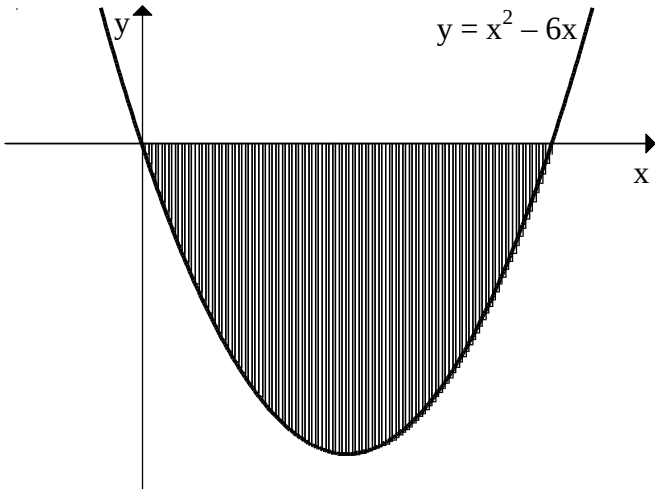
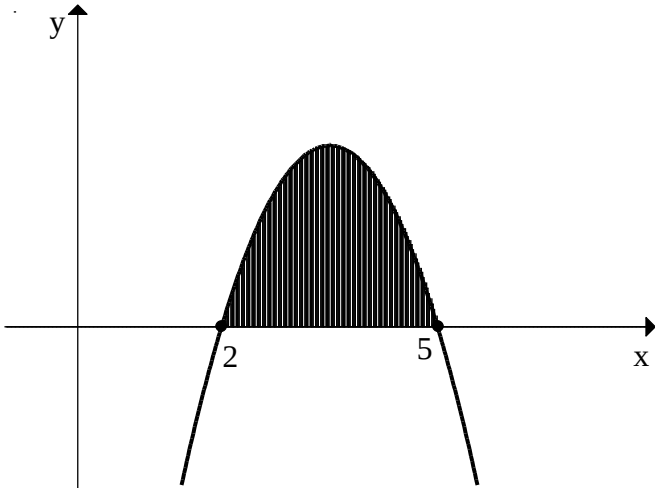
Integration

1. Calculate (a) $\int x^2(x-5)\,dx$ (b) $\int_1^4 \frac{x^2+3}{\sqrt{x}}\,dx$ (c) $\int_{-1}^{2}\left(\frac{x}{2}+\frac{2}{x}\right)^2 dx$

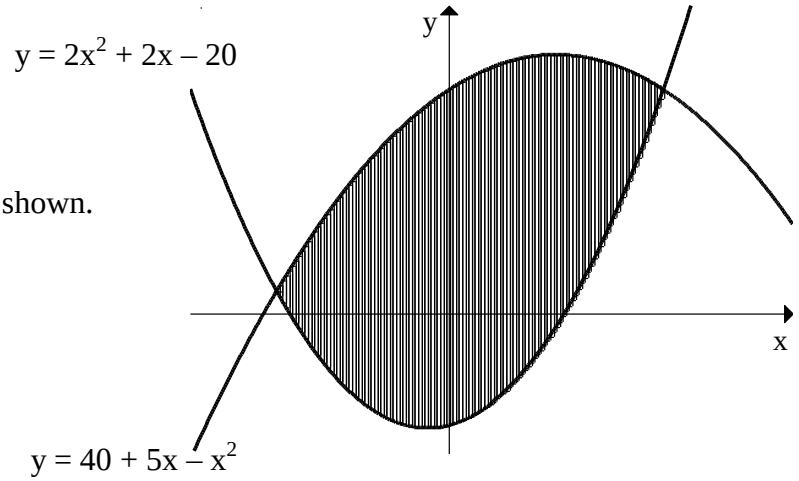
2. Calculate the shaded area in the diagram opposite.



3. The diagram shows part of the graph of $y = 7x - 10 - x^2$.
Calculate the shaded area.

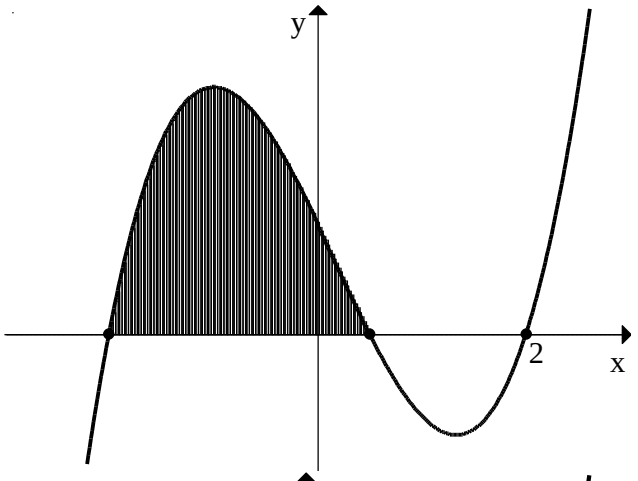


4. Find the area between the curves shown.



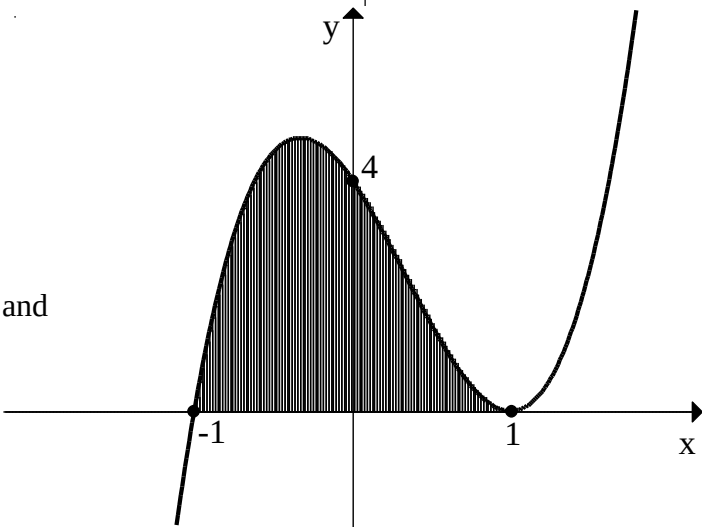
5. The diagram opposite illustrates the graph of $y = f(x)$ where $f(x) = 2x^3 - x^2 - 8x + 4$.

   (a) Show that $x - 2$ is a factor of $f(x)$ and hence fully factorise $f(x)$.
   (b) Calculate the shaded area.

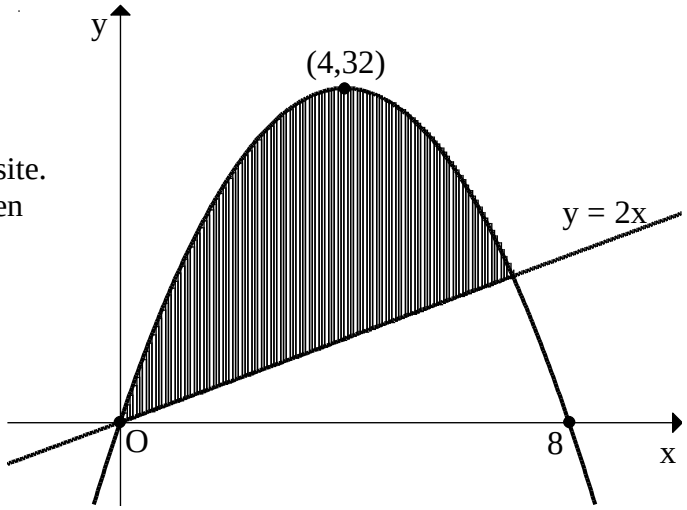


6. The diagram shows part of $y = f(x)$.

   (a) Find a formula for $f(x)$.
   (b) Calculate the area enclosed by $f(x)$ and the x-axis.

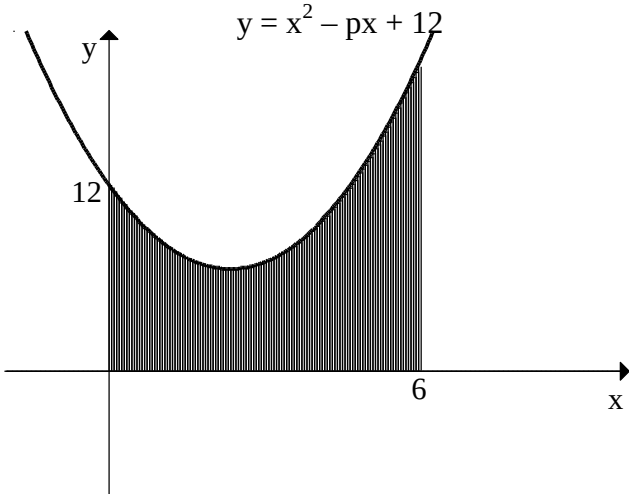


7. (a) Find the equation of the parabola opposite.
   (b)..Hence calculate the shaded area between this parabola and the line $y = 2x$.



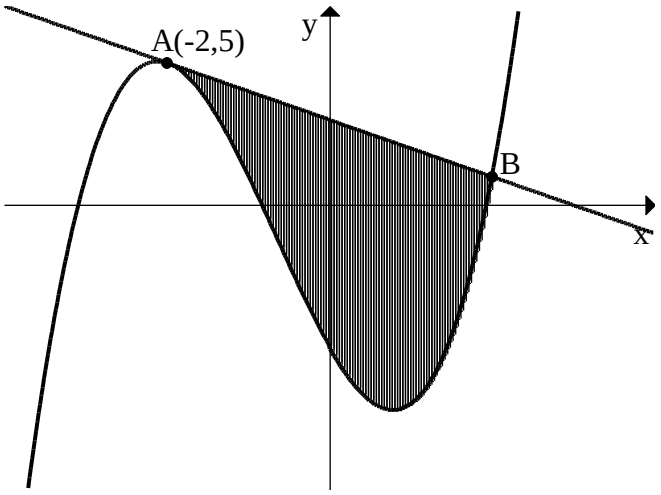
8.In the diagram opposite the area shown is 60. The curve has equation $y = x^2 - px + 12$.

Calculate the value of p.



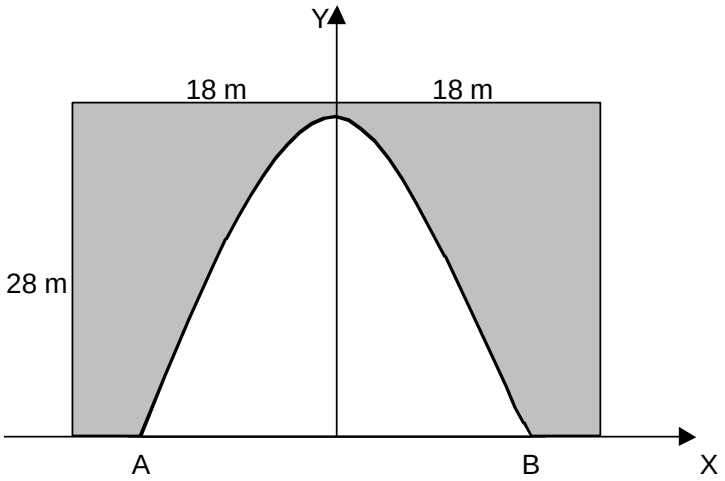
9. The diagram opposite shows the curve $y = x^3 + 2x^2 - 5x - 5$ and the line AB. The line AB is a tangent to the curve at the point A(-2,5).

   (a) Find the equation of the tangent AB.
   (b) Hence find the coordinates of B.
   (c) Calculate the shaded area between the curve and the line.



10. The diagram shows a tunnel 36 metres wide by 28 metres high.
    The roof of the tunnel is in the form of a parabola with equation $y = 24 - \frac{1}{6}x^2$.

    (a) Find the coordinates of A and B.
    (b) Calculate the shaded area.



11. $f'(x) = 3x^2 - 4x + 6$ and $f(2) = 17$.
    Find a formula for f(x).

12. $f'(x) = \dfrac{2x^3 - x^2}{x}$ and $f(6) = 100$.
    Find a formula for f(x).

13. $f'(x) = 4x(x^2 - 1)$ and $f(-1) = 2$. Find a formula for f(x).

14. The graph of $y = g(x)$ passes through the point (3,-1).
    If $\dfrac{dy}{dx} = 3x^2 - \dfrac{1}{x^2}$, express y in terms of x.

15. The graphs of $y = f(x)$ and $y = g(x)$ intersect at the point A on the y-axis.
    If $g(x) = 4x + 2$ and $f'(x) = 2x - 6$, find f(x).