# Logarithms

1. Simplify

(a) $\log_4 2 + \log_4 8$ (b) $\log_3 108 - \log_3 4$ (c) $\log_5 50 + \log_5 2 - \log_5 4$

(d) $\log_9 3 - \log_9 6 + \log_9 18$ (e) $\log_4 10 + 3\log_4 2 - \frac{1}{2}\log_4 25$ (f) $\frac{3}{4}\log_{10} 16 - \frac{2}{3}\log_{10} 8 + \log_{10} 5$

2. Solve for $x > 0$

(a) $\log_a 5 + \log_a 2x = \log_a 60$ (b) $2\log_a 3 + \log_a x = \log_a 36$

(c) $3\log_4 2 + \log_4 x = 1$ (d) $\frac{1}{2}\log_3 16 + 2\log_3 x = 2$

(e) $\log_a x + \log_a (x + 1) = \log_a 2$ (f) $\log_a x + \log_a (x + 4) = \log_a 12$

(g) $\log_2 x + \log_2 (x - 3) = 2$ (h) $\log_5 (x + 3) + \log_5 (x - 1) = 1$

(i) $\log_3 6x - \log_3 (x - 2) = 2$ (j) $2\log_2 x - \log_2 (x - 1) = 2$

3. Find x in each of the following ( $x > 0$)

(a) $4\log_x 6 - 2\log_x 4 = 1$ (b) $2\log_x 3 + \log_x 4 = 2$

(c) $\frac{1}{2}\log_x 64 + 2\log_x 2 = 5$ (d) $2\log_x 6 - \frac{2}{3}\log_x 8 = 2$

(e) $3\log_x 4 - 2\log_x 2 = 2$ (f) $\frac{3}{4}\log_x 81 - \frac{1}{2}\log_x 64 = 3$

4. Given $\frac{1}{2}\log_a y = \log_a (x - 1) + 2\log_a 3$, show that $y = 81(x - 1)^2$.

5. Given $2\log_m n = \log_m 16 + 1$, show that $n = 4\sqrt{m}$

6. Given $3\log_a y = 2\log_a (x - 1) + 6\log_a 2$, show that $y = 4\sqrt[3]{(x-1)^2}$

7. Find where the following curves cut the x-axis.

(a) $y = \log_4 x - 2$ (b) $y = \log_2 (x - 4) - 1$

(c) $y = \log_3 (x + 1) - 2$ (d) $y = \log_2 (x - 2) - 4$

8. Find where the following curves cut the y-axis.

(a) $y = \log_2 (x + 4) + 1$ (b) $y = \log_3 (x + 27) + 5$

(c) $y = -\log_4 (x + 16) - 2$

9. The number of bacteria present in a beaker, during an experiment can be measured using the formula $N(t) = 30e^{1.25t}$ where t is the number of hours passed.

(a) How many bacteria are in the beaker at the start of the experiment?

(b) Calculate the number of bacteria present after 5 hours.

(c) How long will it take for the number of bacteria present to treble?

10. The mass, M grams, of a radioactive isotope after a time of t years, is given by the formula $M = M_o e^{-kt}$ where $M_o$ is the initial mass of the isotope.

In 5 years a mass of 10 grams of the isotope is reduced to 8 grams.

(a) Calculate k.

(b) Calculate the half-life of the substance i.e. the time taken for half the substance to decay.

11. A cup of coffee cools according to the law $P_t = P_o e^{-kt}$, where $P_o$ is the initial temperature of the coffee and $P_t$ is the temperature after t minutes.

(a) A cup of coffee cools from $80^0$ C to $60^0$ C in a time of 15 minutes. Calculate k.

(b) By how many degrees will the cup of coffee cool in the next 15 minutes?

12. A fire spreads according to the law $A = A_o e^{kt}$ where $A_o$ is the area covered by the fire when it is first measured and A is the area covered after t hours.

(a) If it takes $1\frac{1}{2}$ hours for the fire to double in area, find k.

(b) A bush fire covers an area of 800 $km^2$. If not tackled, calculate the area the fire will cover 4 hours later.

13. The value, V (£million), of a container ship is given by the formula $V = 120e^{-0.065t}$ where t is the number of years after the ship is launched.

(a) Calculate the value of the ship when it was launched.

(b) Calculate the percentage reduction in value of the ship after 6 years.

14. A cell culture grows at a rate given by the formula $y(t) = Ae^{kt}$ where A is the initial number of cells and y(t) is the number of cells after t hours.

(a) It takes 24 hours for 500 cells to increase in number to 800. Find k.

(b) Calculate the time taken for the number of cells to double.

15. Dangerous blue algae are spreading over the surface of a lake according to the formula $A_t = A_o e^{kt}$ where $A_o$ is the initial area covered by the algae and $A_t$ is the area covered after t days.
When first noticed the algae covered an area of 200 square metres. One week later the algae covered an area of 320 square metres.

(a) Calculate the value of k.

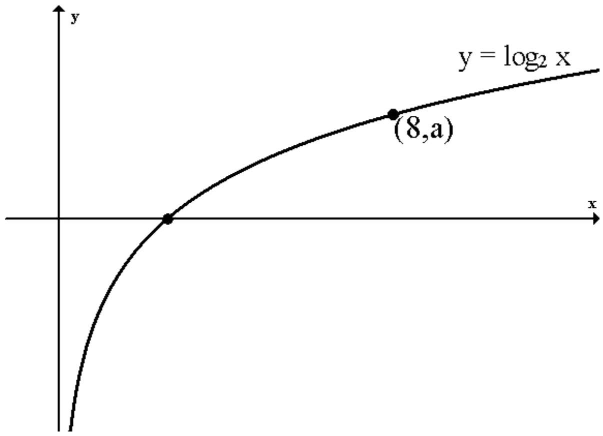
(b) Calculate, to the nearest day, how long it will take for the area of algae to increase by a further 500 square metres.

16. The diagram opposite shows the graph of $y = \log_2 x$.

(a) Find the value of a.

(b) Sketch the graph of $y = \log_2 x - 3$

(c) Sketch the graph of $y = \log_2 4x$

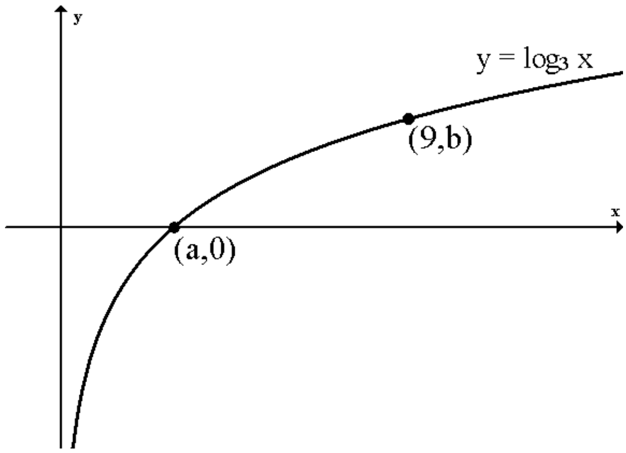



17. The diagram opposite shows the graph of $y = \log_3 x$.

(a) Find a and b.

(b) Sketch the graph of $y = \log_3 27x$

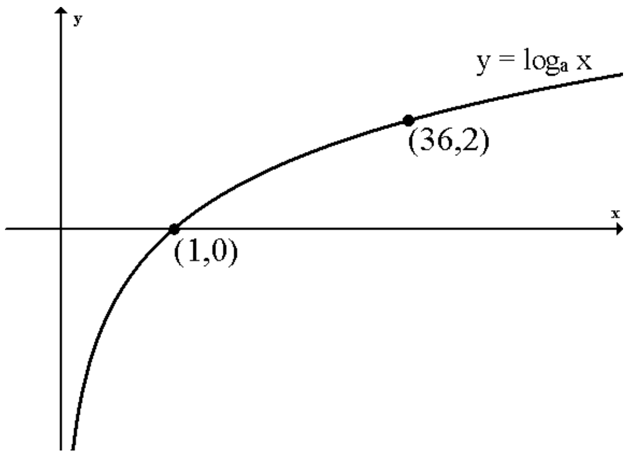
(c) Sketch the graph of $y = \log_3 \dfrac{1}{x}$




18. The diagram shows the graph of $y = \log_a x$

(a) Determine the value of a.

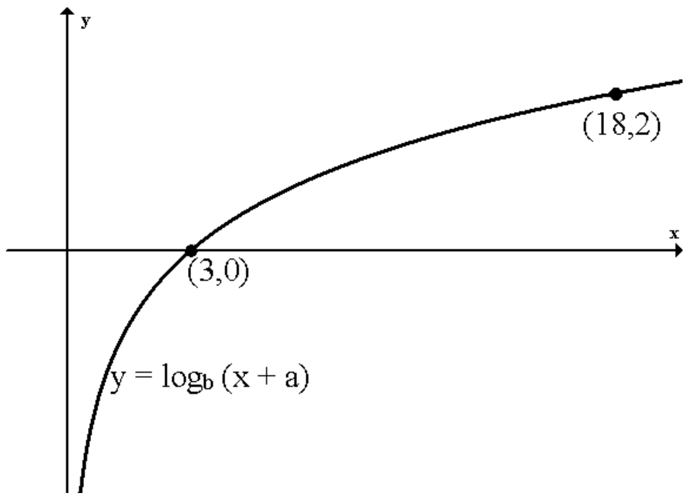
(b) Draw the graph of $y = \log_a 6x^2$ when a takes this value.



19. The diagram opposite shows the graph of $y = \log_b (x + a)$

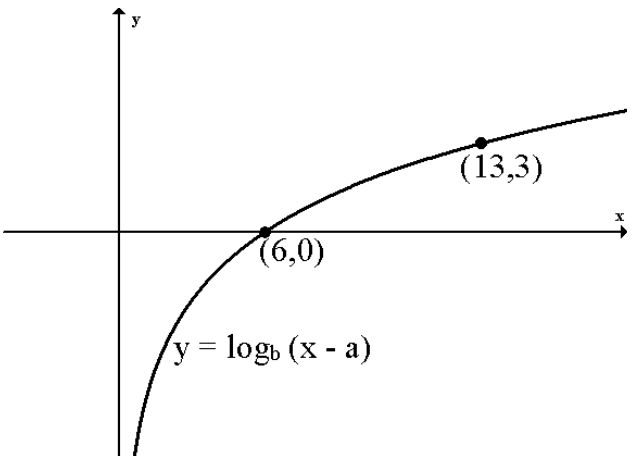
(a) Find the value of a.

(b) Find the value of b.



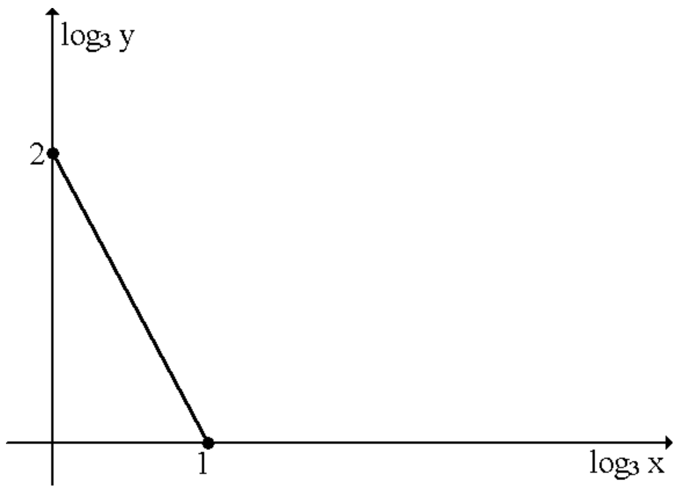

20. The diagram shows the graph of $y = \log_b (x – a)$.

Find a and b.



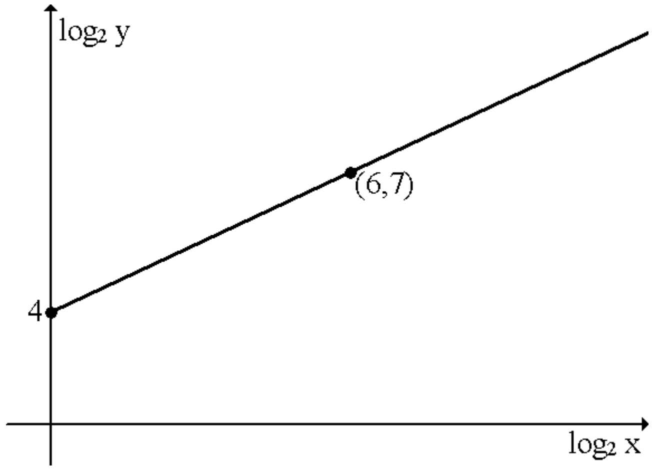

21. The graph opposite illustrates the law $y = kx^n$.

Find the values of k and n.



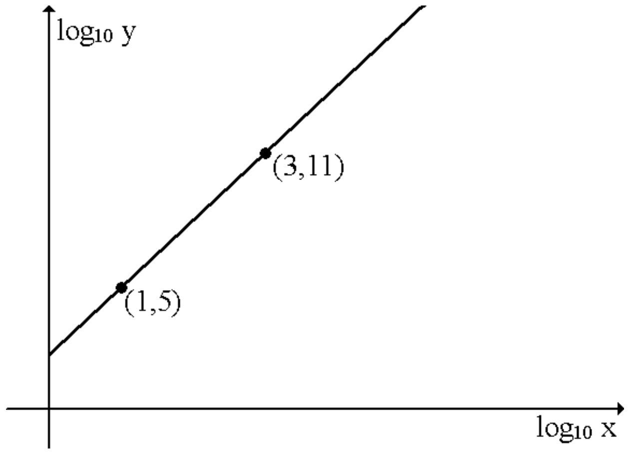
22. The graph opposite illustrates the law $y = kx^n$.

Find the values of k and n.




23. The graph opposite illustrates the law $y = kx^n$.

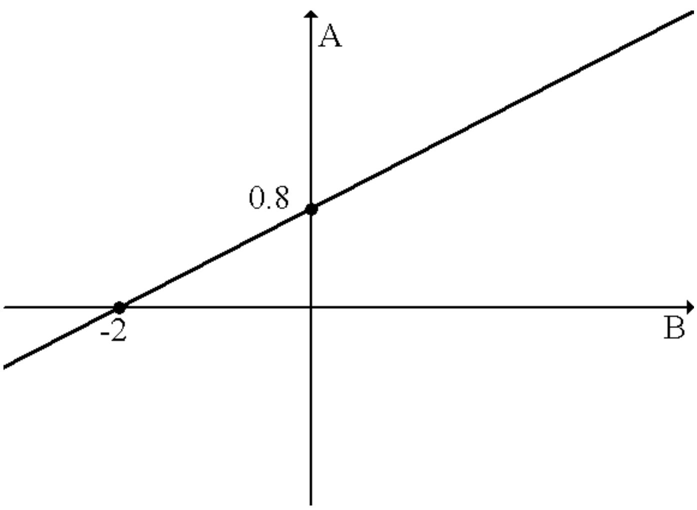
Find the values of k and n.




24. The graph opposite shows the results of an experiment.

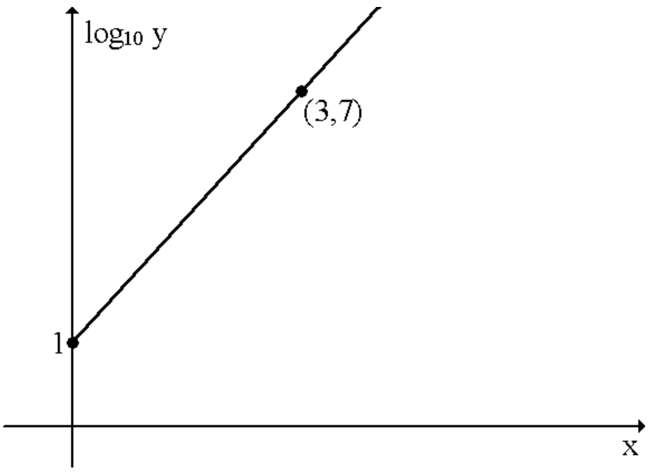
(a) Write down the equation of the line in terms of A and B.

(b) Given $A = \log_e a$ and $B = \log_e b$, show that a and b satisfy a relationship of the form $a = kb^n$ and state the values of k and n.



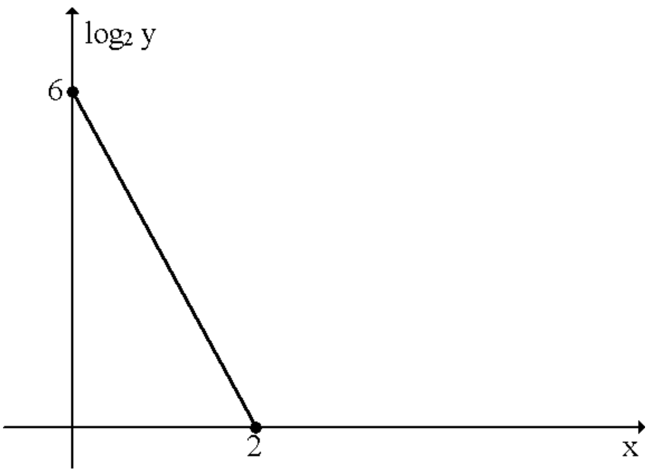
25. The graph opposite illustrates the law $y = ab^x$.

Find the values of a and b.



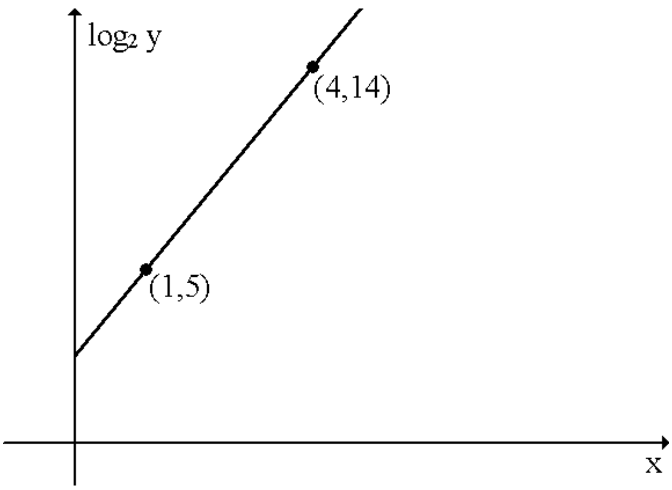

26. The graph opposite illustrates the law $y = ab^x$.

Find the values of a and b.




27. The graph opposite illustrates the law $y = ab^x$.

Find the values of a and b.




28. Given $\log \frac{1}{2}(a + b) = \frac{1}{2}(\log a + \log b)$, show that $(a+ b)^2 = 4ab$.

29. $\log_x a + \log_x b = u$ and $\log_x a - \log_x b = w$. Show that $a = x^{\frac{1}{2}(u + w)}$

30. If $\log_a p = \cos^2 x$ and $\log_a r = \sin^2 x$, show that $pr = a$.