Further Calculus

Differentiation

1. Differentiate

(a) $y = (4x - 2)^3$ (b) $y = \sqrt{6x - 2}$ (c) $f(x) = \frac{2}{5x + 2}$ (d) $y = 3\sin 2x$

(e) $f(x) = \cos^2 x + 2\sin 4x$

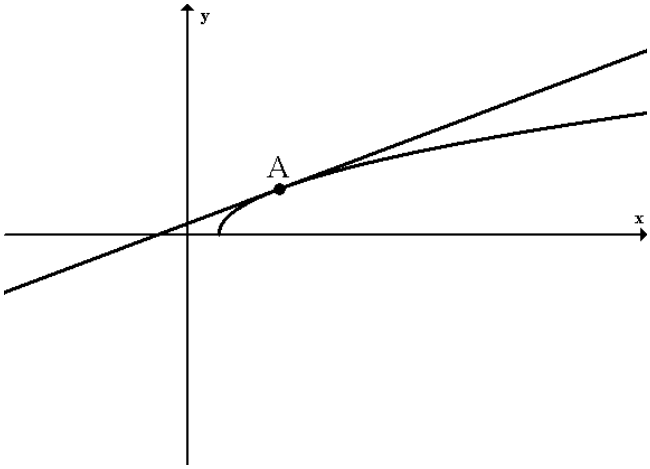
2. $f(x) = 2\cos 2x - \sin 4x$. Show that $f'(\frac{\pi}{6}) = 2(1 - \sqrt{3})$

3. $f(x) = (2 + \cos x)^3$. Show that $f'(\frac{\pi}{2}) = -12$

4. $y = \sqrt{27 + x^2}$. Find the value of $\frac{dy}{dx}$ when x = 3.

5. $f(x) = (9x - 1)^{\frac{1}{3}}$. Find $f'(1)$.

6. Find the equation of the tangent to the curve $y = \sqrt{6x - 2}$ at the point A, where x = 1.

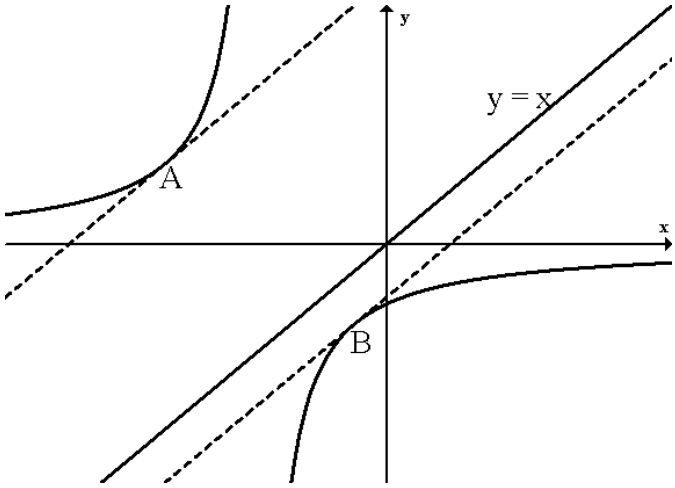


7. Find the equation of the tangent to the curve $y = \frac{4}{3x + 4}$ at the point where x = - 2.

8. Find the points, A and B, on the curve

$$y = \frac{-9}{x + 4}$$

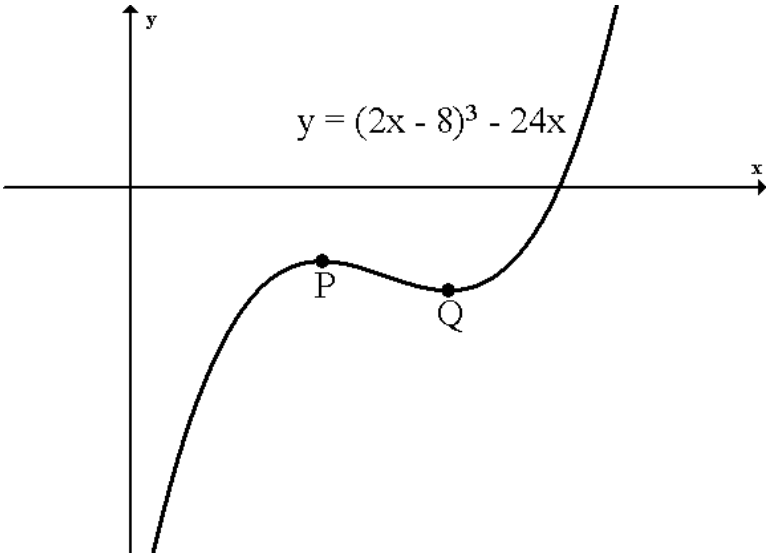
where the tangents to the curve are parallel to the line y = x.



9. Find the equation of the tangent to the curve $y = 2\sin\left(x - \frac{\pi}{6}\right)$ at the point where $x = \frac{\pi}{3}$.

10. Find the equation of the tangent to the curve $y = 4\cos\left(2x - \frac{\pi}{6}\right)$ at the point where $x = \frac{\pi}{2}$

11. The curve $y = (2x - 8)^3 - 24x$ has stationary points at P and Q. Find the coordinates of P and Q.



12. (a) Show that the curve $y = 2\sin 2x - 4x$ is never increasing.

(b) Determine the coordinates of the stationary points of this curve in the interval $0 \le x \le 180$.

Integration

13. Integrate

(a) $\int (2x - 4)^3\, dx$ (b) $\int \sqrt{6x + 1}\, dx$ (c) $\int \frac{6}{(1 - 3x)^2}\, dx$ (d) $\int \sin(4x - 2)\, dx$

(e) $\int 4\cos(2x + 1)\, dx$

14. Evaluate (a) $\int_0^2 \sqrt{4x + 1}\, dx$ (b) $\int_1^2 \frac{8}{(1 - 2x)^3}\, dx$

15. $\frac{dy}{dx} = 10(2x - 1)^4$ and the curve passes through the point (1,6). Find a formula for y.

16. $\frac{dy}{dx} = \frac{1}{\sqrt{2x - 4}}$ and the curve passes through the point (10,3). Find a formula for y.

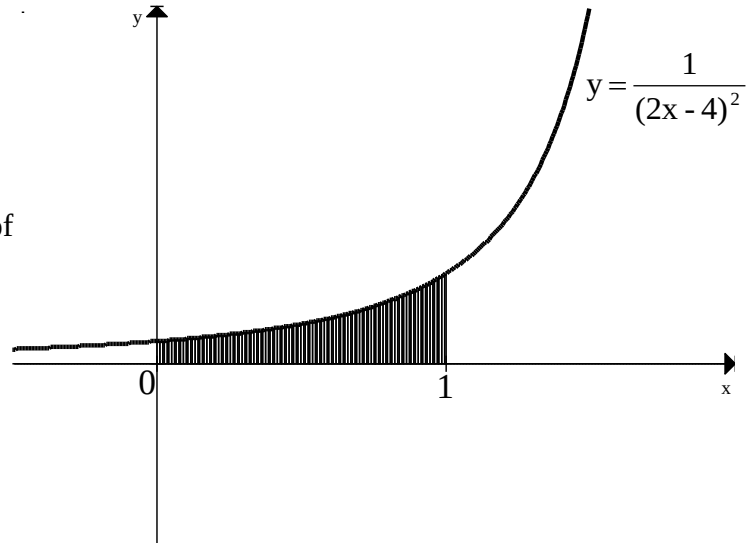
17. $\frac{dy}{dx}$ = 3sin 2x. This curve passes through the point $\left(\frac{5}{12}\pi, \sqrt{3}\right)$. Find a formula for y.

18. $\frac{dy}{dx}$ = 10cos 5x. This curve passes through the point $\left(\frac{\pi}{6},4\right)$. Find y.

19. The diagram shows part of the graph of

$$y = \frac{1}{(2x - 4)^2}$$

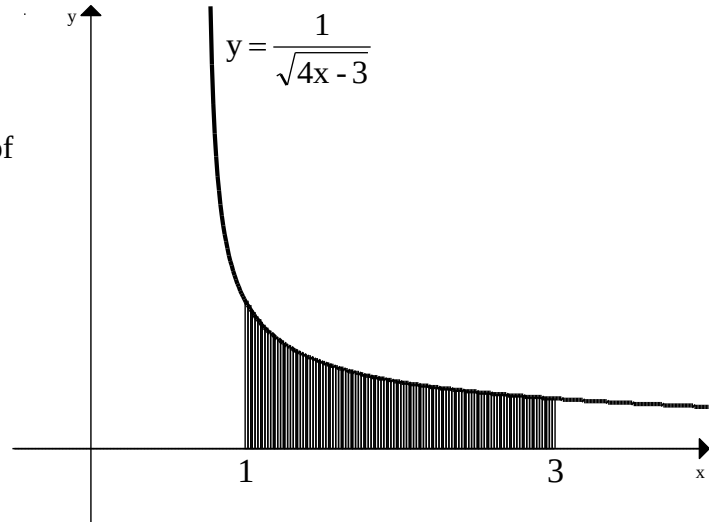
Calculate the shaded area.



20. The diagram shows part of the graph of

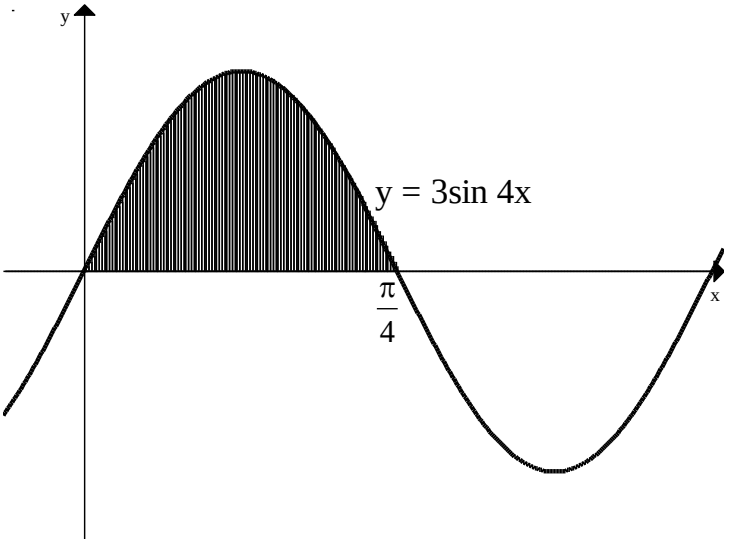
$$y = \frac{1}{\sqrt{4x - 3}}$$

Calculate the shaded area.

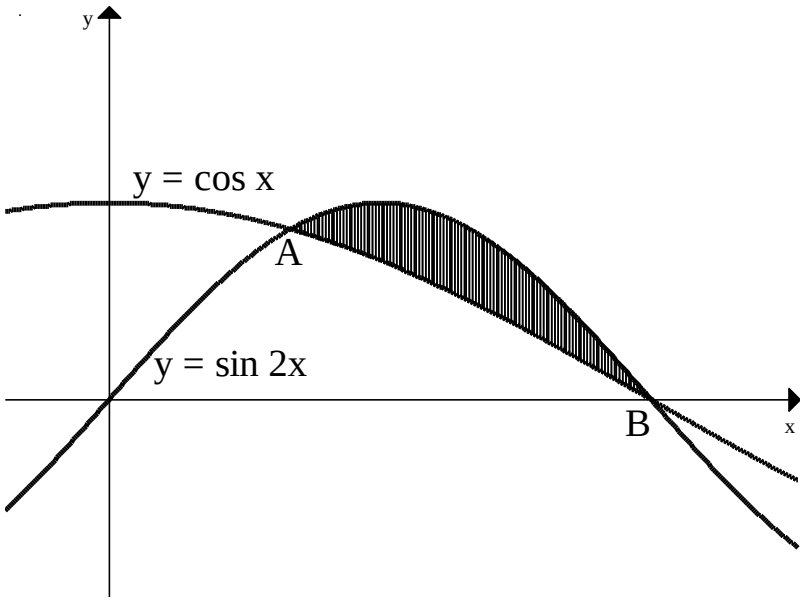


21. The diagram shows part of the graph of y = 3sin 4x.

Calculate the shaded area.



22. The diagram below shows part of the graphs of $y = \sin 2x$ and $y = \cos x$.



(a) Find the x-coordinates of A and B.

(b) Calculate the shaded area.