Differentiation – 3

1. Find the derivative of

(a) $y = x^2 + 3\sqrt{x}$ (b) $f(x) = \dfrac{x^2 - 4}{\sqrt{x}}$ (c) $y = \dfrac{(x - 2)(x + 1)}{\sqrt{x}}$

(d) $y = (4x - 2)^3$ (e) $y = \sqrt{6x - 4}$ (f) $f(x) = \sin 4x$

(g) $y = 2\cos^2 x$ (h) $y = \sin^3 x$

2. The height of a ball projected upwards is calculated using the formula $h(t) = 30t - t^2$, where t is the time in seconds after being projected.

(a) Find the height of the ball after 10 seconds.
(b) Find the speed of the ball after 12 seconds.

3. Find the equation of the tangent to the curve $y = x^3 - x^2 - 1$ at the point (2,3).

4. Find the equation of the tangent to the curve $y = 6\sqrt{x} - \dfrac{2}{x^2}$ at the point where x = 1.

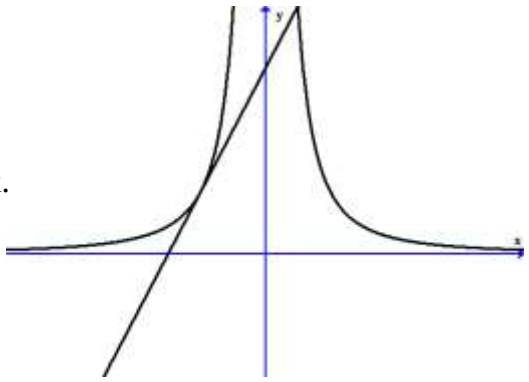
5. Find the equation of the tangent to the curve $y = \sin^2 x$ at the point where $x = \dfrac{\pi}{6}$.

6. A curve has equation $y = (3x - 2)^4$. A tangent to this curve has gradient 12.

(a) Find the point of contact of the tangent and the curve.
(b) Find the equation of this tangent.

7. A tangent to the curve $y = \dfrac{4}{x^2}$ is parallel to the line y = x.

Find the equation of this tangent.



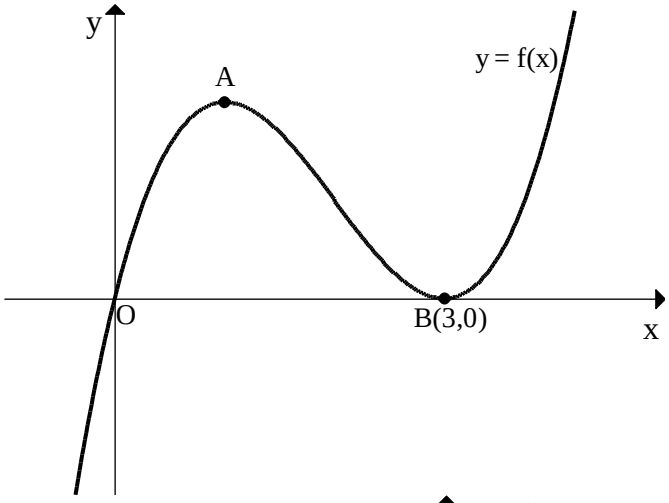
8. Show that the function $f(x) = 6x^2 - x^3 - 12x$ is never increasing.

9. Show that $y = x^3 + 4x + 1$ is always increasing.

10. Find the values of x for which $y = x^3 + 6x^2 - 36x$ is increasing.

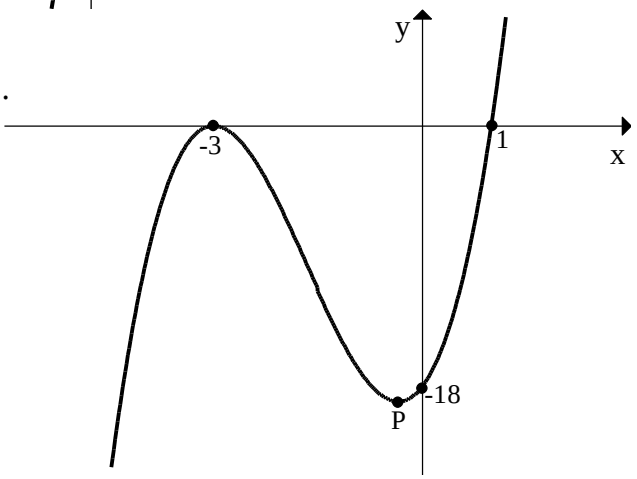
11. Find the values of x for which $y = x^3 + 3x^2 - 9x + 1$ is decreasing.

12. A sketch of the graph of $y = f(x)$ where $f(x) = x^3 - 6x^2 + 9x$ is shown opposite. The graph has a maximum at A and a minimum at (3,0). Find the coordinates of A.



13. The diagram opposite shows the graph of $y = f(x)$.

    (a) Find a formula for f(x).
    (b) Find the coordinates of the turning point at P.



14. A curve has equation $y = x^3 - 3x^{2.}$

    (a) Find where this curve cuts the x and y axes.
    (b) Find the stationary points of the curve and determine their nature.
    (c) Sketch the curve.

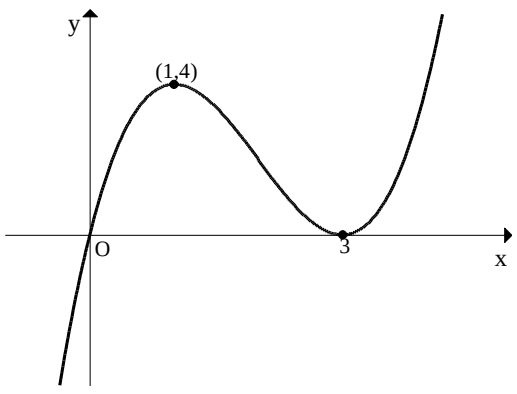
15. Find the minimum and maximum values of $y = 8x^3 - 3x^2$ in the interval $-2 \leq x \leq 1$.

16. Show that the curve $f(x) = x^3 - 4x^2 + 7x$ has no stationary points.

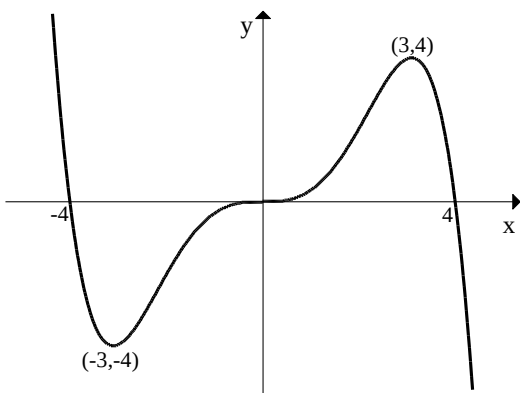
17. Show that the curve $y = \frac{1}{2}x^4 + x^2 - 20x + 15$ has a single stationary point at the point (2, -13).

18. In each example below sketch the graph of $y = f'(x)$.

(a)



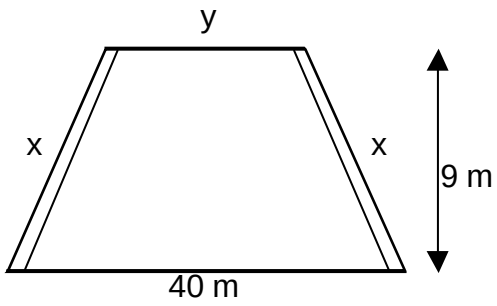
(b)



19. Find the coordinates of the points where the curves $y = x^3 + 2x^2 - 8x$ and $y = x^3 + x^2 + 2x$ have the same gradient.

20. $y = x^2 - 4x$. Show that $\left(\frac{dy}{dx}\right)^2 - 4y - 16 = 0$.

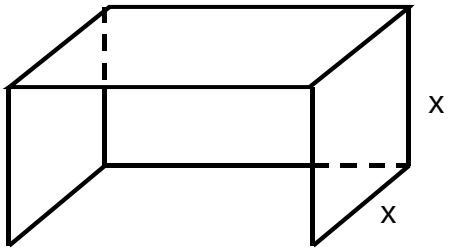
21. The diagram shows the end view of an aircraft hangar. The sloping sides and roof of the hangar are reinforced with metal beams.
The roof beam is of length y metres and there are 2 beams of length x metres at each sloping side.



(a) Show that $y = 40 - 2(x^2 - 81)^{\frac{1}{2}}$

(b) The length of metal needed for the supporting beams is $L = 4x + y$. Find the value of x which minimises this length.

22. A wind shelter, as shown opposite, has a back, top and two square sides. The total amount of canvas used in the shelter is 96 m^2 and the length of each square side is x metres.



(a) If the volume of the shelter is V cm^3, show that $V = x(48 - x^2)$.

(b) Find the dimensions of the shelter which give a maximum volume.