The Circle

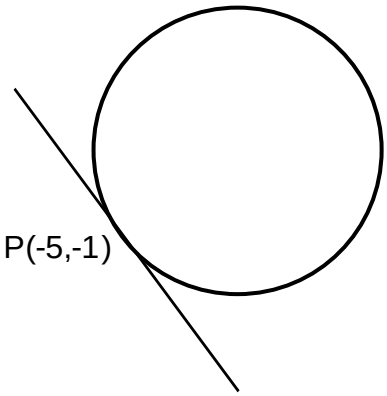
1. Find the equation of the circle centre (-4,7) which has the x-axis as a tangent.

2. Find the equation of the circle which has the lines x = -4, x = 8, y = -2 and y = 10 as tangents.

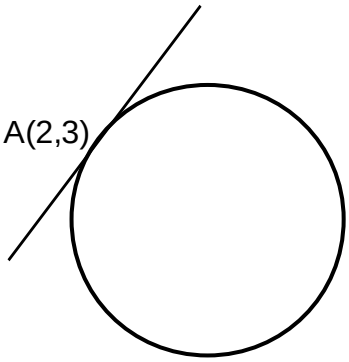
3. A circle has equation $x^2 + y^2 – 4x – 8y – 5 = 0$. Write down the equation of the tangent to this circle at the point (-3,4).

4. A circle has equation $(x + 5)^2 + (y – 1)^2 = 16$. Write down the equation of the tangent to this circle at the point A(-5,-3).

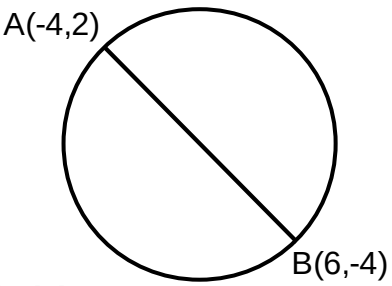
5. A circle has equation $x^2 + y^2 + 6x + 4 = 0$. Find the equation of the tangent to this circle at the point P(-5,-1).



6. Find the equation of the tangent to the circle $x^2 + y^2 – 8x + 2y – 3 = 0$ at the point A(2,3).



7. A is the point (-4,2) and B is (6,-4). Find the equation of the circle which has AB as a diameter.



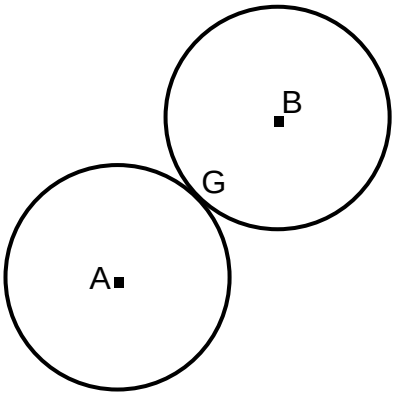
8. P is the point (-5,3) and Q is (5,-21). Find the equation of the circle which has PQ as diameter.

9. Two congruent circles with centres A and B touch at G.
The equations of the circles are

$x^2 + y^2 + 8x - 4y - 5 = 0$ and $x^2 + y^2 - 4x - 20y + 79 = 0$

(a) Find the coordinates of G.

(b) Find the length of AB.



10. Two circles have equations

$(x + 1)^2 + (y + 3)^2 = 20$ and $x^2 + y^2 - 10x - 18y + 26 = 0$

(a) Write down the centre and radius of each circle.

(b) Show that the circles touch at a single point.

(c) Find P, the point of contact of the circles.

11. Two circles have equations

$x^2 + y^2 + 4x + 16y - 60 = 0$ and $x^2 + y^2 - 8x + 4y + 12 = 0$

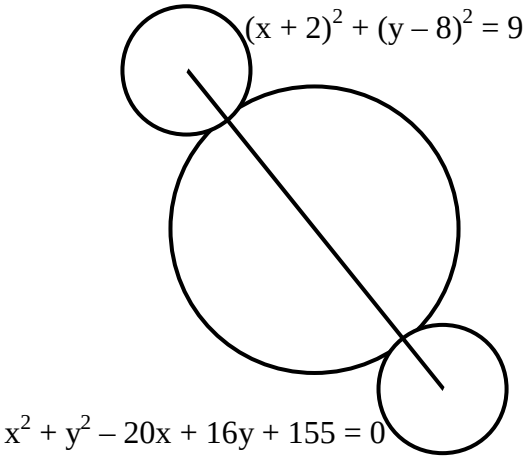
Show that these circles touch at a single point.

12. Three circles touch externally as shown.
The centres of the circles are collinear and the equations of the two smaller circles are

$(x + 2)^2 + (y - 8)^2 = 9$ and

$x^2 + y^2 - 20x + 16y + 155 = 0$

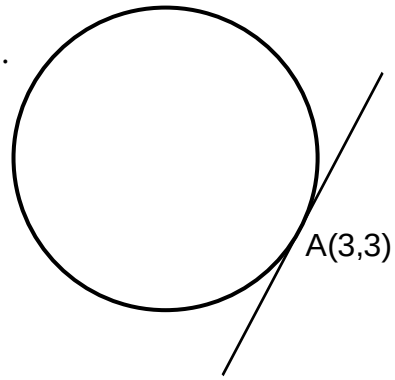
Find the equation of the larger circle.



13. The circle $x^2 + y^2 + 4x - 7y - 8 = 0$ cuts the y-axis at two points.
Find the coordinates of these points.

14. The circle $x^2 + y^2 - 2x + 10y - 24 = 0$ cuts the x-axis at the points A and B. Find the length of AB.

15. (a) A circle has equation $(x + 3)^2 + (y - 6)^2 = 61$.
Find the equation of the tangent to this circle at the point A(3,3).



(b) Show that this tangent is also a tangent to the circle with equation $x^2 + y^2 + 6x - 7y - 10 = 0$ and find the point of contact.

16. Show that the line $y = -3x - 10$ is a tangent to the circle with equation $x^2 + y^2 - 8x + 4y - 20 = 0$ and find the point of contact.

17. (a) Find the equation of the tangent to the curve $y = 2x^3 - 4x^2 - 7x + 12$ at the point where $x = 2$.

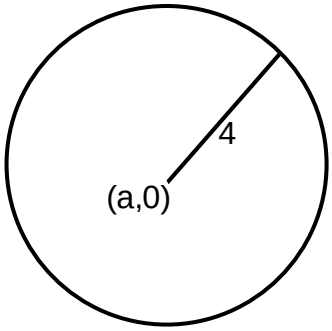
(b) Show that this tangent is also a tangent to the circle $x^2 + y^2 - 6x + 2y + 10 = 0$ and find the point of contact.

18. Show that the line $y = 2x + 1$ does not intersect the circle with equation $x^2 + y^2 - 2x + 4y + 1 = 0$.

19. For what range of values of p does the equation $x^2 + y^2 + 2px + 2py + 6p + 8 = 0$ represent a circle.

20. For what range of values of k does the equation $x^2 + y^2 - 2kx + 4ky + 4 - k = 0$ represent a circle.

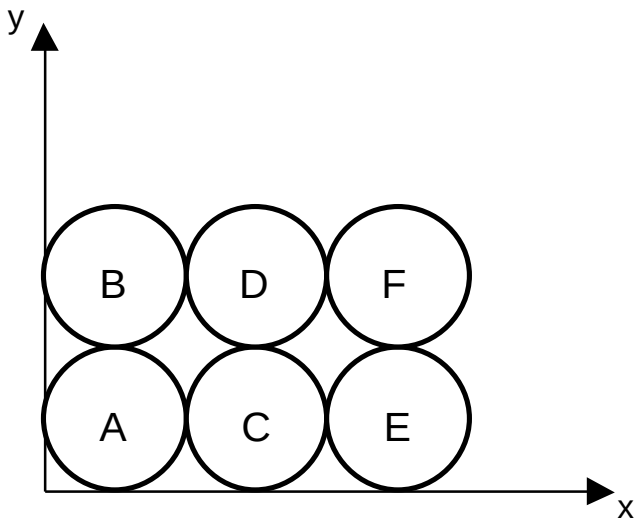
21. (a) A circle has centre (a,0), $a > 0$ and radius 4 units.
Write down the equation of this circle.



(b) Show that if $y = x$ is a tangent to this circle then $a = 4\sqrt{2}$.

22. The diagram shows six identical circles.
Circle A has equation
$x^2 + y^2 - 6x - 6y + 9 = 0$.

(a) Write down the equation of circle F.
(b) Find the point of contact between the the circles C and D.

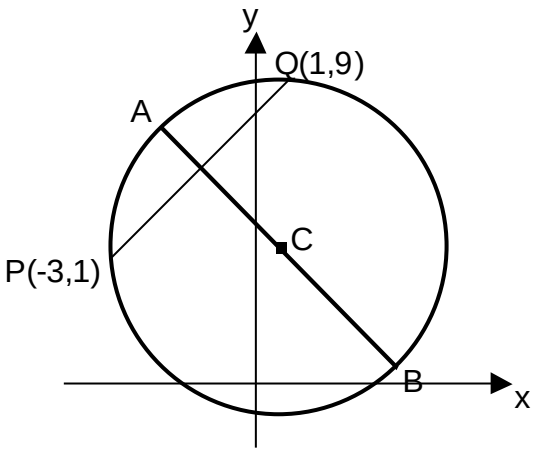


23. (a) Find the equation of AB, the perpendicular bisector of the line joining the points P(-3,1) and Q(1,9).

(b) C is the centre of a circle passing through P and Q. Given that QC is parallel to the y-axis, determine the equation of the circle.

(c) The tangents at P and Q intersect at T.

Write down
(i) the equation of the tangent at Q
(ii) the coordinates of T.



24. The diagram shows a tangent kite ABCD and a circle centre C.
A is the point (-8,0) and B is (4,9).
The radius CD is parallel to the y-axis.

(a) Find the coordinates of D and write down the equation of CD.

(b) Find the equation of the line BC.

(c) Find the coordinates of C and hence determine the equation of the circle.