Polynomials

1. (a) Show that $(x - 4)$ is a factor of $x^3 - 4x^2 - x + 4$.
   (b) Hence factorise $x^3 - 4x^2 - x + 4$ fully.

2. (a) Show that $(x + 1)$ is a factor of $x^3 - 7x - 6$.
   (b) Hence factorise $x^3 - 7x - 6$ fully.

3. (a) $f(x) = x^3 + 3x^2 - 9x + 5$. Factorise $f(x)$ fully.
   (b) Hence solve $f(x) = 0$.

4. $f(x) = 4x^3 - 8x^2 - x + 2$. Given $(x - 2)$ is a factor of $f(x)$, solve $f(x) = 0$.

5. (a) $f(x) = 2x^3 + x^2 - 5x + 2$. Show that $(2x - 1)$ is a factor of $f(x)$.
   (b) Factorise $f(x)$ completely.

6. (a) $g(x) = 3x^3 + 4x^2 - 5x - 2$. Show that $(3x + 1)$ is a factor of $g(x)$.
   (b) Hence solve $g(x) = 0$.

7. (a) Show that $x = 1$ is a solution of the equation $x^3 + x^2 - 10x + 8 = 0$.
   (b) Hence solve the equation $x^3 + x^2 - 10x + 8 = 0$ completely.

8. Show that $x = 2$ is a solution to the equation $6x^3 - 5x^2 - 17x + 6 = 0$ and hence solve the equation completely.

9. (a) Given that $(x + 3)$ is a factor of $x^3 + 3x^2 - x + p$, find the value of p.
   (b) Hence solve the equation $x^3 + 3x^2 - x + p = 0$ when p takes this value.

10. (a) Given $(x + 1)$ is a factor of $2x^3 - x^2 + kx - 9$, find the value of k.
    (b) Hence factorise fully $2x^3 - x^2 + kx - 9$ when k takes this value.

11. (a) $f(x) = x^3 - px^2 - 10x + 8p$. Given $(x - 2)$ is a factor of $f(x)$, find p.
    (b) Hence solve $f(x) = 0$ when p takes this value.

12. $(x - 3)$ and $(x + 3)$ are both factors of $2x^3 - x^2 + px + q$. Find p and q.

13. $x = 1$ and $x = -2$ are both solutions to the equation $4x^3 + x^2 + ax + b = 0$. Find the values of a and b.

14. (a) $f(x) = 4x^3 + 13x^2 + cx + d$. Given $(x - 1)$ and $(x + 5)$ are both factors of $f(x)$, find c and d.
   (b) Hence solve $f(x) = 0$ when c and d take these values.

15. (a) Show that $(x - 1)$ is a factor of $x^3 + 5x^2 + 4x - 10$.
   (b) Hence, or otherwise, show that $x = 1$ is the only real solution to the equation $x^3 + 5x^2 + 4x - 10 = 0$.

16. Show that $x = 4$ is the only real solution to the equation $3x^3 - 11x^2 - 16 = 0$


17. A curve has equation $y = 2x^3 + 3x^2 - 6x - 4$.

   (a) Show that the line $y = 3x + 6$ intersects this curve at the point (2,12).
   (b) Find the other points of intersection of the curve and the line $y = 3x + 6$.
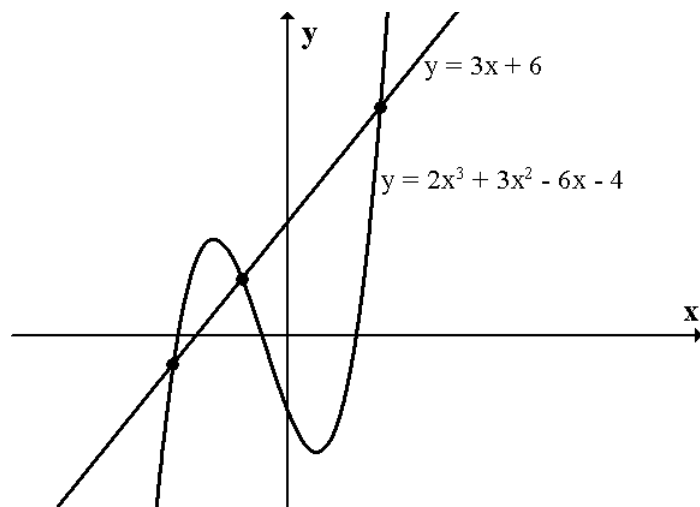
18. A cubic has equation $y = x^3 - 3x^2 - 7x + 18$ and a parabola has equation $y = x^2 + 4x - 12$.

   (a) Show that the cubic and the parabola intersect at the point (2,0)
   (b) Find the other points of intersection of the cubic and the parabola.
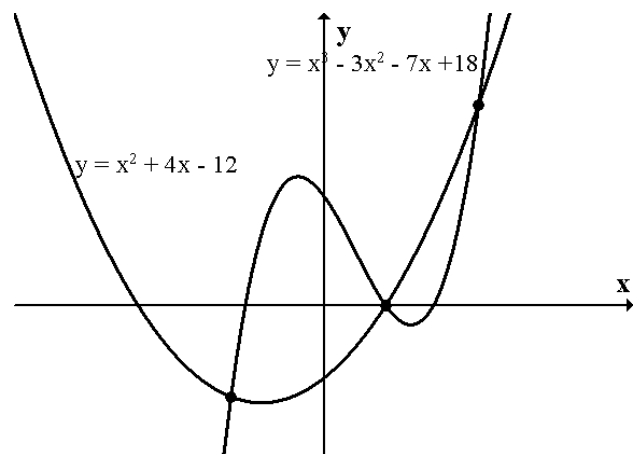
19. A curve has equation $y = 3x^3 + 6x^2 + 9x + 2$.

   (a) Show that the line $y = 2x - 2$ intersects the curve at the point (-1,-4).
   (b) Show that there are no other points of intersection between the curve and the line $y = 2x - 2$.

20. (a) Show that $f(x) = \frac{1}{2}x^4 + 2x^2 + 24x - 1$ has a stationary point when $x = -2$.
    (b) Prove that $f(x)$ has no other stationary points.

21. (a) $y = \frac{3}{2}x^4 + 4x^3 + 6x^2 - 30x + 3$. Show that $\left(1, -\frac{31}{2}\right)$ is a turning point on this
        curve.
    (b) Show that this curve has no other stationary points.

22. (a) Show that $x = -1$ is a solution to the equation $x^3 + px^2 + px + 1 = 0$.
    (b) Hence find the range of values of p for which all the roots of this equation
        are real.

23. (a) Show that $x = 2$ is a solution to the equation $2x^3 + kx^2 - 2kx - 16 = 0$.
    (b) Hence find the range of values of k for which all the roots of this equation
        are real.