Scalar product

1. A is the point (4,3,5), B is (1,0,-4) and C is (2,2,-5).
   Show that angle ABC = 90^0.

2. P, Q and R are the points (3,1,2), (9,2,4) and (1,5,6) respectively.
   Show that the triangle PQR is right-angled at P.

3. $\mathbf{u} = 3\mathbf{i} - 2\mathbf{j} - 4\mathbf{k}$ and $\mathbf{v} = 4\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$.

   Show that the vectors **u** and **v** are perpendicular.

4. $\mathbf{u} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix}$.

   (a) Find the vectors $2\mathbf{u} + \mathbf{v}$ and $\mathbf{u} - \mathbf{v}$
   (b) Show that the vectors $2\mathbf{u} + \mathbf{v}$ and $\mathbf{u} - \mathbf{v}$ are perpendicular.

5. (a) A is the point (2,1,-1) and B is (5,7,11). C divides AB in the ratio 2:1.
   Find the coordinates of C.

   (b) D is the point (6,6,6). Show that the vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are perpendicular.

6. (a) P is the point (1,2,-4) and Q is (6,-3,6). PR:RQ is 3:2.
   Find the coordinates of R.

   (b) T is the point (6,-3.0).
   Show that PQ and RT are perpendicular.

7. A is the point (-2,2,0) and B is (13,-8,20).

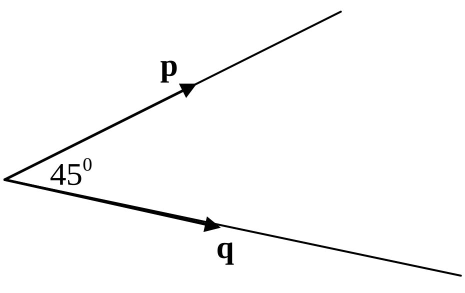
   (a) C divides AB in the ratio 2:3. Find the coordinates of C.

   (b) D has coordinates (1,k,9).
   Given DC is perpendicular to AB, find k.

8. (a) A and B are the points (1,2,-1) and B(2,0,-4).
   Given AC = 3AB, find the coordinates of C.

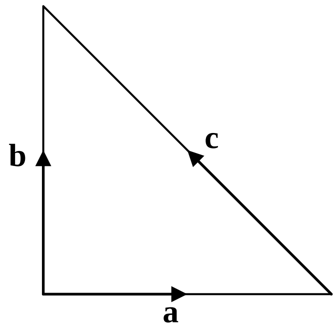
   (b) D is the point (10,-4,-8).
   Show that AB and CD are perpendicular.

9. For vectors **p** and **q** calculate **q.**(**q** + **p**) when $|\mathbf{p}| = 3$ and $|\mathbf{q}| = 4$.



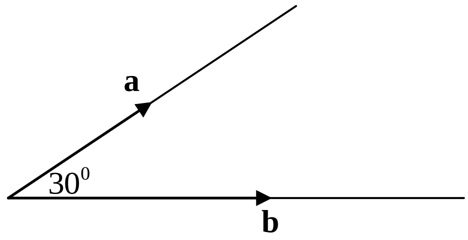
10. In the right-angled isosceles triangle opposite $|\mathbf{a}| = 1$.

Calculate $\mathbf{a}.(\mathbf{a} + \mathbf{b} + \mathbf{c})$.

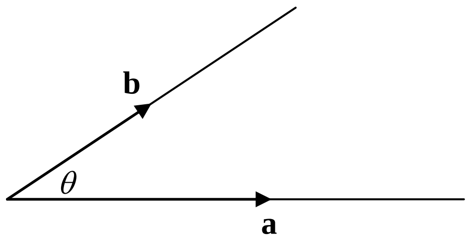


11. In the diagram $|\mathbf{a}| = 2\sqrt{3}$ and $|\mathbf{b}| = 5$.

Calculate (i) $\mathbf{a}.\mathbf{a}$ (ii) $\mathbf{b}.\mathbf{b}$ (iii) $(\mathbf{a} + 2\mathbf{b}).(\mathbf{a} + \mathbf{b})$

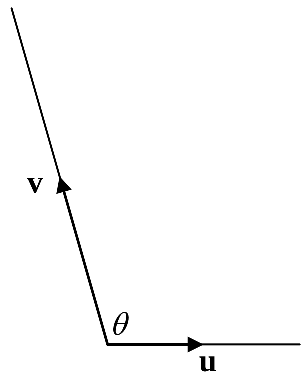


12. The diagram shows the vectors **a** and **b**.
If $|\mathbf{a}| = 5$ and $|\mathbf{b}| = 4$ and $\mathbf{a}.(\mathbf{a} + \mathbf{b}) = 36$,
Find the size of angle θ.



13. Two vectors **u** and **v** are such that $|\mathbf{u}| = 2$ and $|\mathbf{v}| = 6$.

Given that $2\mathbf{u}.(\mathbf{u} + \mathbf{v}) = -4$, show that angle $\theta = 120^0$.



14. **u** and **v** are vectors given by $\mathbf{u} = \begin{pmatrix} k^3 \\ 1 \\ k+2 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 1 \\ 3k^2 \\ -1 \end{pmatrix}$, where $k > 0$.

(a) If $\mathbf{u}.\mathbf{v} = 1$, show that $k^3 + 3k^2 - k - 3 = 0$

(b) Show that $(k + 3)$ is a factor of $k^3 + 3k^2 - k - 3$ and hence factorise $k^3 + 3k^2 - k - 3$ fully.

(c) Deduce the only possible value of k.

(d) The angle between **u** and **v** is θ. Find the exact value of $\cos\theta$.