersection with the axes. 2
- (c) Write down the range of values of k for which $g(x) = k$ has 3 real roots. 1

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
(a)	4	C	NC	C8	A(1,4)	2000 P1 Q2
(b)	2	C	NC	A3	sketch (translate 4 up, 2 left)	
(c)	1	A/B	NC	A2	$4 < k < 8$	

<ul style="list-style-type: none"> •¹ ss: know to differentiate •² pd: differentiate correctly •³ ss: know gradient = 0 •⁴ pd: process •⁵ ic: interpret transformation •⁶ ic: interpret transformation •⁷ ic: interpret sketch 	<ul style="list-style-type: none"> •¹ $\frac{dy}{dx} = \dots$ •² $\frac{dy}{dx} = 3x^2 - 12x + 9$ •³ $3x^2 - 12x + 9 = 0$ •⁴ $A = (1,4)$ <p>translate $f(x)$ 4 units up, 2 units left</p> <ul style="list-style-type: none"> •⁵ sketch with coord. of $A'(-1,8)$ •⁶ sketch with coord. of $B'(1,4)$ •⁷ $4 < k < 8$ (accept $4 \leq k \leq 8$)
--	--

22. Find $\int (4x^{\frac{1}{2}} + x^{-3}) dx$, where $x > 0$.

A. $2x^{-\frac{1}{2}} - 3x^{-4} + c$

B. $2x^{-\frac{1}{2}} - \frac{1}{2}x^{-2} + c$

C. $\frac{8}{3}x^{\frac{3}{2}} - 3x^{-4} + c$

D. $\frac{8}{3}x^{\frac{3}{2}} - \frac{1}{2}x^{-2} + c$

2

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
D	2.2	C	0	0	CN	C12, C13	2011 P1 Q11

[SQA] 23. Evaluate $\int_1^2 \left(x^2 + \frac{1}{x}\right)^2 dx$.

5

Part	Marks	Level	Calc.	Content	Answer	U2 OC2
	5	C	NC	C12, C13, C15		1998 P1 Q12

• ¹	know to expand brackets	• ⁴	$-\frac{1}{x}$
• ²	$x^4 + 2x + x^{-2}$	• ⁵	$9\frac{7}{10}$
• ³	$\frac{1}{5}x^5 + x^2$		

24. Find $\int \frac{1}{3x^4} dx$, where $x \neq 0$.

A. $-\frac{1}{9x^3} + c$

B. $-\frac{1}{x^3} + c$

C. $\frac{1}{x^3} + c$

D. $\frac{1}{12x^3} + c$

2

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
A	2.2	C	0	0	CN	C13, C14	2011 P1 Q16

25. Find $\int \left(\frac{1}{6x^2}\right) dx$, $x \neq 0$.

A. $-12x^{-3} + c$

B. $-6x^{-1} + c$

C. $-\frac{1}{3}x^{-3} + c$

D. $-\frac{1}{6}x^{-1} + c$

2

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
D	2.2	C	0	0	NC	C13, C14	2012 P1 Q11

[SQA] 26. Find $\int \frac{x^2 - 5}{x\sqrt{x}} dx$.

4

Part	Marks	Level	Calc.	Content	Answer	U2 OC2
	2	C	NC	C14		1999 P1 Q20
	2	A/B	NC	C13		

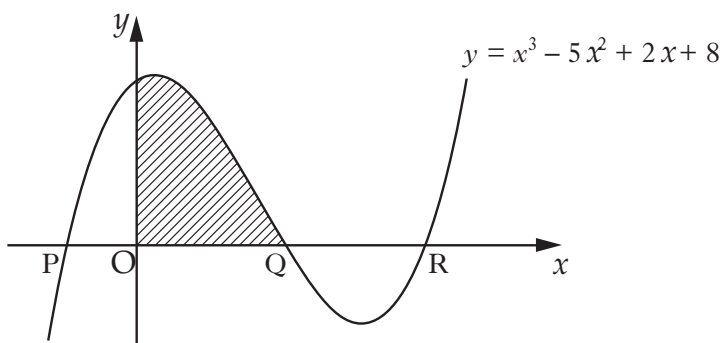
•¹ $\left(\frac{x^2}{x\sqrt{x}} = \right) x^{\frac{1}{2}}$ •³ $\frac{x^{\frac{3}{2}}}{\frac{3}{2}}$

•² $\left(\frac{-5}{x\sqrt{x}} = \right) -5x^{-\frac{3}{2}}$ •⁴ $\frac{-5}{-\frac{1}{2}} x^{-\frac{1}{2}}$

27. (a) (i) Show that $(x - 4)$ is a factor of $x^3 - 5x^2 + 2x + 8$.
 (ii) Factorise $x^3 - 5x^2 + 2x + 8$ fully.
 (iii) Solve $x^3 - 5x^2 + 2x + 8 = 0$.

6

(b) The diagram shows the curve with equation $y = x^3 - 5x^2 + 2x + 8$.



The curve crosses the x -axis at P, Q and R.

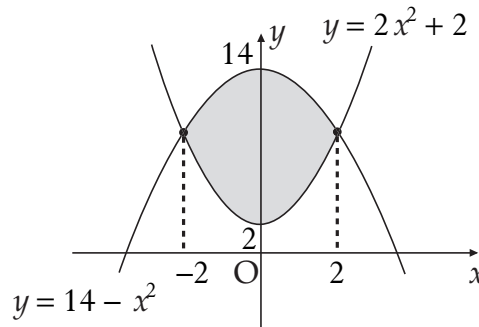
Determine the shaded area.

6

Part	Marks	Level	Calc.	Content	Answer	U2 OC2
(a)	6	C	CN	A21, A22	-1, 2, 4	2012 P1 Q21
(b)	6	C	CN	C16, C12	$\frac{32}{3}$	

<ul style="list-style-type: none"> •¹ ss know to use $x = 4$ •² pd complete evaluation •³ ic state conclusion •⁴ ic find quadratic factor •⁵ pd factorise completely •⁶ ic state solutions •⁷ ic identify x_Q from working in (a) •⁸ ic interpret appropriate limits •⁹ ss know and start to integrate •¹⁰ pd complete integration •¹¹ ic substitute limits •¹² pd state area 	<p>Method 1 : Using synthetic division</p> <ul style="list-style-type: none"> •¹ $\begin{array}{r rrrr} 4 & 1 & -5 & 2 & 8 \\ & & 4 & -4 & -8 \\ \hline & 1 & -1 & -2 & 0 \end{array}$ •² $\begin{array}{r rrrr} 4 & 1 & -5 & 2 & 8 \\ & & 4 & -4 & -8 \\ \hline & 1 & -1 & -2 & 0 \end{array}$ •³ 'remainder is zero so $(x - 4)$ is a factor' •⁴ $x^2 - x - 2$ stated, or implied by •⁵ •⁵ $(x - 4)(x - 2)(x + 1)$ stated explicitly in any order •⁶ -1, 2, 4 <p>Method 2 : Using substitution and inspection</p> <ul style="list-style-type: none"> •¹ know to use $x = 4$ •² $64 - 80 + 8 + 8 = 0$ •³ $(x - 4)$ is a factor •⁴ $(x - 4)(x^2 - x - 2)$ stated, or implied by •⁵ •⁵ $(x - 4)(x - 2)(x + 1)$ stated explicitly in any order •⁶ -1, 2, 4 •⁷ 2 •⁸ 0, 2 •⁹ integrate one term correctly (but see Note 10) •¹⁰ $\frac{1}{4}x^4 - \frac{5}{3}x^3 + \frac{2}{2}x^2 + 8x$ or equivalent •¹¹ $\left(\frac{1}{4}(2)^4 - \frac{5}{3}(2)^3 + 2^2 + 8 \times 2\right) - 0$ •¹² $\frac{32}{3}$ or $10\frac{2}{3}$ but not a decimal approximation
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28. The diagram shows graphs with equations $y = 14 - x^2$ and $y = 2x^2 + 2$.



Which of the following represents the shaded area?

- A. $\int_2^{14} (12 - 3x^2) dx$
- B. $\int_2^{14} (3x^2 - 12) dx$
- C. $\int_{-2}^2 (12 - 3x^2) dx$
- D. $\int_{-2}^2 (3x^2 - 12) dx$

2

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
C	2.2	C	0	0	CN	C17	2010 P1 Q14

[SQA] 29.

- (a) Find the coordinates of the points of intersection of the curves with equations $y = 2x^2$ and $y = 4 - 2x^2$. 2
- (b) Find the area completely enclosed between these two curves. 3

Part	Marks	Level	Calc.	Content	Answer	U2 OC2
(a)	2	C	NC	A25		1990 P1 Q13
(b)	3	C	NC	C17		

- ¹ $2x^2 = 4x - 2x^2$ or $y = 4 - y$
- ³ $\int_{-1}^1 (4 - 2x^2 - 2x^2) dx$
- ² $x = 1$ and $x = -1$
- ⁴ $4x - \frac{2}{3}x^3 - \frac{2}{3}x^3$
- ⁵ $5\frac{1}{3}$

[SQA] 30. The graph of $y = g(x)$ passes through the point $(1, 2)$.

If $\frac{dy}{dx} = x^3 + \frac{1}{x^2} - \frac{1}{4}$, express y in terms of x .

4

Part	Marks	Level	Calc.	Content	Answer	U2 OC2
	4	C	NC	C18		1999 P1 Q11

<ul style="list-style-type: none"> •¹ x^{-2} stated or implied by •² or •³ •² $y = \int (x^3 + x^{-2} - \frac{1}{4}) dx$ or the appearance of any term of $\frac{1}{4}x^4 - \frac{1}{4}x - x^{-1}$ •³ the remaining two terms •⁴ $c = 3$
--

[SQA] 31. A curve with equation $y = f(x)$ passes through the point $(2, -1)$ and is such that $f'(x) = 4x^3 - 1$.

Express $f(x)$ in terms of x .

5

Part	Marks	Level	Calc.	Content	Answer	U2 OC2
	5	C	NC	C18		1991 P1 Q10

<ul style="list-style-type: none"> •¹ $\int (4x^3 - 1) dx = \dots\dots$ •² $x^4 - x$ •³ $+c$ •⁴ $f(2) = 14 + c$ •⁵ $c = -15$

[SQA] 32. A curve for which $\frac{dy}{dx} = 3x^2 + 1$ passes through the point $(-1, 2)$.

Express y in terms of x .

4

Part	Marks	Level	Calc.	Content	Answer	U2 OC2
	4	C	NC	C18		1992 P1 Q4

<ul style="list-style-type: none"> •¹ $\int (3x^2 + 1) dx$ •² $x^3 + x$ •³ $+c$ •⁴ $y = x^3 + x + 4$
--

- [SQA] 33. A curve for which $\frac{dy}{dx} = 6x^2 - 2x$ passes through the point $(-1, 2)$.

Express y in terms of x .

3

Part	Marks	Level	Calc.	Content	Answer	U2 OC2
	3	C	NC	C18		1998 P1 Q10

- ¹ $y = 2x^3 - x^2$
- ² $y = 2x^3 - x^2 + k$ and substituting
- ³ $k = 5$

- [SQA] 34.

(a) Evaluate $\int_0^{\frac{\pi}{2}} \cos 2x \, dx$.

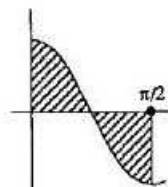
3

(b) Draw a sketch and explain your answer.

2

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
(a)	3	A/B	NC	C23		1992 P1 Q14
(b)	1	C	NC	T1, C16		
(b)	1	A/B	NC	T1, C16		

- ¹ $\frac{1}{2}$
- ² $\sin 2x$
- ³ 0
- ⁴ diagram
- ⁵ +ve and -ve cancel out



35. Find $\int (2x^{-4} + \cos 5x) dx$.

- A. $-\frac{2}{5}x^{-5} - 5 \sin 5x + c$
 B. $-\frac{2}{5}x^{-5} + \frac{1}{5} \sin 5x + c$
 C. $-\frac{2}{3}x^{-3} + \frac{1}{5} \sin 5x + c$
 D. $-\frac{2}{3}x^{-3} - 5 \sin 5x + c$

2

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
C	3.2	C	0	0	CN	C13, C23	2010 P1 Q9

36. Given that $f(x) = 4 \sin 3x$, find $f'(0)$.

- A. 0
 B. 1
 C. 12
 D. 36

2

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
C	3.2	C	0	0	NC	C20, T3	2011 P1 Q13

37. Given that $f(x) = (4 - 3x^2)^{-\frac{1}{2}}$ on a suitable domain, find $f'(x)$.

- A. $-3x(4 - 3x^2)^{-\frac{1}{2}}$
 B. $-\frac{1}{2}(4 - 6x)^{-\frac{3}{2}}$
 C. $2(4 - 3x^3)^{\frac{1}{2}}$
 D. $3x(4 - 3x^2)^{-\frac{3}{2}}$

2

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
D	3.2	C	0	0	NC	C21	2009 P1 Q18

38. If $y = 3 \cos^4 x$, find $\frac{dy}{dx}$.

- A. $12 \cos^3 x \sin x$
 B. $12 \cos^3 x$
 C. $-12 \cos^3 x \sin x$
 D. $-12 \sin^3 x$

2

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
C	3.2	C	0	0	NC	C21, C23	2012 P1 Q16

39. Find $\int (2x - 1)^{\frac{1}{2}} dx$ where $x > \frac{1}{2}$.

A. $\frac{1}{3}(2x - 1)^{\frac{3}{2}} + c$

B. $\frac{1}{2}(2x - 1)^{-\frac{1}{2}} + c$

C. $\frac{1}{2}(2x - 1)^{\frac{3}{2}} + c$

D. $\frac{1}{3}(2x - 1)^{-\frac{1}{2}} + c$

2

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
A	3.2	C	0	0	NC	C22	2012 P1 Q14

[END OF QUESTIONS]