

## Functions

1.  $f(x) = 2x - 5$  and  $g(x) = x^2 + 1$ . Find a formula for  $f(g(x))$ .

2.  $f(x) = x^2 - 2$ . Find a formula for  $f(f(x))$ .

3. The functions  $f$  and  $g$  are defined on suitable domains with

$$f(x) = \frac{2}{x+1} \quad \text{and} \quad g(x) = \frac{2x+4}{x}$$

Show that  $g(f(x)) = 2x + 4$ .

4.  $f(x) = \frac{4}{3x+1}$   $x \neq -\frac{1}{3}$  and  $g(x) = 4x - 3$

(a) Find a formula for  $f(g(x))$ .

(b) State a suitable domain for  $f(g(x))$ .

5. The functions  $f$  and  $g$  are defined on suitable domains with

$$f(x) = \frac{1}{x^2 - 1} \quad \text{and} \quad g(x) = x + 1$$

(a)  $h(x) = g(f(x))$ . Find an expression for  $h(x)$ . Give your answer as a single fraction.

(b) State a suitable domain for  $h(x)$ .

6.  $f(x) = \frac{1}{2x+4}$   $x \neq -2$  and  $g(x) = \frac{1}{x-1}$   $x \neq 1$

(a) Find a formula for  $f(g(x))$ .

(b) State a suitable domain for  $f(g(x))$ .

7. The function  $f$ , defined on a suitable domain, is  $f(x) = \frac{x}{x-1}$

(a) Find a formula for  $f(f(x))$ .

(b) What can you say about the function  $f$ ?

8.  $f(x) = 2\sin x$  and  $g(x) = (x + \frac{\pi}{2})$

- (a) Given  $k(x) = f(g(x))$ , find a formula for  $k(x)$ .
- (b) Solve the equation  $k(x) = -1$ , for  $0 \leq x \leq 2\pi$

9.  $f(x) = 2x - 1$  and  $g(x) = x^2 + x$ .

- (a) Find a formula for  $g(f(x))$  in its simplest form.
- (b)  $h(x) = g(f(x)) - f(g(x))$ . Find a formula for  $h(x)$ .
- (c) Solve the equation  $h(x) = 7$ .

10.  $f(x) = x^2 - 2$  and  $g(x) = 2x + 1$

- (a) Find expressions for  $f(g(x))$  and  $g(f(x))$ .
- (b) There is only one value of  $x$  for which  $f(g(x)) = g(f(x))$ , find this value of  $x$ .

11.  $f(x) = 3x - 10$      $g(x) = 4 - 2x$      $h(x) = \frac{1}{6}(2 - x)$

- (a)  $k(x) = f(g(x))$ . Find  $k(x)$ .
- (b) Find a formula for  $h(k(x))$ .
- (c) What is the connection between  $h$  and  $k$ ?

12.  $f(x) = 3x - 2$  and  $g(x) = 3x + 2$

- (a) Find formulae for  $f(g(x))$  and  $g(f(x))$ .
- (b) Find the least value of the product  $f(g(x)) \times g(f(x))$ .

13.  $f(x) = x^2 + 1, x \geq 0$  and  $g(x) = \sqrt{x - 1}, x \geq 1$

- (a) Sketch  $f(x)$  for the given domain.
- (b) Find an expression for  $f(g(x))$ .
- (c) Hence, or otherwise sketch  $g(x)$  for the given domain.

14.  $f(x) = 2x^2 + 1, x \geq 0$  and  $g(x) = \sqrt{\frac{1}{2}(x - 1)}, x \geq 1$ .

- (a) Sketch  $f(x)$  for the given domain.
- (b) Find an expression for  $g(f(x))$ .
- (c) Hence, or otherwise, sketch the graph of  $g(x)$  for the given domain.