

## The Discriminant

1. State the nature of the roots of each of the following
  - (a)  $x^2 + 3x - 7 = 0$
  - (b)  $3x^2 = 5x - 2$
  - (c)  $5x^2 - x = -11$
2. Show that the roots of the equation  $x^2 + px - 1 = 0$  are always real.
3. Show that the roots of the equation  $mx^2 + m = 2mx$  are equal.
4. For what value of  $k$  does the equation  $2x^2 - 7x + k + 5 = 0$  have equal roots?
5. The roots of the equation  $(x + 2)(2x - n) = -2$  are equal. Find the values of  $n$ .
6. Show that the roots of  $(p - 2)x^2 - (3p - 2)x + 2p = 0$  are always real.
7. Show that  $x(x + 12) + 9 = 2(x - 8)$  has equal roots and find the corresponding value of  $x$ .
8. Show that the roots of  $2x(x - 1) + 1 = 6x - 7$  are equal and find  $x$ .
9. Show that the line  $y = 2x - 21$  is a tangent to the parabola  $y = x^2 - 10x + 15$  and find the point of contact.
10. Show that the line  $y = 10x - 2$  is a tangent to the curve  $y = 2x(x + 3)$  and find the point of contact.
11. Is the line  $y = 4x - 1$  a tangent to the curve  $y = 3x^2 - 11x + 5$ ?
12. Show that there are two values of  $m$  for which  $(x - m)^2 = 3x(2 - x)$  has equal roots.
13. The roots of the equation  $\frac{p-1}{x} + \frac{x}{4} = 1$  are equal. Find  $p$ .
14. (a) If  $\frac{(x-2)^2}{x^2+2} = k$ , show that  $(k-1)x^2 + 4x + (2k-4) = 0$ .  
  
(b) Find the range of values of  $k$  such that the equation  $\frac{(x-2)^2}{x^2+2} = k$  has real roots.