Trigonometry Non-Calculator C Grade

1. If $f(x) = 2\sin\left(3x - \frac{\pi}{2}\right) + 5$, what is the range of values of $f(x)$?

   A. $-1 \leq f(x) \leq 11$

   B. $2 \leq f(x) \leq 8$

   C. $3 \leq f(x) \leq 7$

   D. $-3 \leq f(x) \leq 7$ **2**

2. Functions f and g are defined on suitable domains by $f(x) = \cos x$ and $g(x) = x + \frac{\pi}{6}$.

   What is the value of $f\left(g\left(\frac{\pi}{6}\right)\right)$?

   A. $\frac{1}{2} + \frac{\pi}{6}$

   B. $\frac{\sqrt{3}}{2} + \frac{\pi}{6}$

   C. $\frac{\sqrt{3}}{2}$

   D. $\frac{1}{2}$ **2**

3. Find the maximum value of

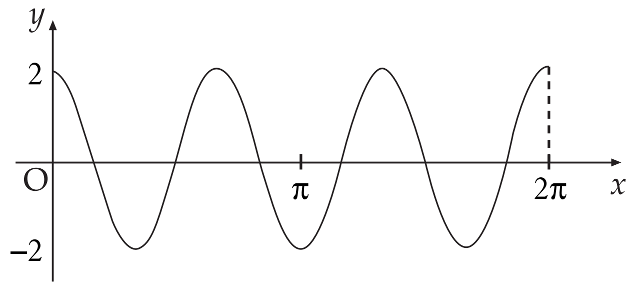
$$2 - 3\sin\left(x - \frac{\pi}{3}\right)$$

   and the value of x where this occurs in the interval $0 \leq x \leq 2\pi$.

	max value	x
A.	-1	$\frac{11\pi}{6}$
B.	5	$\frac{11\pi}{6}$
C.	-1	$\frac{5\pi}{6}$
D.	5	$\frac{5\pi}{6}$

2

4. The diagram shows the graph with equation of the form $y = a\cos bx$ for $0 \leq x \leq 2\pi$.



What is the equation of this graph?

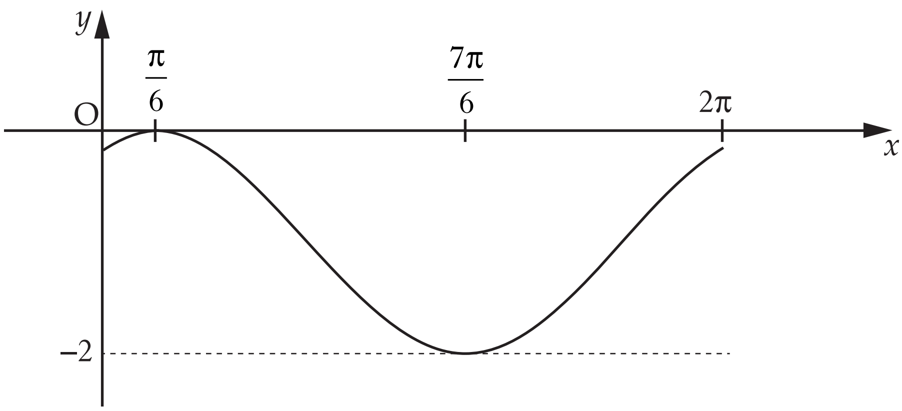
A. $y = 2\cos 3x$

B. $y = 2\cos 2x$

C. $y = 3\cos 2x$

D. $y = 4\cos 3x$ **2**

5. The diagram shows the curve with equation of the form $y = \cos(x + a) + b$ for $0 \leq x \leq 2\pi$.



What is the equation of this curve?

A. $y = \cos\left(x - \frac{\pi}{6}\right) - 1$

B. $y = \cos\left(x - \frac{\pi}{6}\right) + 1$

C. $y = \cos\left(x + \frac{\pi}{6}\right) - 1$

D. $y = \cos\left(x + \frac{\pi}{6}\right) + 1$ **2**

6. (*a*) Diagram 1 shows a right angled triangle, where the line OA has equation $3x - 2y = 0$.

   (i) Show that $\tan a = \frac{3}{2}$.

   (ii) Find the value of $\sin a$. **4**

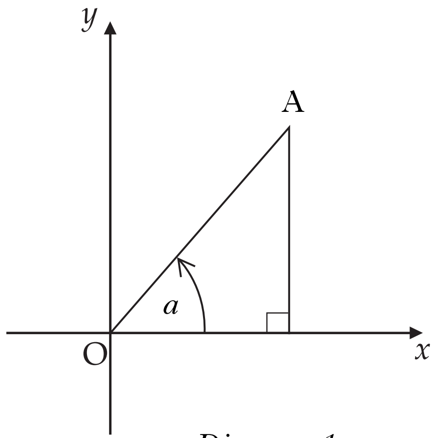


Diagram 1

(*b*) A second right angled triangle is added as shown in Diagram 2.

The line OB has equation $3x - 4y = 0$.

Find the values of $\sin b$ and $\cos b$. **4**

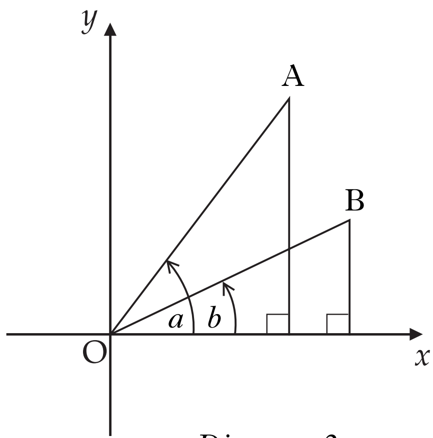


Diagram 2

(*c*) (i) Find the value of $\sin(a - b)$.

   (ii) State the value of $\sin(b - a)$. **4**

[SQA] 7. Functions $f(x) = \sin x$, $g(x) = \cos x$ and $h(x) = x + \frac{\pi}{4}$ are defined on a suitable set of real numbers.

(*a*) Find expressions for:

   (i) $f(h(x))$;

   (ii) $g(h(x))$. **2**

(*b*) (i) Show that $f(h(x)) = \frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x$.

   (ii) Find a similar expression for $g(h(x))$ and hence solve the equation $f(h(x)) - g(h(x)) = 1$ for $0 \leq x \leq 2\pi$. **5**

[SQA] 8. Functions f and g are defined on suitable domains by $f(x) = \sin(x^\circ)$ and $g(x) = 2x$.

(*a*) Find expressions for:

(i) $f(g(x))$;

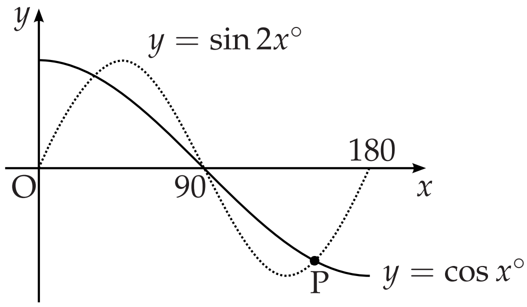
(ii) $g(f(x))$. **2**

(*b*) Solve $2f(g(x)) = g(f(x))$ for $0 \leq x \leq 360$. **5**

[SQA] 9. (*a*) Solve the equation $\sin 2x^\circ - \cos x^\circ = 0$ in the interval $0 \leq x \leq 180$. **4**

(*b*) The diagram shows parts of two trigonometric graphs, $y = \sin 2x^\circ$ and $y = \cos x^\circ$.

Use your solutions in (*a*) to write down the coordinates of the point P. **1**

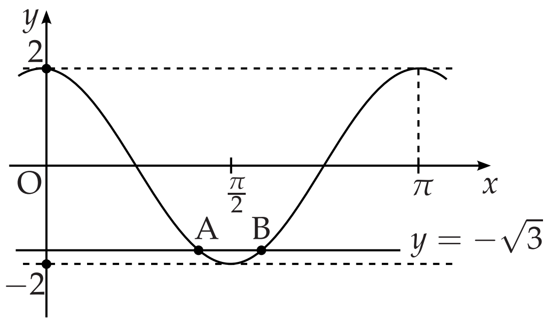


[SQA] 10. The diagram shows the graph of a cosine function from 0 to π.

(*a*) State the equation of the graph. **1**

(*b*) The line with equation $y = -\sqrt{3}$ intersects this graph at point A and B.

Find the coordinates of B. **3**



11. Solve $2\cos x = \sqrt{3}$ for x, where $0 \leq x < 2\pi$.

A. $\frac{\pi}{3}$ and $\frac{5\pi}{3}$

B. $\frac{\pi}{3}$ and $\frac{2\pi}{3}$

C. $\frac{\pi}{6}$ and $\frac{5\pi}{6}$

D. $\frac{\pi}{6}$ and $\frac{11\pi}{6}$ **2**

[SQA] 12.

(a) Using the fact that $\frac{7\pi}{12} = \frac{\pi}{3} + \frac{\pi}{4}$, find the exact value of $\sin\left(\frac{7\pi}{12}\right)$. **3**

(b) Show that $\sin(A + B) + \sin(A - B) = 2\sin A\cos B$. **2**

(c) (i) Express $\frac{\pi}{12}$ in terms of $\frac{\pi}{3}$ and $\frac{\pi}{4}$.

(ii) Hence or otherwise find the exact value of $\sin\left(\frac{7\pi}{12}\right) + \sin\left(\frac{\pi}{12}\right)$. **4**

[SQA] 13.

(a) Show that $2\cos(x^\circ + 30^\circ) - \sin x^\circ$ can be written as $\sqrt{3}\cos x^\circ - 2\sin x^\circ$. **3**

(b) Express $\sqrt{3}\cos x^\circ - 2\sin x^\circ$ in the form $k\cos(x^\circ + \alpha^\circ)$ where $k > 0$ and $0 \leq \alpha \leq 360$ and find the values of k and α. **4**

(c) Hence, or otherwise, solve the equation $2\cos(x^\circ + 30^\circ) = \sin x^\circ + 1$, $0 \leq x \leq 360$. **3**

[END OF QUESTIONS]