Functions

1. $f(x) = 2x^2$ and $g(x) = 5x - 4$.

   (a) Find f(g(2)).
   (b) Find a formula for f(g(x)).

2. $f(x) = \frac{2}{3x} - 1$ and $g(x) = \frac{2}{3x + 3}$ $\quad x \neq -1, 0$

   (a) Given h(x) = f(g(x)), find a formula for h(x).
   (b) State the connection between f(x) and g(x).

3. $f(x) = 6x^2 - 4x$ and $g(x) = \frac{1}{3x - 6}, x \neq 2$

   (a) Show that $g(f(x)) = \frac{1}{6(3x + 1)(x - 1)}$.
   (b) State a suitable domain for g(f(x)).

4. $f(x) = \frac{2}{1 - x}$ and $g(x) = 1 - \frac{2}{x}, x \neq 0, 1$

   (a) find f(g(x))
   (b) State the connection between f and g.

5. $f(x) = (x - 1)(x + 3)$ and $g(x) = x^2 + 3$.

   Show that $f(g(x)) - g(g(x)) = 2x^2$

6. The functions f and g, defined on suitable domains, are given by

   $f(x) = \frac{1}{x^2 - 4}$ and $g(x) = x + 1$.

   (a) Find an expression for h(x), where h(x) = f(g(x)).
   Give your answer as a single fraction.
   (b) State a suitable domain for h.

7. On a suitable set of real numbers, functions f and g are defined by

$$f(x) = \frac{1}{x+3} \text{ and } g(x) = \frac{1}{x} - 3$$

Find f(g(x)) in its simplest form.

8. A function f is defined on the set of real numbers by $f(x) = \frac{4-x}{x}, x \neq 0$

Find in its simplest form an expression for f(f(x)).

9. $f(x) = \frac{4}{x+2}$ and $g(x) = \frac{2}{x} - 2, x \neq -2, 0$

Find f(g(x)) in its simplest form.

10. $f(x) = \frac{x-5}{x}$ and $g(x) = 3x - \frac{12}{x},\ x \neq 0$

(a) Show that $f(g(x)) = \frac{(3x+4)(x-3)}{3(x-2)(x+2)}$

(b) State a suitable domain for f(g(x)).

11. Two functions are defined as $f(x) = x^2 + 1$ and $g(x) = 2 - x^2$.

(a) Find an expression for f(f(x)).
(b) Find a similar expression for g(g(x)) and hence show that $f(f(x)) + g(g(x)) = 6x^2$.

12. $f(x) = 2x + 1$ and $g(x) = x^2 + k$, where k is a constant.

(a) Find an expression for (i) g(f(x)) (ii) f(g(x)).

(b) Show that g(f(x)) – f(g(x)) = 0 simplifies to $2x^2 + 4x - k = 0$.

(c) Find the value of k for which $2x^2 + 4x - k = 0$ has equal roots.