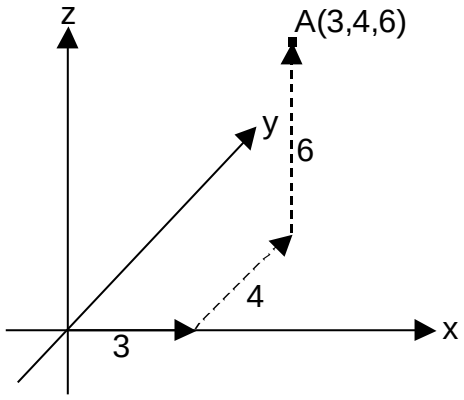
Three Dimensional Vectors

Vectors in 3-dimensions have an x, y and z coordinate.

x-coordinate: along **y-coordinate:** in (or out) **z-coordinate:** up (or down)

For example the point A(3,4,6) will be 3 along, 4 in and 6 up.



Note: All rules that are true for 2-dimensional coordinates are also true for 3-dimensional coordinates.

Examples:

1. If $\mathbf{a} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -4 \\ 0 \\ 2 \end{pmatrix}$ then

(i) $4\mathbf{a} = \begin{pmatrix} 12 \\ -4 \\ 8 \end{pmatrix}$ (ii) $\frac{1}{2}\mathbf{b} = \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}$ (iii) $2\mathbf{a} - 3\mathbf{b} = \begin{pmatrix} 18 \\ -2 \\ -2 \end{pmatrix}$ (iv) $|2a - 3b|^2 = 18^2 + -2^2 + -2^2$

$= 342$

$|2a - 3b| = \sqrt{342}$

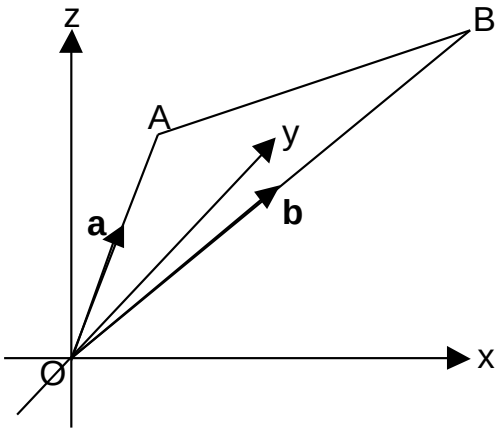
$= \sqrt{9 \times 38} = 3\sqrt{38}$

2. $\begin{pmatrix} p \\ q \\ r \end{pmatrix} + \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ -4 \\ 3 \end{pmatrix}$ gives $\begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} -2 \\ -5 \\ 1 \end{pmatrix}$

Position Vectors

The position vector of a point is the vector from an origin to that point.
i.e. If A has coordinates (3,1,4) and B is (5,3,9)

the position vector of A is $\mathbf{a} = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}$ and of B is $\mathbf{b} = \begin{pmatrix} 5 \\ 3 \\ 9 \end{pmatrix}$



Note: The vectors $\overrightarrow{AB}$ and $\overrightarrow{BA}$ can be found as

$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$$
$$= \mathbf{-a} + \mathbf{b}$$
$$= \mathbf{b - a}$$

$$= \begin{pmatrix} 5 \\ 3 \\ 9 \end{pmatrix} - \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ 2 \\ 5 \end{pmatrix}$$

$$\overrightarrow{BA} = \overrightarrow{BO} + \overrightarrow{OA}$$
$$= \mathbf{-b} + \mathbf{a}$$
$$= \mathbf{a - b}$$

$$= \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} - \begin{pmatrix} 5 \\ 3 \\ 9 \end{pmatrix}$$

$$= \begin{pmatrix} -2 \\ -2 \\ -5 \end{pmatrix}$$

This is the most important rule that you will use in vectors.

$\overrightarrow{AB} = \mathbf{b - a}$	$\overrightarrow{CD} = \mathbf{d - c}$	$\overrightarrow{PQ} = \mathbf{q - p}$	$\overrightarrow{UW} = \mathbf{w - u}$