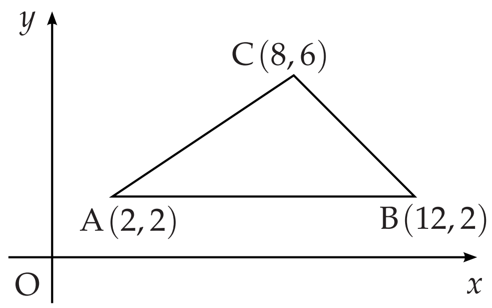
Geometry Calculator AB Grade

[SQA] 1. Triangle ABC has vertices A(2, 2), B(12, 2) and C(8, 6).



(*a*) Write down the equation of l_1, the perpendicular bisector of AB. **1**

(*b*) Find the equation of l_2, the perpendicular bisector of AC. **4**

(*c*) Find the point of intersection of lines l_1 and l_2. **1**

(*d*) Hence find the equation of the circle passing through A, B and C. **2**

Part	Marks	Level	Calc.	Content	Answer	U2 OC4
(*a*)	1	C	CN	G3, G7	$x = 7$	2001 P2 Q7
(*b*)	4	C	CN	G7	$3x + 2y = 23$	
(*c*)	1	C	CN	G8	$(7, 1)$	
(*d*)	2	A/B	CN	G8, G9, G10	$(x-7)^2 + (y-1)^2 = 26$	

•1 ic: state equation of a vertical line	•1 $x = 7$
•2 pd: process coord. of a midpoint	•2 midpoint $= (5, 4)$
•3 ss: find gradient of AC	•3 $m_{AC} = \frac{2}{3}$
•4 ic: state gradient of perpendicular	•4 $m_{\perp} = -\frac{3}{2}$
•5 ic: state equation of straight line	•5 $y - 4 = -\frac{3}{2}(x - 5)$
•6 pd: find pt of intersection	•6 $x = 7, y = 1$
•7 ss: use standard form of circle equ.	•7 $(x-7)^2 + (y-1)^2$
•8 ic: find radius and complete	•8 $(x-7)^2 + (y-1)^2 = 26$
	or
	•7 $x^2 + y^2 - 14x - 2y + c = 0$
	•8 $c = 24$

2. Circle C_1 has equation $(x+1)^2 + (y-1)^2 = 121$.

   A circle C_2 with equation $x^2 + y^2 - 4x + 6y + p = 0$ is drawn inside C_1.

   The circles have no points of contact.

   What is the range of values of p? **9**

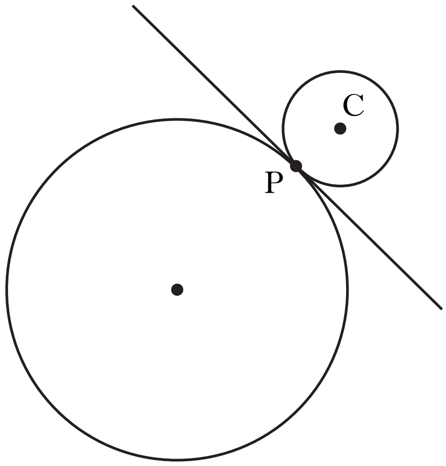
Part	Marks	Level	Calc.	Content	Answer	U2 OC4
	9	A	CN	G9, G15	$-23 < p < 13$	2011 P2 Q7

•[1] ic: state centre of C_1	•[1] $(-1, 1)$
•[2] ic: state radius of C_1	•[2] 11 ($\sqrt{121}$ not accepted)
•[3] ic: state centre of C_2	•[3] $(2, -3)$
•[4] pd: find radius of C_2 in terms of p	•[4] $\sqrt{13-p}$
•[5] ic: interpret upper bound for p	•[5] $p < 13$
•[6] ic: find distance between centres, d	•[6] 5
•[7] ss: identify relevant relationship	•[7] $\sqrt{13-p} < 6$ or $r_2 + d < 11$ or $r_2 < 6$
•[8] ic: develop relationship by squaring	•[8] $13 - p < 36$
•[9] pd: find lower bound for p	•[9] $p > -23$

3. (*a*) (i) Show that the line with equation $y = 3 - x$ is a tangent to the circle with equation $x^2 + y^2 + 14x + 4y - 19 = 0$.

(ii) Find the coordinates of the points of contact, P. **5**

(*b*) Relative to a suitable set of coordinate axes, the diagram below shows the circle from (*a*) and a second smaller circle with centre C.



The line $y = 3 - x$ is a common tangent at the point P.

The radius of the larger circle is three times the radius of the smaller circle.

Find the equation of the smaller circle. **6**

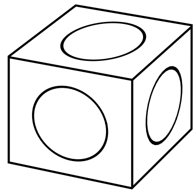
Part	Marks	Level	Calc.	Content	Answer	U2 OC4
(*ai*)	4	C	CN	G13	proof	2010 P2 Q3
(*aii*)	1	C	CN	G12	$P(-1, 4)$	
(*b*)	6	B	CN	G9, G15	$(x-1)^2 + (y-6)^2 = 8$	

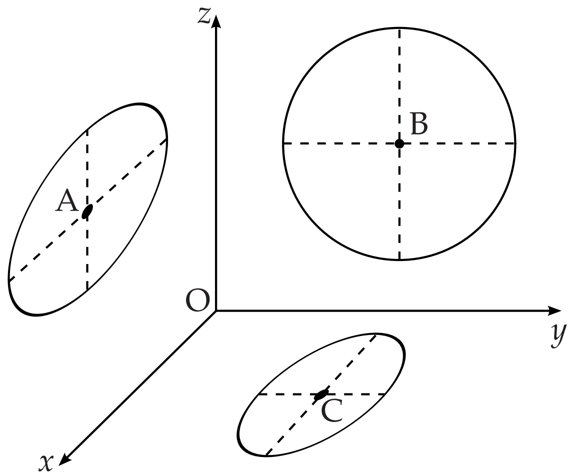
•1 ss: substitute	•1 $x^2 + (3-x)^2 + 14x + 4(3-x) - 19 = 0$
•2 pd: express in standard form	•2 $2x^2 + 4x + 2 = 0$
•3 ic: start proof	•3 $2(x+1)(x+1)$
•4 ic: complete proof	•4 equal roots so line is a tangent
•5 pd: coordinates of P	•5 $x = -1, y = 4$
•6 ic: state centre of larger circle	•6 $(-7, -2)$
•7 ss: find radius of larger circle	•7 $\sqrt{72}$
•8 pd: find radius of smaller circle	•8 $\sqrt{8}$
•9 ss: strategy for finding centre	•9 e.g. "Stepping out"
•10 ic: interpret centre of smaller circle	•10 $(1, 6)$
•11 ic: state equation	•11 $(x-1)^2 + (y-6)^2 = 8$

[SQA] 4. A box in the shape of a cuboid is designed with **circles** of different sizes on each face.

The diagram shows three of the circles, where the origin represents one of the corners of the cuboid. The centres of the circles are $A(6,0,7)$, $B(0,5,6)$ and $C(4,5,0)$.

Find the size of angle ABC. **7**





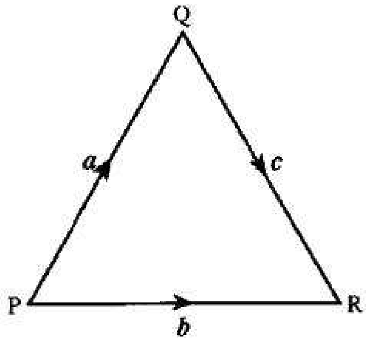
Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	5	C	CR	G17, G16, G22		2001 P2 Q4
	2	A/B	CR	G26, G28	$71{\cdot}5°$	

•[1] ss: use $\frac{\overrightarrow{BA}.\overrightarrow{BC}}{\lvert\overrightarrow{BA}\rvert\lvert\overrightarrow{BC}\rvert}$	•[1] use $\frac{\overrightarrow{BA}.\overrightarrow{BC}}{\lvert\overrightarrow{BA}\rvert\lvert\overrightarrow{BC}\rvert}$ *stated or implied by* •[7]
•[2] ic: state vector e.g. $\overrightarrow{BA}$	•[2] $\overrightarrow{BA} = \begin{pmatrix} 6 \\ -5 \\ 1 \end{pmatrix}$
•[3] ic: state a consistent vector e.g. $\overrightarrow{BC}$	•[3] $\overrightarrow{BC} = \begin{pmatrix} 4 \\ 0 \\ -6 \end{pmatrix}$
•[4] pd: process $\lvert\overrightarrow{BA}\rvert$	•[4] $\lvert\overrightarrow{BA}\rvert = \sqrt{62}$
•[5] pd: process $\lvert\overrightarrow{BC}\rvert$	•[5] $\lvert\overrightarrow{BC}\rvert = \sqrt{52}$
•[6] pd: process scalar product	•[6] $\overrightarrow{BA}.\overrightarrow{BC} = 18$
•[7] pd: find angle	•[7] $A\hat{B}C = 71{\cdot}5°$

[SQA] 5. PQR is an equilateral triangle of side 2 units.

$\overrightarrow{PQ} = \boldsymbol{a}$, $\overrightarrow{PR} = \boldsymbol{b}$ and $\overrightarrow{QR} = \boldsymbol{c}$.

Evaluate $\boldsymbol{a}.(\boldsymbol{b}+\boldsymbol{c})$ and hence identify two vectors which are perpendicular. **4**



Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	1	C	CN	G26		1997 P1 Q13
	3	A/B	CN	G29, G27		

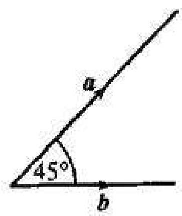
•[1] $a.b + a.c$

•[2] $a.b = 2 \times 2 \times \frac{1}{2}$

•[3] $a.c = 2 \times 2 \times -\frac{1}{2}$

•[4] 0 and a is perpendicular to $(b+c)$

[SQA] 6. The diagram shows two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, with $|\boldsymbol{a}| = 3$ and $|\boldsymbol{b}| = 2\sqrt{2}$.
These vectors are inclined at an angle of 45° to each other.



(*a*) Evaluate (i) $\boldsymbol{a}.\boldsymbol{a}$
(ii) $\boldsymbol{b}.\boldsymbol{b}$
(iii) $\boldsymbol{a}.\boldsymbol{b}$ **2**

(*b*) Another vector $\boldsymbol{p}$ is defined by $\boldsymbol{p} = 2\boldsymbol{a} + 3\boldsymbol{b}$.
Evaluate $\boldsymbol{p}.\boldsymbol{p}$ and hence write down $|\boldsymbol{p}|$. **4**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	2	C	CN	G26		1999 P1 Q17
(*b*)	4	A/B	CN	G29, G30		

•[1] $a.a = 9$ and $b.b = 8$

•[2] $a.b = 6$

•[3] $(2a+3b).(2a+3b)$

•[4] $4a.a + 9b.b + 12a.b$

•[5] 180

•[6] $\sqrt{180}$

[END OF QUESTIONS]