Recurrence Relations

1. Given the recurrence relation $u_{n+1} = 0.8u_n + 6$, $u_o = 19$

   (a) State why the sequence generated by it has a limit.
   (b) Calculate the value of this limit.

2. A sequence is defined by the recurrence relation $u_{n+1} = 0.4u_n + 8$.

   (a) Explain why this sequence has a limit as n tends to infinity.
   (b) Find the exact value of this limit.

3. Two sequences are defined by these recurrence relations

   $u_{n+1} = 3u_n - 0.6$ with $u_o = 1$ $\quad$ $v_{n+1} = 0.3v_n + 5$ with $v_o = 1$

   (a) Explain why only one of these sequences approaches a limit as $n \to \infty$
   (b) Find algebraically the exact value of this limit.

4. A sequence is defined by the recurrence relation $u_n = 0.9u_{n-1} + 2$, $u_1 = 3$

   (a) Calculate the value of u_2 and u_3
   (b) What is the smallest value of n for which $u_n > 8$
   (c) Find the limit of this sequence as $n \to \infty$

5. A sequence is defined by the recurrence relation $V_n = 0.7V_{n-1} + 3$, $V_1 = 6$

   (a) Calculate the value of V_2
   (b) What is the smallest value of n for which $V_n > 8.5$
   (c) Find the limit of this sequence as $n \to \infty$

6. Two sequences are defined by the recurrence relations

   $u_{n+1} = 0.3u_n + p$ $\quad$ $v_{n+1} = 0.9v_n + q$

   If both sequences have the same limit, express p in terms of q.

7. Two sequences are defined by the recurrence relations

   $u_{n+1} = au_n + 6$ $\quad$ $v_{n+1} = a^2v_n + 10$

   If both sequences approach the same limit as $n \to \infty$, calculate a and hence evaluate this limit.

8. For the recurrence relation $u_{n+1} = au_n + b$, it is known that $u_o = 6$, $u_1 = 12$ and $u_2 = 21$.

   (a) Find the values of a and b.
   (b) Hence find the value of u_3.

9. For the recurrence relation

$$u_{n+1} = mu_n + c$$

$u_2 = 20$, $u_3 = 16$ and $u_4 = 14$

(a) Find the values of m and c.
(b) Hence find the value of u_o
(c) Find the limit of the sequence.

10. The first three terms of the recurrence relation $u_{n+1} = pu_n + q$ are 14,12 and 10 respectively.
Find the values of p and q.

11. The terms of a sequence satisfy the relation $u_{n+1} = ku_n + 6$. Find the value of k which produces a limit of 9.

12. A recurrence relation is defined as $u_{n+1} = tu_n + 8$. Find the value of t which produces a limit of 12.

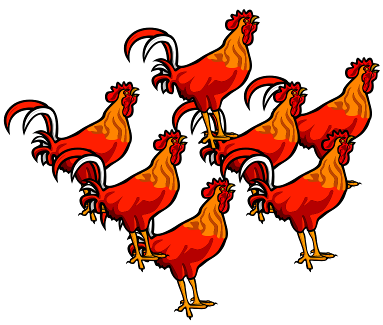
13. A sequence satisfies the relation $u_{n+1} = mu_n + 3$, $u_o = 2$.

a. Express u_1 and u_2 in terms of m.
b. Given that $u_2 = 5$, find the value of m that produces a sequence with a limit.

14. A sequence satisfies the relation $v_{n+1} = pv_n + 4$, $v_o = 3$.

a. Express v_1 and v_2 in terms of p.
b. Given that $v_2 = 8$, find the value of p that produces a sequence with no limit.

15. A farmer has 160 hens. Foxes attack the hens and kill 30% of the remaining hens each month.
At the end of each month the farmer buys 30 new hens to replenish his stock.



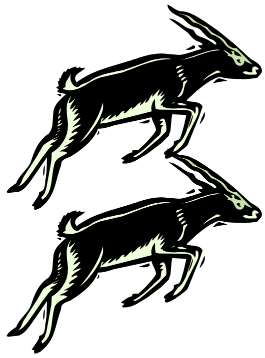
(a) Set up a recurrence relation to show the number of hens present at the start of each month, just after he restocks his farm.

(b) find the limit of this sequence and use this to explain what happens in the long run to his initial stock of 160 hens.

16. A patient is injected with 80 ml of an antibiotic drug. Every 4 hours 40% of the drug passes out of her bloodstream. To compensate for this an extra 15ml of antibiotic is given every 4 hours.

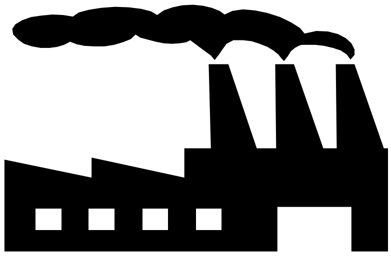
(a) Find a recurrence relation for the amount of drug in the patient's bloodstream.
(b) Calculate the amount of antibiotic remaining in the bloodstream after one day.

17. A game reserve in Kenya has a population of 4000 antelope. Due to poaching and other factors 20% of the antelope are killed each year. On average, in the same period, 650 baby antelope are born in the reserve



(a) Set up a recurrence relation to describe this situation.
(b) What will happen in the long term to the number of antelope in the reserve?

18. A lake next to a chemical factory is found to contain an estimated 20 tonnes of pollutant. Through filtration, the factory are able to remove 85% of the pollutant annually but an extra 2 tonnes is also released into the lake over the same period.



(a) Find a recurrence relation to describe this situation.
(b) Health inspectors inform the factory that a level of 2.5 tonnes of pollutant or less in the lake would be acceptable.
In the long run, will the factory attain an acceptable level of pollutant in the lake?

19. A man plants a hedge round the outside of his lawn. The hedge is estimated to grow at a rate of 1.2 metres per year. He decides to trim the hedge in December each year by 40% of its height.

(a) To what height will the hedge grow in the long run?
(b) He wants the hedge to grow to a height of no more than 2 metres. What is the minimum percentage he must trim the hedge to ensure that this happens?

20. Once a month the cleansing department in a Scottish city remove chewing gum from city streets. The cleaning operation removes 40 % of the gum present. Each month the public drop 10 kg of gum on the streets.

(a) In the long run what will happen to the mass of chewing gum on the streets?

(b) The council initiate a poster campaign to encourage the public not to drop chewing gum. They estimate that this campaign should cut the amount of gum dropped to 6 kg per month.

How will this affect the chewing gum problem in the long run?