Trigonometry Non-Calculator C Grade

1. If $f(x) = 2\sin\left(3x - \frac{\pi}{2}\right) + 5$, what is the range of values of $f(x)$?

   A. $-1 \leq f(x) \leq 11$

   B. $2 \leq f(x) \leq 8$

   C. $3 \leq f(x) \leq 7$

   D. $-3 \leq f(x) \leq 7$ **2**

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
C	1.2	C	0	0	NC	A1	2009 P1 Q14

2. Functions f and g are defined on suitable domains by $f(x) = \cos x$ and $g(x) = x + \frac{\pi}{6}$.

   What is the value of $f\left(g\left(\frac{\pi}{6}\right)\right)$?

   A. $\frac{1}{2} + \frac{\pi}{6}$

   B. $\frac{\sqrt{3}}{2} + \frac{\pi}{6}$

   C. $\frac{\sqrt{3}}{2}$

   D. $\frac{1}{2}$ **2**

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
D	1.2	C	0	0	NC	A4, T3	2010 P1 Q11

3. Find the maximum value of

$$2 - 3\sin\left(x - \frac{\pi}{3}\right)$$

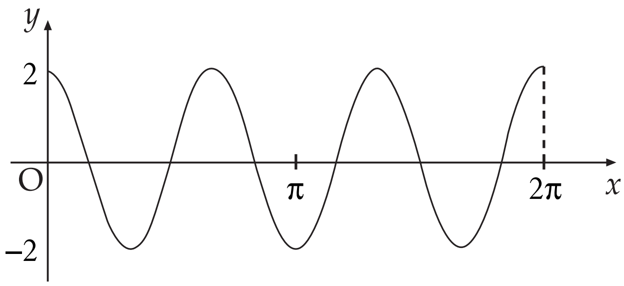
and the value of x where this occurs in the interval $0 \leq x \leq 2\pi$.

	max value	x
A.	-1	$\frac{11\pi}{6}$
B.	5	$\frac{11\pi}{6}$
C.	-1	$\frac{5\pi}{6}$
D.	5	$\frac{5\pi}{6}$

2

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
B	1.2	C	0	0	NC	T1, T6	2012 P1 Q12

4. The diagram shows the graph with equation of the form $y = a\cos bx$ for $0 \leq x \leq 2\pi$.



What is the equation of this graph?

A. $y = 2\cos 3x$

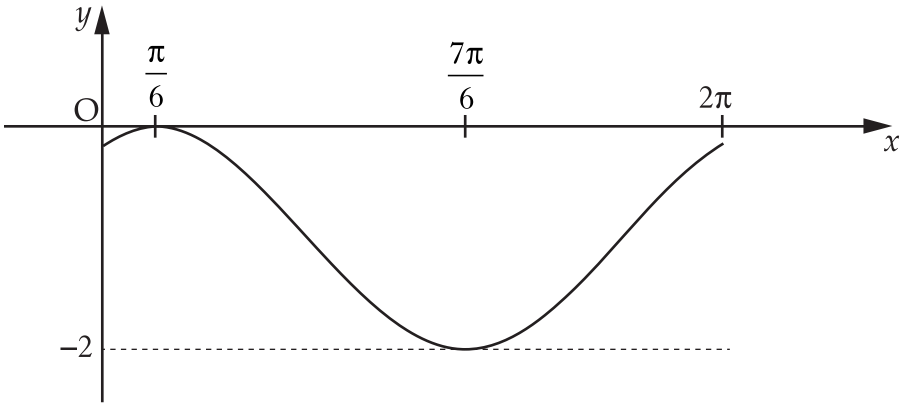
B. $y = 2\cos 2x$

C. $y = 3\cos 2x$

D. $y = 4\cos 3x$ **2**

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
A	1.2	C	0	0	CN	T4	2010 P1 Q4

5. The diagram shows the curve with equation of the form $y = \cos(x + a) + b$ for $0 \le x \le 2\pi$.



What is the equation of this curve?

A. $y = \cos\left(x - \frac{\pi}{6}\right) - 1$

B. $y = \cos\left(x - \frac{\pi}{6}\right) + 1$

C. $y = \cos\left(x + \frac{\pi}{6}\right) - 1$

D. $y = \cos\left(x + \frac{\pi}{6}\right) + 1$ **2**

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
A	1.2	C	0	0	NC	T4, T1	2012 P1 Q9

6. (*a*) Diagram 1 shows a right angled triangle, where the line OA has equation $3x - 2y = 0$.

(i) Show that $\tan a = \frac{3}{2}$.

(ii) Find the value of $\sin a$. **4**

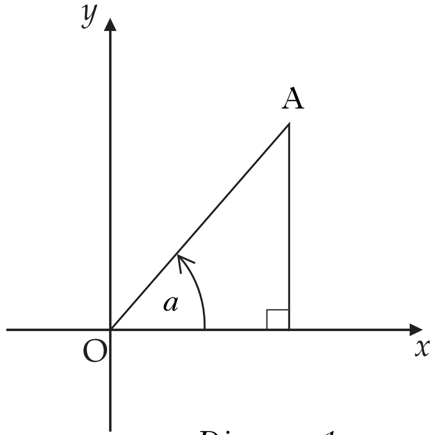


Diagram 1

(*b*) A second right angled triangle is added as shown in Diagram 2.

The line OB has equation $3x - 4y = 0$.

Find the values of $\sin b$ and $\cos b$. **4**

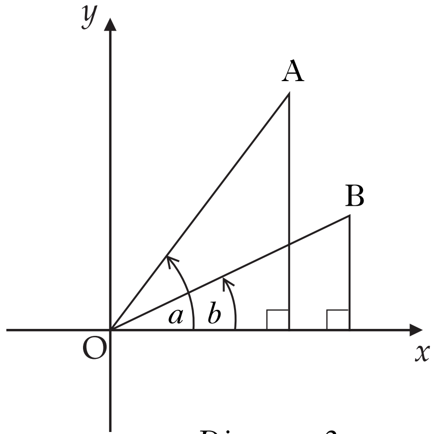


Diagram 2

(*c*) (i) Find the value of $\sin(a - b)$.

(ii) State the value of $\sin(b - a)$. **4**

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
(*a*)	4	C	CN	G2, T5	proof, $\frac{3}{\sqrt{13}}$	2010 P1 Q23
(*b*)	4	C	CN	G2, T5	$\sin b = \frac{3}{5}, \cos b = \frac{4}{5}$	
(*c*)	4	B	CN	T8, T1	$\frac{6}{5\sqrt{13}}, -\frac{6}{5\sqrt{13}}$	

•1 ss: write in slope/intercept form	•1 $y = \frac{3}{2}x$
•2 ic: connect gradient and $\tan a$	•2 $m = \frac{3}{2}$ and $\tan a = \frac{3}{2}$
•3 pd: calculate hypotenuse	•3 $\sqrt{13}$
•4 ic: state value of sine ratio	•4 $\frac{3}{\sqrt{13}}$ or $\frac{3\sqrt{13}}{13}$
•5 ss: determine $\tan b$	•5 $\tan b = \frac{3}{4}$
•6 ss: know to complete triangle	•6 right-angled triangle with 3, 4
•7 pd: determine hypotenuse	•7 5
•8 ic: state values of sine and cosine ratios	•8 $\sin b = \frac{3}{5}$ and $\cos b = \frac{4}{5}$
•9 ss: know to use addition formula	•9 $\sin a \cos b - \cos a \sin b$
•10 ic: substitute into expansion	•10 $\frac{3}{\sqrt{13}} \times \frac{4}{5} - \frac{2}{\sqrt{13}} \times \frac{3}{5}$
•11 pd: evaluate sine of compound angle	•11 $\frac{6}{5\sqrt{13}}$
•12 ss: use $\sin(-x) = -\sin x$	•12 $-\frac{6}{5\sqrt{13}}$

[SQA] 7. Functions $f(x) = \sin x$, $g(x) = \cos x$ and $h(x) = x + \frac{\pi}{4}$ are defined on a suitable set of real numbers.

(*a*) Find expressions for:

(i) $f(h(x))$;

(ii) $g(h(x))$. **2**

(*b*) (i) Show that $f(h(x)) = \frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x$.

(ii) Find a similar expression for $g(h(x))$ and hence solve the equation $f(h(x)) - g(h(x)) = 1$ for $0 \le x \le 2\pi$. **5**

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
(*a*)	2	C	NC	A4	(i) $sin(x + \frac{\pi}{4})$, (ii) $cos(x + \frac{\pi}{4})$	2001 P1 Q7
(*b*)	5	C	NC	T8, T7	(i) proof, (ii) $x = \frac{\pi}{4}, \frac{3\pi}{4}$	

•[1] ic: interpret composite functions	•[1] $\sin(x + \frac{\pi}{4})$
•[2] ic: interpret composite functions	•[2] $\cos(x + \frac{\pi}{4})$
•[3] ss: expand $\sin(x + \frac{\pi}{4})$	•[3] $\sin x \cos \frac{\pi}{4} + \cos x \sin \frac{\pi}{4}$ **and complete**
•[4] ic: interpret	•[4] $g(h(x)) = \frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x$
•[5] ic: substitute	•[5] $(\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x) - (\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x)$
•[6] pd: start solving process	•[6] $\frac{2}{\sqrt{2}}\sin x$
•[7] pd: process	•[7] $x = \frac{\pi}{4}, \frac{3\pi}{4}$ *accept only radians*

[SQA] 8. Functions f and g are defined on suitable domains by $f(x) = \sin(x^\circ)$ and $g(x) = 2x$.

(*a*) Find expressions for:

(i) $f(g(x))$;

(ii) $g(f(x))$. **2**

(*b*) Solve $2f(g(x)) = g(f(x))$ for $0 \leq x \leq 360$. **5**

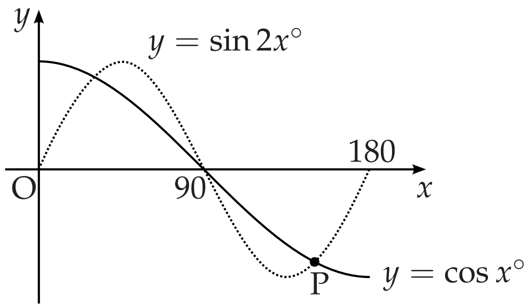
Part	Marks	Level	Calc.	Content	Answer	U2 OC3
(*a*)	2	C	CN	A4	(i) $\sin(2x^\circ)$, (ii) $2\sin(x^\circ)$	2002 P1 Q3
(*b*)	5	C	CN	T10	$0^\circ, 60^\circ, 180^\circ, 300^\circ, 360^\circ$	

•[1] ic: interpret $f(g(x))$	•[1] $\sin(2x^\circ)$
•[2] ic: interpret $g(f(x))$	•[2] $2\sin(x^\circ)$
•[3] ss: equate for intersection	•[3] $2\sin(2x^\circ) = 2\sin(x^\circ)$
•[4] ss: substitute for $\sin 2x$	•[4] appearance of $2\sin(x^\circ)\cos(x^\circ)$
•[5] pd: extract a common factor	•[5] $2\sin(x^\circ)\,(2\cos(x^\circ) - 1)$
•[6] pd: solve a 'common factor' equation	•[6] $\sin(x^\circ) = 0$ **and** $0, 180, 360$
•[7] pd: solve a 'linear' equation	•[7] $\cos(x^\circ) = \frac{1}{2}$ **and** $60, 300$
	or
	•[6] $\sin(x^\circ) = 0$ **and** $\cos(x^\circ) = \frac{1}{2}$
	•[7] $0, 60, 180, 300, 360$

[SQA] 9. (*a*) Solve the equation $\sin 2x° - \cos x° = 0$ in the interval $0 \leq x \leq 180$. **4**

(*b*) The diagram shows parts of two trigonometric graphs, $y = \sin 2x°$ and $y = \cos x°$.

Use your solutions in (*a*) to write down the coordinates of the point P. **1**



Part	Marks	Level	Calc.	Content	Answer	U2 OC3
(*a*)	4	C	NC	T10	$30, 90, 150$	2001 P1 Q5
(*b*)	1	C	NC	T3	$(150, -\frac{\sqrt{3}}{2})$	

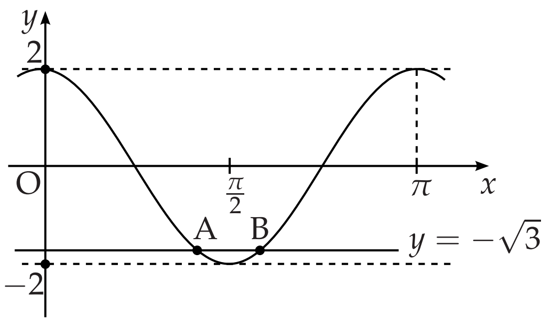
•[1] ss: use double angle formula •[2] pd: factorise •[3] pd: process •[4] pd: process •[5] ic: interpret graph	•[1] $2\sin x° \cos x°$ •[2] $\cos x°(2\sin x° - 1)$ •[3] $\cos x° = 0, \sin x° = \frac{1}{2}$ •[4] $90, 30, 150$ **or** •[3] $\sin x° = \frac{1}{2}$ and $x = 30, 150$ •[4] $\cos x° = 0$ and $x = 90$ •[5] $\left(150, -\frac{\sqrt{3}}{2}\right)$

[SQA] 10. The diagram shows the graph of a cosine function from 0 to π.

(*a*) State the equation of the graph. **1**

(*b*) The line with equation $y = -\sqrt{3}$ intersects this graph at point A and B.

Find the coordinates of B. **3**



Part	Marks	Level	Calc.	Content	Answer	U2 OC3
(*a*)	1	C	NC	T4	$y = 2\cos 2x$	2002 P1 Q8
(*b*)	3	C	NC	T7	$B(\frac{7\pi}{12}, -\sqrt{3})$	

•[1] ic: interpret graph •[2] ss: equate equal parts •[3] pd: solve linear trig equation in radians •[4] ic: interpret result	•[1] $2\cos 2x$ •[1] $2\cos 2x = -\sqrt{3}$ •[2] $2x = \frac{5\pi}{6}, \frac{7\pi}{6}$ •[3] $x = \frac{7\pi}{12}$

11. Solve $2\cos x = \sqrt{3}$ for x, where $0 \leq x < 2\pi$.

A. $\frac{\pi}{3}$ and $\frac{5\pi}{3}$

B. $\frac{\pi}{3}$ and $\frac{2\pi}{3}$

C. $\frac{\pi}{6}$ and $\frac{5\pi}{6}$

D. $\frac{\pi}{6}$ and $\frac{11\pi}{6}$ **2**

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
D	2.3	C	0	0	NC	T7, T2, T3	2011 P1 Q10

[SQA] 12.

(a) Using the fact that $\frac{7\pi}{12} = \frac{\pi}{3} + \frac{\pi}{4}$, find the exact value of $\sin\left(\frac{7\pi}{12}\right)$. **3**

(b) Show that $\sin(A+B) + \sin(A-B) = 2\sin A\cos B$. **2**

(c) (i) Express $\frac{\pi}{12}$ in terms of $\frac{\pi}{3}$ and $\frac{\pi}{4}$.

(ii) Hence or otherwise find the exact value of $\sin\left(\frac{7\pi}{12}\right) + \sin\left(\frac{\pi}{12}\right)$. **4**

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
(a)	3	C	NC	T8, T3	$\frac{\sqrt{3}+1}{2\sqrt{2}}$	2009 P1 Q24
(b)	2	C	CN	T8	proof	
(c)	3	B	NC	T11	$\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$	
(c)	1	C	NC	T11	$\frac{\sqrt{6}}{2}$ or $\sqrt{\frac{3}{2}}$	

•1 ss: expand compound angle	•1 $\sin\frac{\pi}{3}\cos\frac{\pi}{4} + \cos\frac{\pi}{3}\sin\frac{\pi}{4}$
•2 ic: substitute exact values	•2 $\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \frac{1}{\sqrt{2}}$
•3 pd: process to a single fraction	•3 $\frac{\sqrt{3}+1}{2\sqrt{2}}$ or equivalent
•4 ic: start proof	•4 $\sin A\cos B + \cos A\sin B + \cdots$
•5 ic: complete proof	•5 $\cdots + \sin A\cos B - \cos A\sin B$ and complete
•6 ss: identify steps	•6 $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$
•7 ic: start process (identify 'A' & 'B')	•7 $2 \times \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}$
•8 ic: substitute	•8 $\frac{\sqrt{6}}{2}$ or $\sqrt{\frac{3}{2}}$
•9 pd: process	

[SQA] 13.

(*a*) Show that $2\cos(x^\circ + 30^\circ) - \sin x^\circ$ can be written as $\sqrt{3}\cos x^\circ - 2\sin x^\circ$. **3**

(*b*) Express $\sqrt{3}\cos x^\circ - 2\sin x^\circ$ in the form $k\cos(x^\circ + \alpha^\circ)$ where $k > 0$ and $0 \leq \alpha \leq 360$ and find the values of k and α. **4**

(*c*) Hence, or otherwise, solve the equation $2\cos(x^\circ + 30^\circ) = \sin x^\circ + 1$, $0 \leq x \leq 360$. **3**

Part	Marks	Level	Calc.	Content	Answer	U3 OC4
(*a*)	3	C	CR	T8, T3		1990 P2 Q5
(*b*)	4	C	CR	T13		
(*c*)	3	A/B	CR	T16		

(*a*)
- •1 $\cos(x+30)^\circ = \cos x^\circ \cos 30^\circ - \sin x^\circ \sin 30^\circ$
- •2 $\frac{\sqrt{3}}{2}\cos x^\circ - \frac{1}{2}\sin x^\circ$
- •3 $2 \times \left(\frac{\sqrt{3}}{2}\cos x^\circ - \frac{1}{2}\sin x^\circ\right) - \sin x^\circ$

(*b*)
- •4 $k\cos x^\circ \cos\alpha^\circ - k\sin x^\circ \sin\alpha^\circ$
- •5 $k\sin\alpha^\circ = \sqrt{3}$ and $k\sin\alpha^\circ = 1$
- •6 $k = \sqrt{7}\ \vec{OG} = 426$
- •7 $\alpha = 49.1$

(*c*)
- •8 $\sqrt{7}\cos(x + 49.1)^\circ = 1$
- •9 $x = 18.7^\circ$
- •10 $x = 243.1^\circ$

[END OF QUESTIONS]