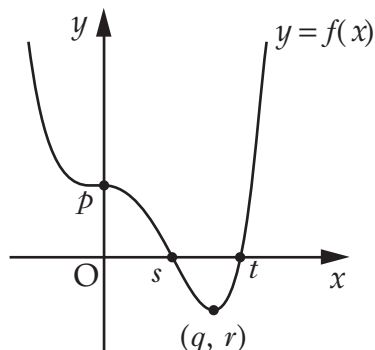


## Calculus Non Calculator C Grade

1. The graph of  $y = f(x)$  shown has stationary points at  $(0, p)$  and  $(q, r)$ .



Here are two statements about  $f(x)$ :

- I.  $f(x) < 0$  for  $s < x < t$ ;
- II.  $f'(x) < 0$  for  $x < q$ .

Which of the following is true?

- A. neither statement is correct
- B. only statement I is correct
- C. only statement II is correct
- D. both statements are correct

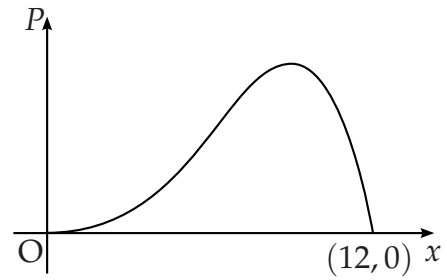
2

2. If  $y = 3x^{-2} + 2x^{\frac{3}{2}}$ ,  $x > 0$ , determine  $\frac{dy}{dx}$ .

- A.  $-6x^{-3} + \frac{4}{5}x^{\frac{5}{2}}$
- B.  $-3x^{-1} + 3x^{\frac{1}{2}}$
- C.  $-6x^{-3} + 3x^{\frac{1}{2}}$
- D.  $-3x^{-1} + \frac{4}{5}x^{\frac{5}{2}}$

2

- [SQA] 3. A company spends  $x$  thousand pounds a year on advertising and this results in a profit of  $P$  thousand pounds. A mathematical model, illustrated in the diagram, suggests that  $P$  and  $x$  are related by  $P = 12x^3 - x^4$  for  $0 \leq x \leq 12$ .



Find the value of  $x$  which gives the maximum profit.

5

4. Differentiate  $2\sqrt[3]{x}$  with respect to  $x$ .

- A.  $6\sqrt{x}$   
 B.  $\frac{3}{2}\sqrt[3]{x^4}$   
 C.  $-\frac{4}{3\sqrt[3]{x^2}}$   
 D.  $\frac{2}{3\sqrt[3]{x^2}}$

2

5. If  $f(x) = \frac{1}{\sqrt[5]{x}}$ ,  $x \neq 0$ , what is  $f'(x)$ ?

- A.  $-\frac{1}{5}x^{-\frac{6}{5}}$   
 B.  $-\frac{1}{5}x^{-\frac{4}{5}}$   
 C.  $-\frac{5}{2}x^{-\frac{7}{2}}$   
 D.  $-\frac{5}{2}x^{-\frac{3}{2}}$

2

6. What is the derivative of  $\frac{1}{4x^3}$ ,  $x \neq 0$ ?

- A.  $\frac{1}{12x^2}$   
 B.  $-\frac{1}{12x^2}$   
 C.  $\frac{4}{x^4}$   
 D.  $-\frac{3}{4x^4}$

2

7. A tangent to the curve with equation  $y = x^3 - 2x$  is drawn at the point  $(2, 4)$ .

What is the gradient of this tangent?

- A. 2
- B. 3
- C. 4
- D. 10

2

8. What is the gradient of the tangent to the curve with equation  $y = x^3 - 6x + 1$  at the point where  $x = -2$ ?

- A. -24
- B. 3
- C. 5
- D. 6

2

9. A curve has equation  $y = 5x^3 - 12x$ .

What is the gradient of the tangent at the point  $(1, -7)$ ?

- A. -7
- B. -5
- C. 3
- D. 5

2

[SQA] 10. Find the  $x$ -coordinate of each of the points on the curve  $y = 2x^3 - 3x^2 - 12x + 20$  at which the tangent is parallel to the  $x$ -axis.

4

[SQA] 11. The point  $P(-1, 7)$  lies on the curve with equation  $y = 5x^2 + 2$ . Find the equation of the tangent to the curve at  $P$ .

4

[SQA] 12. Find the equation of the tangent to the curve  $y = 4x^3 - 2$  at the point where  $x = -1$ .

4

[SQA] 13. Find the equation of the tangent to the curve  $y = 3x^2 + 2$  at the point where  $x = 1$ . 4

[SQA] 14. Find the equation of the tangent to the curve with equation  $y = 5x^3 - 6x^2$  at the point where  $x = 1$ . 4

15. The derivative of a function  $f$  is given by  $f'(x) = x^2 - 9$ .

Here are two statements about  $f$ :

- (1)  $f$  is increasing at  $x = 1$ ;
- (2)  $f$  is stationary at  $x = -3$ .

Which of the following is true?

- A. Neither statement is correct.
- B. Only statement (1) is correct.
- C. Only statement (2) is correct.
- D. Both statements are correct. 2

16. If  $s(t) = t^2 - 5t + 8$ , what is the rate of change of  $s$  with respect to  $t$  when  $t = 3$ ?

- A.  $-5$
- B.  $1$
- C.  $2$
- D.  $9$  2

17. The volume of a sphere is given by the formula  $V = \frac{4}{3}\pi r^3$ . What is the rate of change of  $V$  with respect to  $r$ , at  $r = 2$ ?

- A.  $\frac{16\pi}{3}$
- B.  $\frac{32\pi}{3}$
- C.  $16\pi$
- D.  $32\pi$  2

18.  $A = 2\pi r^2 + 6\pi r$

What is the rate of change of  $A$  with respect to  $r$  when  $r = 2$ ?

- A.  $10\pi$   
 B.  $12\pi$   
 C.  $14\pi$   
 D.  $20\pi$

2

[SQA] 19. For what values of  $x$  is the function  $f(x) = \frac{1}{3}x^3 - 2x^2 - 5x - 4$  increasing?

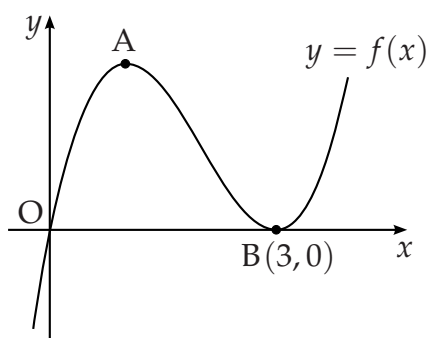
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[SQA] 20. The point  $P(-2, b)$  lies on the graph of the function  $f(x) = 3x^3 - x^2 - 7x + 4$ .(a) Find the value of  $b$ .

1

(b) Prove that this function is increasing at  $P$ .

3

[SQA] 21. A sketch of the graph of  $y = f(x)$  where  $f(x) = x^3 - 6x^2 + 9x$  is shown below.  
The graph has a maximum at  $A$  and a minimum at  $B(3, 0)$ .(a) Find the coordinates of the turning point at  $A$ .

4

(b) Hence sketch the graph of  $y = g(x)$  where  $g(x) = f(x + 2) + 4$ .

Indicate the coordinates of the turning points. There is no need to calculate the coordinates of the points of intersection with the axes.

2

(c) Write down the range of values of  $k$  for which  $g(x) = k$  has 3 real roots.

1

22. Find  $\int (4x^{\frac{1}{2}} + x^{-3}) dx$ , where  $x > 0$ .

A.  $2x^{-\frac{1}{2}} - 3x^{-4} + c$

B.  $2x^{-\frac{1}{2}} - \frac{1}{2}x^{-2} + c$

C.  $\frac{8}{3}x^{\frac{3}{2}} - 3x^{-4} + c$

D.  $\frac{8}{3}x^{\frac{3}{2}} - \frac{1}{2}x^{-2} + c$

2

[SQA] 23. Evaluate  $\int_1^2 \left(x^2 + \frac{1}{x}\right)^2 dx$ .

5

24. Find  $\int \frac{1}{3x^4} dx$ , where  $x \neq 0$ .

A.  $-\frac{1}{9x^3} + c$

B.  $-\frac{1}{x^3} + c$

C.  $\frac{1}{x^3} + c$

D.  $\frac{1}{12x^3} + c$

2

25. Find  $\int \left(\frac{1}{6x^2}\right) dx$ ,  $x \neq 0$ .

A.  $-12x^{-3} + c$

B.  $-6x^{-1} + c$

C.  $-\frac{1}{3}x^{-3} + c$

D.  $-\frac{1}{6}x^{-1} + c$

2

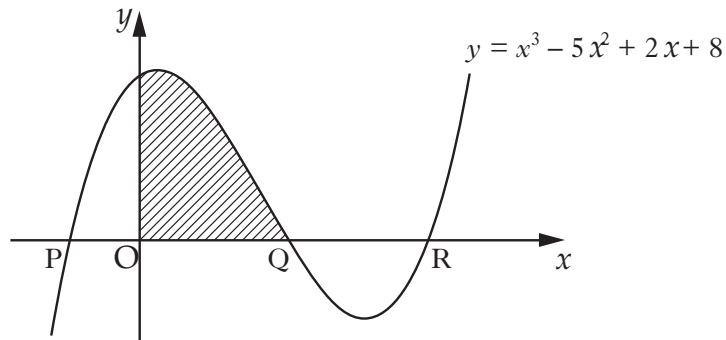
[SQA] 26. Find  $\int \frac{x^2 - 5}{x\sqrt{x}} dx$ .

4

27. (a) (i) Show that  $(x - 4)$  is a factor of  $x^3 - 5x^2 + 2x + 8$ .  
 (ii) Factorise  $x^3 - 5x^2 + 2x + 8$  fully.  
 (iii) Solve  $x^3 - 5x^2 + 2x + 8 = 0$ .

6

- (b) The diagram shows the curve with equation  $y = x^3 - 5x^2 + 2x + 8$ .

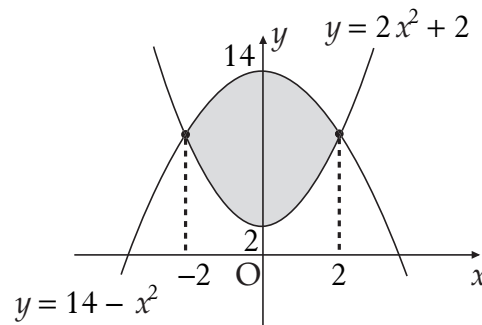


The curve crosses the  $x$ -axis at P, Q and R.

Determine the shaded area.

6

28. The diagram shows graphs with equations  $y = 14 - x^2$  and  $y = 2x^2 + 2$ .



Which of the following represents the shaded area?

- A.  $\int_2^{14} (12 - 3x^2) dx$   
 B.  $\int_2^{14} (3x^2 - 12) dx$   
 C.  $\int_{-2}^2 (12 - 3x^2) dx$   
 D.  $\int_{-2}^2 (3x^2 - 12) dx$

2

- [SQA] 29.
- (a) Find the coordinates of the points of intersection of the curves with equations  $y = 2x^2$  and  $y = 4 - 2x^2$ . 2
- (b) Find the area completely enclosed between these two curves. 3
- [SQA] 30. The graph of  $y = g(x)$  passes through the point  $(1, 2)$ .
- If  $\frac{dy}{dx} = x^3 + \frac{1}{x^2} - \frac{1}{4}$ , express  $y$  in terms of  $x$ . 4
- [SQA] 31. A curve with equation  $y = f(x)$  passes through the point  $(2, -1)$  and is such that  $f'(x) = 4x^3 - 1$ .
- Express  $f(x)$  in terms of  $x$ . 5
- [SQA] 32. A curve for which  $\frac{dy}{dx} = 3x^2 + 1$  passes through the point  $(-1, 2)$ .
- Express  $y$  in terms of  $x$ . 4
- [SQA] 33. A curve for which  $\frac{dy}{dx} = 6x^2 - 2x$  passes through the point  $(-1, 2)$ .
- Express  $y$  in terms of  $x$ . 3
- [SQA] 34.
- (a) Evaluate  $\int_0^{\frac{\pi}{2}} \cos 2x \, dx$ . 3
- (b) Draw a sketch and explain your answer. 2
35. Find  $\int (2x^{-4} + \cos 5x) \, dx$ .
- A.  $-\frac{2}{5}x^{-5} - 5 \sin 5x + c$
- B.  $-\frac{2}{5}x^{-5} + \frac{1}{5} \sin 5x + c$
- C.  $-\frac{2}{3}x^{-3} + \frac{1}{5} \sin 5x + c$
- D.  $-\frac{2}{3}x^{-3} - 5 \sin 5x + c$  2



36. Given that  $f(x) = 4 \sin 3x$ , find  $f'(0)$ .

- A. 0
- B. 1
- C. 12
- D. 36

2

37. Given that  $f(x) = (4 - 3x^2)^{-\frac{1}{2}}$  on a suitable domain, find  $f'(x)$ .

- A.  $-3x(4 - 3x^2)^{-\frac{1}{2}}$
- B.  $-\frac{1}{2}(4 - 6x)^{-\frac{3}{2}}$
- C.  $2(4 - 3x^3)^{\frac{1}{2}}$
- D.  $3x(4 - 3x^2)^{-\frac{3}{2}}$

2

38. If  $y = 3 \cos^4 x$ , find  $\frac{dy}{dx}$ .

- A.  $12 \cos^3 x \sin x$
- B.  $12 \cos^3 x$
- C.  $-12 \cos^3 x \sin x$
- D.  $-12 \sin^3 x$

2

39. Find  $\int (2x - 1)^{\frac{1}{2}} dx$  where  $x > \frac{1}{2}$ .

- A.  $\frac{1}{3}(2x - 1)^{\frac{3}{2}} + c$
- B.  $\frac{1}{2}(2x - 1)^{-\frac{1}{2}} + c$
- C.  $\frac{1}{2}(2x - 1)^{\frac{3}{2}} + c$
- D.  $\frac{1}{3}(2x - 1)^{-\frac{1}{2}} + c$

2

[END OF QUESTIONS]