The Discriminant

1. Show that $x = 2$ is the only real solution to the equation $x^3 - x^2 + x - 6 = 0$.

2. Show that $x = -4$ is the only real solution to the equation $x^3 + 2x^2 - 3x + 20 = 0$.

3. (a) Show that the line $y = 2x - 2$ and the curve $y = 2x^3 + 5x^2 + 12x + 5$ intersect at the point $(-1,-4)$.

   (b) Show that there are no other points of intersection between the line and the curve.

4. (a) Show that the line $y = 3x - 2$ and the curve $y = x^3 + 5x + 10$ intersect at the point $(-2,-8)$.

   (b) Prove that there are no other points of intersection between the curve and the line.

5. Show that the curve with equation $f(x) = x^3 + x^2 + 5x - 4$ has no stationary points.

6. (a) Show that the curve $y = \frac{1}{4}x^4 + 2x^2 - 16x + 5$ has a stationary point when $x = 2$.

   (b) Prove that the curve has no other stationary points.

7. (a) $f(x) = x^4 + 8x^3 + 24x^2 + 28x + 3$. Show that $f(x)$ has a stationary point when $x = -1$.

   (b) Show that the curve has no other stationary points.

8. The equation $2x^2 + 4px + p^2 - 2p - 1 = 0$ has equal roots.
   Find the value of p.

9. (a) Prove that the roots of $mx^2 - (2m + 4)x + 8 = 0$ are always real.

   (b) If the roots of $mx^2 - (2m + 4)x + 8 = 0$ are in fact equal, write down the value of m.