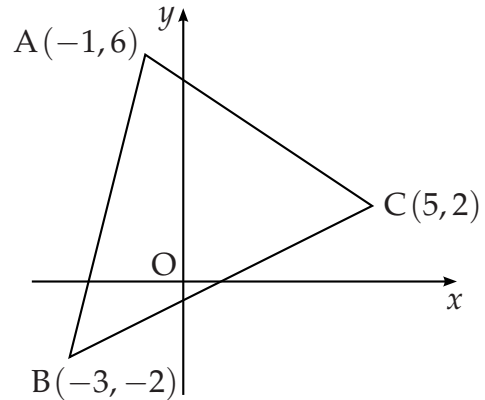


Geometry Calculator C Grade

[SQA] 1. Triangle ABC has vertices A(-1, 6), B(-3, -2) and C(5, 2).

Find

- (a) the equation of the line p , the median from C of triangle ABC.
- (b) the equation of the line q , the perpendicular bisector of BC.
- (c) the coordinates of the point of intersection of the lines p and q .



3
4
1

Part	Marks	Level	Calc.	Content	Answer	U1 OC1
(a)	3	C	CN	G7	$y = 2$	2002 P2 Q1
(b)	4	C	CN	G7	$y = -2x + 2$	
(c)	1	C	CN	G8	(0, 2)	

<ul style="list-style-type: none"> •¹ ss: determine midpoint coordinates •² pd: determine gradient thro' 2 pts •³ ic: state equation of straight line •⁴ ss: determine midpoint coordinates •⁵ pd: determine gradient thro' 2 pts •⁶ ss: determine gradient perp. to •⁵ •⁷ ic: state equation of straight line •⁸ pd: process intersection 	<ul style="list-style-type: none"> •¹ F = mid_{AB} = (-2, 2) •² $m_{FC} = 0$ stated or implied by •³ •³ equ. FC is $y = 2$ •⁴ M = mid_{BC} = (1, 0) •⁵ $m_{BC} = \frac{1}{2}$ •⁶ $m_{\perp} = -2$ •⁷ $y - 0 = -2(x - 1)$ •⁸ (0, 2)
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[SQA] 2. (a) Find the equation of AB, the perpendicular bisector of the line joining the points $P(-3,1)$ and $Q(1,9)$.

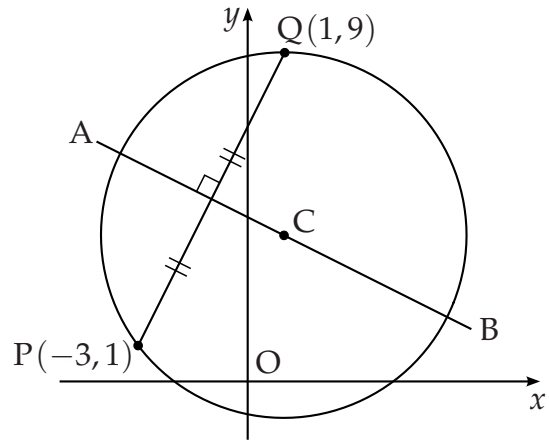
(b) C is the centre of a circle passing through P and Q. Given that QC is parallel to the y -axis, determine the equation of the circle.

(c) The tangents at P and Q intersect at T.

Write down

(i) the equation of the tangent at Q

(ii) the coordinates of T.



4

3

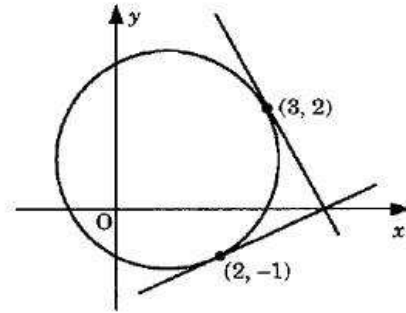
2

Part	Marks	Level	Calc.	Content	Answer	U2 OC4
(a)	4	C	CN	G7	$x + 2y = 9$	2000 P2 Q2
(b)	3	C	CN	G10	$(x - 1)^2 + (y - 4)^2 = 25$	
(c)	2	C	CN	G11, G8	(i) $y = 9$, (ii) $T(-9, 9)$	

<ul style="list-style-type: none"> •¹ ss: know to use midpoint •² pd: process gradient of PQ •³ ss: know how to find perp. gradient •⁴ ic: state equ. of line •⁵ ic: interpret "parallel to y-axis" •⁶ pd: process radius •⁷ ic: state equ. of circle •⁸ ic: interpret diagram •⁹ ss: know to use equ. of AB 	<ul style="list-style-type: none"> •¹ midpoint = $(-1, 5)$ •² $m_{PQ} = \frac{9-1}{1-(-1)}$ •³ $m_{\perp} = -\frac{1}{2}$ •⁴ $y - 5 = -\frac{1}{2}(x - (-1))$ •⁵ $y_C = 4$ stated or implied by •⁷ •⁶ radius = 5 or equiv. stated or implied by •⁷ •⁷ $(x - 1)^2 + (y - 4)^2 = 25$ •⁸ $y = 9$ •⁹ $T = (-9, 9)$
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[SQA]

3. The circle shown in the diagram has equation $(x-1)^2 + (y-1)^2 = 5$.
Tangents are drawn at the points $(3, 2)$ and $(2, -1)$.
Write down the coordinates of the centre of the circle and hence show that the tangents are perpendicular to each other.



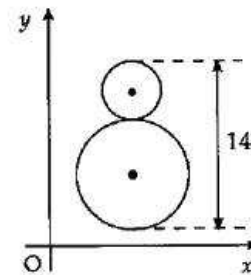
4

Part	Marks	Level	Calc.	Content	Answer	
	4	C	CN	G9, G5		U2 OC4 1994 P1 Q5

<ul style="list-style-type: none"> •¹ centre = $(1, 1)$ •² $m_{\text{radii}} = \frac{1}{2}, -2$ •³ $m_{\text{tgts}} = -2, \frac{1}{2}$ •⁴ $-2 \times \frac{1}{2} = -1 \Rightarrow \text{tgts are } \perp$ 	OR	<ul style="list-style-type: none"> •¹ centre = $(1, 1)$ •² $r = \sqrt{5}, d = \sqrt{10}$ •³ Show $\hat{ACB} = 90^\circ$ •⁴ State tangents \perp to radii 	
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[SQA]

4. A bakery firm makes gingerbread men each 14cm high with a circular "head" and "body".
The equation of the "body" is $x^2 + y^2 - 10x - 12y + 45 = 0$ and the line of centres is parallel to the y -axis. Find the equation of the "head".

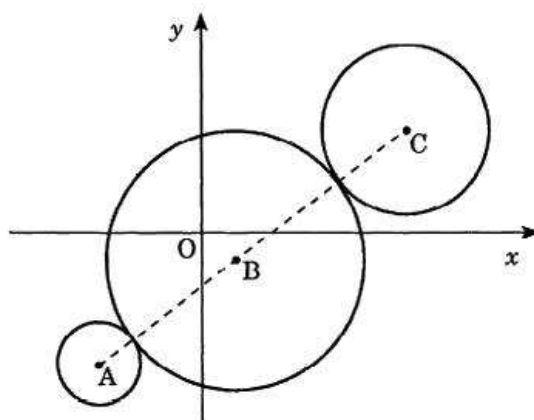


5

Part	Marks	Level	Calc.	Content	Answer	
	5	C	CN	G9, G10		U2 OC4 1990 P1 Q7

<ul style="list-style-type: none"> •¹ centre of body = $(5, 6)$ •² radius of body = 4 •³ radius of head = 3 •⁴ centre of head = $(5, 13)$ •⁵ $(x-5)^2 + (y-13)^2 = 9$

- [SQA] 5. When newspapers were printed by lithograph, the newsprint had to run over three rollers, illustrated in the diagram by three circles. The centres A, B and C of the three circles are collinear.



The equations of the circumferences of the outer circles are

$$(x + 12)^2 + (y + 15)^2 = 25 \text{ and } (x - 24)^2 + (y - 12)^2 = 100.$$

Find the equation of the central circle.

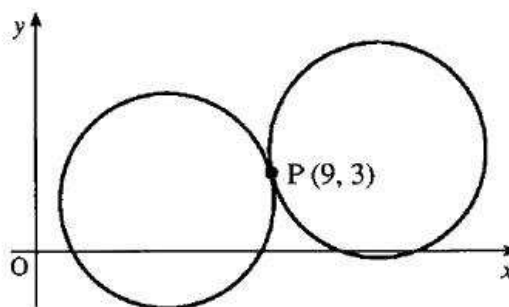
(8)

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	8	C	CN	G9, G10, G25		1995 P2 Q8

- ¹ $(-12, -15)$ and $(24, 12)$
- ² radii are 5 and 10
- ³ $AC = 45$
- ⁴ radius = 15
- ⁵ B divides AC in ratio 4:5
- ⁶ $\vec{OB} = \frac{1}{9} \left[4\vec{OC} + 5\vec{OA} \right]$ stated or implied
- ⁷ $\vec{OB} = \frac{1}{9} \left[4 \begin{pmatrix} 24 \\ 12 \end{pmatrix} + 5 \begin{pmatrix} -12 \\ -15 \end{pmatrix} \right]$
- ⁸ $(x - 4)^2 + (y + 3)^2 = 15^2$

- [SQA] 6. Two identical circles touch at the point P (9, 3) as shown in the diagram. One of the circles has equation $x^2 + y^2 - 10x - 4y + 12 = 0$.

Find the equation of the other circle.



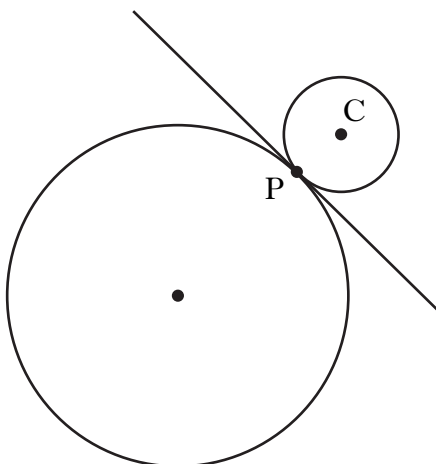
5

Part	Marks	Level	Calc.	Content	Answer	
	5	C	CN	G9, G25		U3 OC1 1997 P1 Q12

- ¹ use P as midpoint of C_1C_2
- ² $C_1 = (5, 2)$
- ³ $C_2 = (13, 4)$
- ⁴ radius = $\sqrt{17}$
- ⁵ $(x - 13)^2 + (y - 4)^2 = 17$

7. (a) (i) Show that the line with equation $y = 3 - x$ is a tangent to the circle with equation $x^2 + y^2 + 14x + 4y - 19 = 0$.
- (ii) Find the coordinates of the points of contact, P.
- (b) Relative to a suitable set of coordinate axes, the diagram below shows the circle from (a) and a second smaller circle with centre C.

5



The line $y = 3 - x$ is a common tangent at the point P.

The radius of the larger circle is three times the radius of the smaller circle.

Find the equation of the smaller circle.

6

Part	Marks	Level	Calc.	Content	Answer	U2 OC4
(ai)	4	C	CN	G13	proof	2010 P2 Q3
(aii)	1	C	CN	G12	$P(-1, 4)$	
(b)	6	B	CN	G9, G15	$(x - 1)^2 + (y - 6)^2 = 8$	

<ul style="list-style-type: none"> •¹ ss: substitute •² pd: express in standard form •³ ic: start proof •⁴ ic: complete proof •⁵ pd: coordinates of P •⁶ ic: state centre of larger circle •⁷ ss: find radius of larger circle •⁸ pd: find radius of smaller circle •⁹ ss: strategy for finding centre •¹⁰ ic: interpret centre of smaller circle •¹¹ ic: state equation 	<ul style="list-style-type: none"> •¹ $x^2 + (3 - x)^2 + 14x + 4(3 - x) - 19 = 0$ •² $2x^2 + 4x + 2 = 0$ •³ $2(x + 1)(x + 1)$ •⁴ equal roots so line is a tangent •⁵ $x = -1, y = 4$ •⁶ $(-7, -2)$ •⁷ $\sqrt{72}$ •⁸ $\sqrt{8}$ •⁹ e.g. "Stepping out" •¹⁰ $(1, 6)$ •¹¹ $(x - 1)^2 + (y - 6)^2 = 8$
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[SQA] 8. VABCD is a pyramid with rectangular base ABCD.

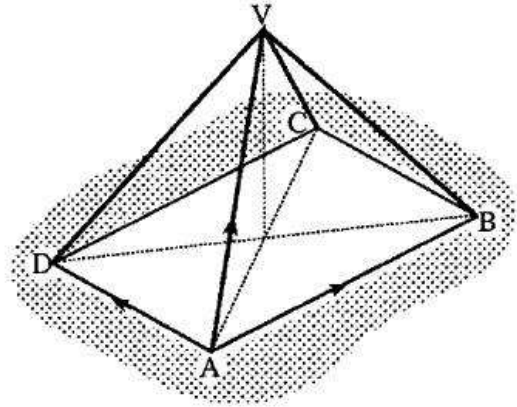
The vectors \vec{AB} , \vec{AD} and \vec{AV} are given by

$$\vec{AB} = 8i + 2j + 2k$$

$$\vec{AD} = -2i + 10j - 2k \quad \text{and}$$

$$\vec{AV} = i + 7j + 7k.$$

Express \vec{CV} in component form.



3

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	3	C	CN	G18		1999 P1 Q6

<ul style="list-style-type: none"> •¹ pathway for \vec{CV}: $\vec{CV} = \vec{CA} + \vec{AV}$ •² e.g. $\vec{CB} = 2i - 10j + 2k$ or $\vec{BA} = -8i - 2j - 2k$ or $\vec{AC} = 6i + 12j$ 	<ul style="list-style-type: none"> •³ $\begin{pmatrix} -5 \\ -5 \\ 7 \end{pmatrix}$
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[SQA] 9. The vector $ai + bj + k$ is perpendicular to both the vectors $i - j + k$ and $-2i + j + k$.

Find the values of a and b .

3

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	3	C	CN	G18	$a = 2, b = 3$	1990 P1 Q12

<ul style="list-style-type: none"> •¹ $\begin{pmatrix} a \\ b \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = a - b + 1$ or $\begin{pmatrix} a \\ b \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} = -2a + b + 1$ •² $a - b + 1 = 0$ or $-2a + b + 1 = 0$ •³ $a = 2$ and $b = 3$

[SQA] 10. Calculate the length of the vector $2i - 3j + \sqrt{3}k$.

2

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	2	C	CN	G18	4	1995 P1 Q1

<ul style="list-style-type: none"> •¹ $\sqrt{2^2 + (-3)^2 + (\sqrt{3})^2}$ stated or implied by •² 4

- [SQA] 11. Show that the vectors $a = 2i + 3j - k$ and $b = 3i - j + 3k$ are perpendicular. 3

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	3	C	CN	G18, G27	$a \cdot b = \dots = 0$	1991 P1 Q3

\bullet^1 strat: $a \cdot b = \dots$ \bullet^2 $a \cdot b = 0 \Rightarrow$ perpendicularity explicitly stated \bullet^3 $\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -1 \\ 3 \end{pmatrix} = 6 - 3 - 3 = 0$

- [SQA] 12. The position vectors of the points P and Q are $p = -i + 3j + 4k$ and $q = 7i - j + 5k$ respectively.

(a) Express \vec{PQ} in component form. 2

(b) Find the length of PQ. 1

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(a)	2	C	CN	G18, G16		1997 P1 Q4
(b)	1	C	CN	G16		

\bullet^1 $q - p = 8i - 4j + k$ or $p = \begin{pmatrix} -1 \\ 3 \\ 4 \end{pmatrix}, q = \begin{pmatrix} 7 \\ -1 \\ 5 \end{pmatrix}$	\bullet^2 $\vec{PQ} = \begin{pmatrix} 8 \\ -4 \\ 1 \end{pmatrix}$ \bullet^3 9
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- [SQA] 13. The vectors a, b and c are defined as follows:

$$a = 2i - k, \quad b = i + 2j + k, \quad c = -j + k.$$

(a) Evaluate $a \cdot b + a \cdot c$. 3

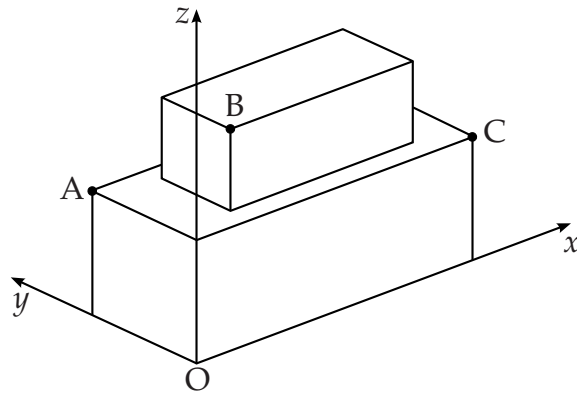
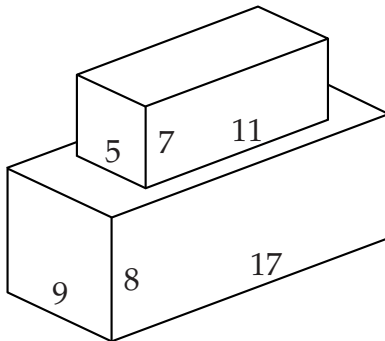
(b) From your answer to part (a), make a deduction about the vector $b + c$. 2

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(a)	3	C	CN	G18, G26		1993 P1 Q12
(b)	2	A/B	CN	G27		

\bullet^1 $a = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}, b = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, c = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$ \bullet^2 $a \cdot b = 1$ \bullet^3 $a \cdot c = -1$	\bullet^4 $a \cdot b + a \cdot c = a \cdot (b + c)$ \bullet^5 $a \perp b + c$
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- [SQA] 14. A cuboid measuring 11 cm by 5 cm by 7 cm is placed centrally on top of another cuboid measuring 17 cm by 9 cm by 8 cm.

Coordinates axes are taken as shown.



- (a) The point A has coordinates $(0, 9, 8)$ and C has coordinates $(17, 0, 8)$.

Write down the coordinates of B.

1

- (b) Calculate the size of angle ABC.

6

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(a)	1	C	CN	G22	$B(3, 2, 15)$	2000 P2 Q9
(b)	6	C	CR	G28	92.5°	

<ul style="list-style-type: none"> •¹ ic: interpret 3-d representation •² ss: know to use scalar product •³ pd: process vectors •⁴ pd: process vectors •⁵ pd: process lengths •⁶ pd: process scalar product •⁷ pd: evaluate scalar product 	<ul style="list-style-type: none"> •¹ $B = (3, 2, 15)$ treat $\begin{pmatrix} 3 \\ 2 \\ 15 \end{pmatrix}$ as bad form •² $\cos \widehat{ABC} = \frac{\vec{BA} \cdot \vec{BC}}{ \vec{BA} \vec{BC} }$ •³ $\vec{BA} = \begin{pmatrix} -3 \\ 7 \\ -7 \end{pmatrix}$ •⁴ $\vec{BC} = \begin{pmatrix} 14 \\ -2 \\ -7 \end{pmatrix}$ •⁵ $\vec{BA} = \sqrt{107}, \vec{BC} = \sqrt{249}$ •⁶ $\vec{BA} \cdot \vec{BC} = -7$ •⁷ $\widehat{ABC} = 92.5^\circ$
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[SQA] 15. The diagram shows a square-based pyramid of height 8 units.

Square OABC has a side length of 6 units.

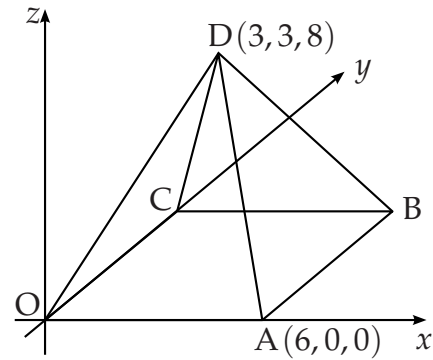
The coordinates of A and D are (6, 0, 0) and (3, 3, 8).

C lies on the y -axis.

(a) Write down the coordinates of B.

(b) Determine the components of \vec{DA} and \vec{DB} .

(c) Calculate the size of angle ADB.

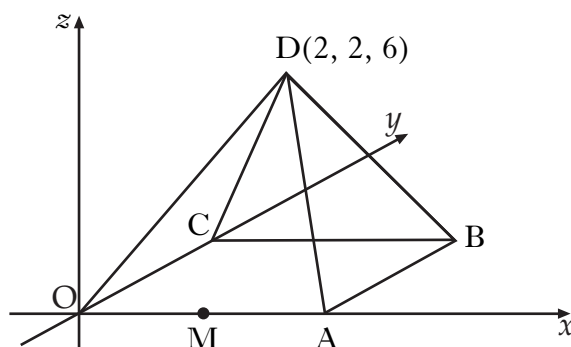


1
2
4

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(a)	1	C	CN	G22	(6, 6, 0)	2002 P2 Q2
(b)	2	C	CN	G17	$\vec{DA} = \begin{pmatrix} 3 \\ -3 \\ -8 \end{pmatrix},$ $\vec{DB} = \begin{pmatrix} 3 \\ 3 \\ -8 \end{pmatrix}$	
(c)	4	C	CR	G28	38.7°	

<ul style="list-style-type: none"> •¹ ic: interpret diagram •² ic: write down components of a vector •³ ic: write down components of a vector •⁴ ss: use e.g. scalar product formula •⁵ pd: process lengths •⁶ pd: process scalar product •⁷ pd: process angle 	<ul style="list-style-type: none"> •¹ B = (6, 6, 0) •² $\vec{DA} = \begin{pmatrix} 3 \\ -3 \\ -8 \end{pmatrix}$ •³ $\vec{DB} = \begin{pmatrix} 3 \\ 3 \\ -8 \end{pmatrix}$ •⁴ $\cos \widehat{ADB} = \frac{\vec{DA} \cdot \vec{DB}}{ \vec{DA} \vec{DB} }$ •⁵ $\vec{DA} = \sqrt{82}, \vec{DB} = \sqrt{82}$ •⁶ $\vec{DA} \cdot \vec{DB} = 64$ •⁷ $\widehat{ADB} = 38.7^\circ$
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16. D,OABC is a square based pyramid as shown in the diagram below.



O is the origin, D is the point $(2, 2, 6)$ and $OA = 4$ units.

M is the mid-point of OA.

(a) State the coordinates of B.

1

(b) Express \vec{DB} and \vec{DM} in component form.

3

(c) Find the size of angle BDM.

5

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(a)	1	C	CN	G22	$(4, 4, 0)$	2011 P2 Q1
(b)	3	C	CN	G20, G22	$\vec{DB} = \begin{pmatrix} 2 \\ 2 \\ -6 \end{pmatrix}, \vec{DM} = \begin{pmatrix} 0 \\ -2 \\ -6 \end{pmatrix}$	
(c)	5	C	CN	G28	40.3°	

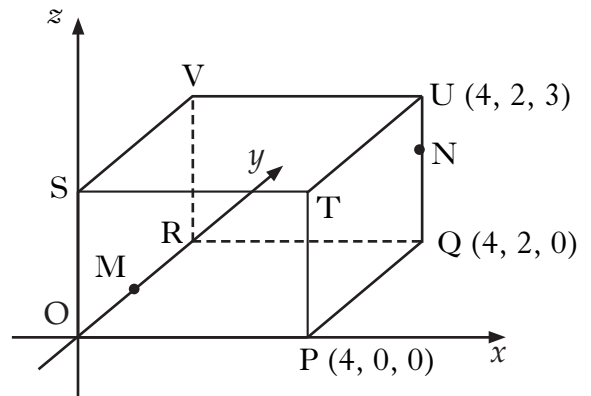
<ul style="list-style-type: none"> •¹ ic: state coordinates of B •² pd: state components of \vec{DB} •³ ic: state coordinates of M •⁴ pd: state components of \vec{DM} •⁵ ss: know to use scalar product •⁶ pd: find scalar product •⁷ pd: find magnitude of a vector •⁸ pd: find magnitude of a vector •⁹ pd: evaluate angle BDM 	<ul style="list-style-type: none"> •¹ $(4, 4, 0)$ •² $\begin{pmatrix} 2 \\ 2 \\ -6 \end{pmatrix}$ •³ $(2, 0, 0)$ •⁴ $\begin{pmatrix} 0 \\ -2 \\ -6 \end{pmatrix}$ •⁵ $\cos BDM = \frac{\vec{DB} \cdot \vec{DM}}{ \vec{DB} \vec{DM} }$ •⁶ $\vec{DB} \cdot \vec{DM} = 32$ •⁷ $\vec{DB} = \sqrt{44}$ •⁸ $\vec{DM} = \sqrt{40}$ •⁹ 40.3° or 0.703 rads
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17. The diagram shows a cuboid OPQR,STUV relative to the coordinate axes.

P is the point $(4, 0, 0)$, Q is $(4, 2, 0)$ and U is $(4, 2, 3)$.

M is the midpoint of OR.

N is the point on UQ such that $UN = \frac{1}{3}UQ$.



- (a) State the coordinates of M and N. 2
- (b) Express the vectors \vec{VM} and \vec{VN} in component form. 2
- (c) Calculate the size of angle MVN. 5

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(a)	2	C	CN	G22, G25	$M(0, 1, 0), N(4, 2, 2)$	2010 P2 Q1
(b)	2	C	CN	G17	$\vec{VM} = \begin{pmatrix} 0 \\ -1 \\ -3 \end{pmatrix}, \vec{VN} = \begin{pmatrix} 4 \\ 0 \\ -1 \end{pmatrix}$	
(c)	5	C	CN	G28	76.7° or 1.339 rad	

<ul style="list-style-type: none"> •¹ ic: interpret midpoint for M •² ic: interpret ratio for N •³ ic: interpret diagram •⁴ pd: process vectors •⁵ ss: know to use scalar product •⁶ pd: find scalar product •⁷ pd: find magnitude of a vector •⁸ pd: find magnitude of a vector •⁹ pd: evaluate angle 	<ul style="list-style-type: none"> •¹ $(0, 1, 0)$ •² $(4, 2, 2)$ •³ $\vec{VM} = \begin{pmatrix} 0 \\ -1 \\ -3 \end{pmatrix}$ •⁴ $\vec{VN} = \begin{pmatrix} 4 \\ 0 \\ -1 \end{pmatrix}$ •⁵ $\cos \widehat{MVN} = \frac{\vec{VM} \cdot \vec{VN}}{ \vec{VM} \vec{VN} }$ •⁶ $\vec{VM} \cdot \vec{VN} = 3$ •⁷ $\vec{VM} = \sqrt{10}$ •⁸ $\vec{VN} = \sqrt{17}$ •⁹ 76.7° or 1.339 rads or 85.2 grads
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[END OF QUESTIONS]