Revision Pack

HIGHER MATHEMATICS

BLOCK 1

- Straight Line
- Vectors
- Quadratics & Polynomials
- Basic Differentiation

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}|\,|\boldsymbol{b}| \cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$

or $\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2\cos^2 A - 1$$

$$= 1 - 2\sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

Straight Line

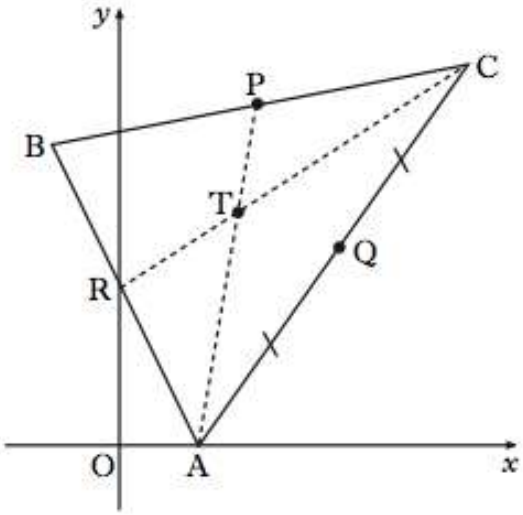
1 A straight line has the equation $2x + y - 7 = 0$.

Write down the equation of the line parallel to the given line, which passes through the point (-3, 5).

2

Triangle ABC has vertices A(4, 0), B(–4, 16) and C(18, 20), as shown in the diagram opposite.

Medians AP and CR intersect at the point T(6, 12).



(*a*) Find the equation of median BQ.

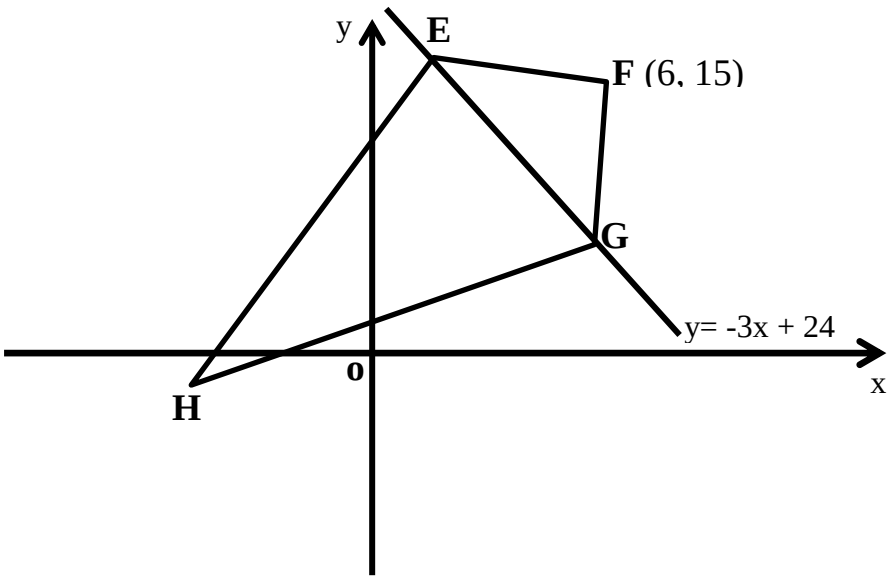
(*b*) Verify that T lies on BQ.

(*c*) Find the ratio in which T divides BQ.

3. EFGH is a kite.

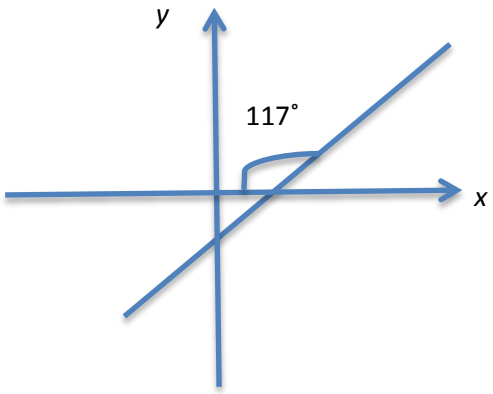
Diagonal EG has equation $y = -3x + 24$ and point **F** has coordinates (6, 15). Note that the diagram is not to scale.

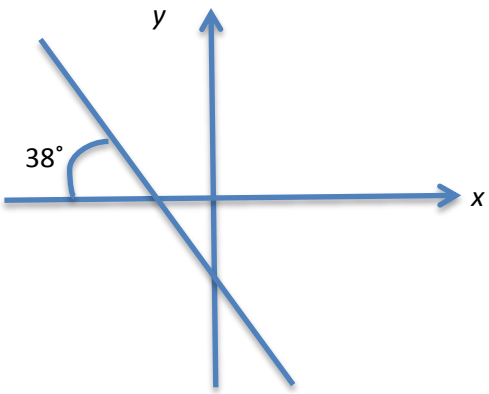
Find the equation of the diagonal HF.



4 Calculate the size of the obtuse angle between the line $y = \frac{1}{2}x - 6$ and the x-axis.

5. Find the gradient of the two lines shown in the diagrams below

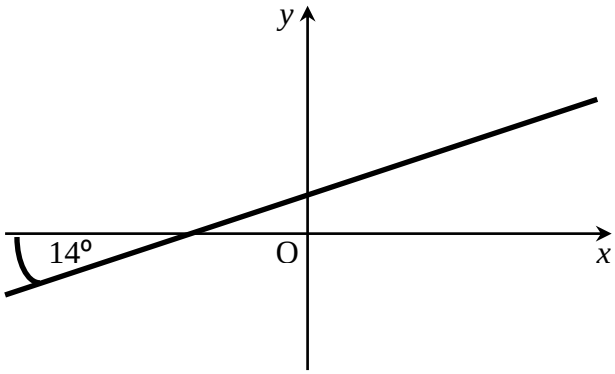




5. A ramp is categorised by its gradient as shown in the table.

Ramp category	**Gradient (m) of ramp**
Wheelchair accessible	$0 < m \leq 0 \cdot 25$
Not wheelchair accessible	$m > 0 \cdot 25$

Which category does the ramp in the diagram below belong to?
Explain your answer fully.



Vectors

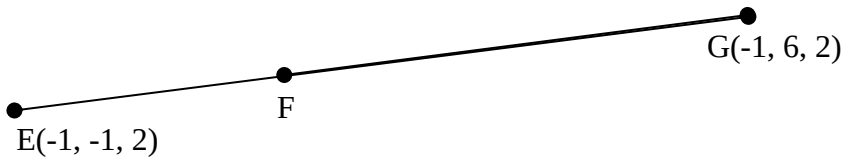
1 An engineer laying flags needs to check that:

- they are in a straight line;
- the distance between Flag 2 and Flag 3 is three times the distance between Flag 1 and Flag 2.

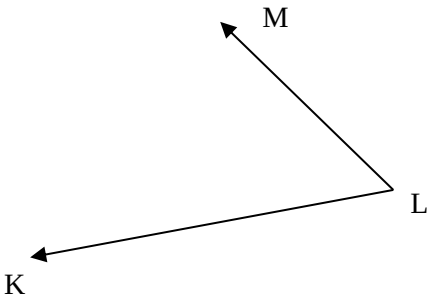
Relative to suitable axes, the top-left corner of each flag can be represented by the points A (4, 1, -2), B (5, -2, 3), and C (8,–11, 18) respectively. All three flags point vertically upwards.

Has the engineer laid the flags correctly? You must justify your answer.

2 The points E, F and G lie in a straight line, as shown. F divides EG in the ratio 2:5. Find the coordinates of F.

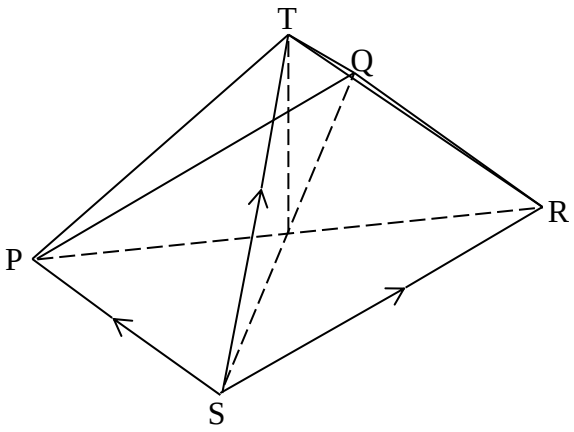


3 Points K, L and M have coordinates K(6, –1, 0), L(4, –3, -2) and M(5, –1, 8).



Find the size of the acute angle KLM.

4 TPQRS is a pyramid with rectangular base PQRS.



The vectors $\overrightarrow{SP}, \overrightarrow{SR}, \overrightarrow{ST}$ are given by

$$\overrightarrow{SP} = -3\mathbf{i} + 16\mathbf{j} - 2\mathbf{k}$$

$$\overrightarrow{SR} = 14\mathbf{i} + 5\mathbf{j} + 7\mathbf{k}$$

$$\overrightarrow{ST} = \mathbf{i} + 14\mathbf{j} + 12\mathbf{k}$$

Express $\overrightarrow{RT}$ in component form.

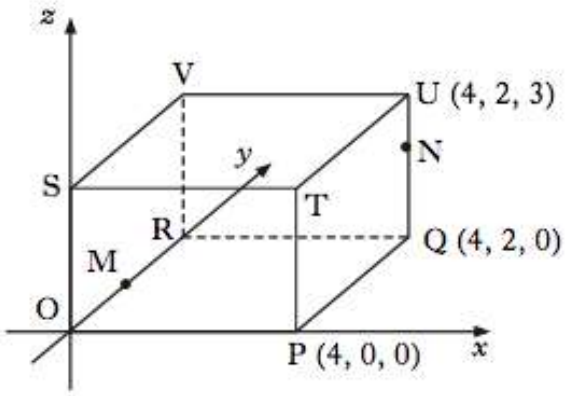
5.

The diagram shows a cuboid OPQR,STUV relative to the coordinate axes.

P is the point (4, 0, 0),
Q is (4, 2, 0) and U is (4, 2, 3).

M is the midpoint of OR.

N is the point on UQ such that $UN = \frac{1}{3}UQ$.

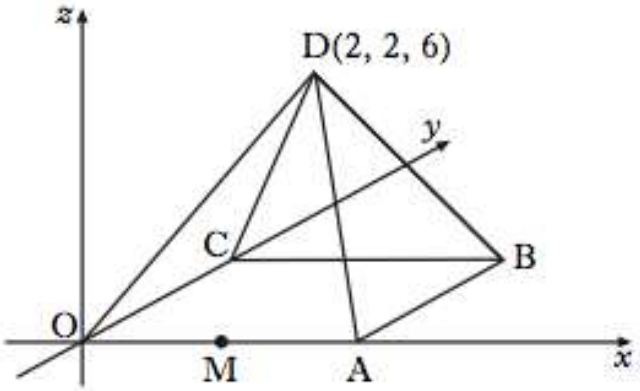


(*a*) State the coordinates of M and N.

(*b*) Express $\overrightarrow{VM}$ and $\overrightarrow{VN}$ in component form.

(*c*) Calculate the size of angle MVN.

6. D,OABC is a square based pyramid as shown in the diagram below.



O is the origin, D is the point (2, 2, 6) and OA = 4 units.

M is the mid-point of OA.

(*a*) State the coordinates of B.

(*b*) Express $\overrightarrow{DB}$ and $\overrightarrow{DM}$ in component form.

(*c*) Find the size of angle BDM.

Polynomials and Quadratics

1. (a) (i) Show that (x – 1) is a factor of $f(x) = 2x^3 + x^2 - 8x + 5$.
(ii) Hence factorise f(x) fully.

(b) Solve $2x^3 + x^2 - 8x + 5 = 0$. [2010]

2. (*a*) (i) Show that $(x-4)$ is a factor of $x^3 - 5x^2 + 2x + 8$.

(ii) Factorise $x^3 - 5x^2 + 2x + 8$ fully.

(iii) Solve $x^3 - 5x^2 + 2x + 8 = 0$.

3. Show that $(x + 1)$ is a factor of $f(x) = 2x^3 - 5x^2 - x + 6$

4. A function *f* is defined by the formula $f(x) = x^3 - 7x^2 + 7x + 15$ where *x* is a real number.
(a) Show that (x + 1) is a factor of $f(x)$.
(b) Hence factorise $f(x)$ fully.
(c) Solve $f(x) = 0$

5. Solve the cubic equation $f(x) = 0$ given the following:
- when $f(x)$ is divided by $x + 6$, the remainder is zero
- when the graph of $y = f(x)$ is drawn, it passes through the point $(1, 0)$
- $(x - 4)$ is a factor of $f(x)$.

6. The graph of the function $f(x) = 2x^2 + 4x - k$ does not touch or cross the *x*-axis.
What are the possible values for *k*?

7. Find the range of values of *k* such that the equations $kx^2 - x - 1 = 0$ has no real roots.

8. (a) Find the stationary points on the curve with equation $y = x^3 + 3x^2 - 9x + 5$ and justify their nature.

(b) The curve passes through the point (–5, 0). Sketch the curve.

9. Prove that the roots of the equation $2x^2 + px - 3 = 0$ are real for all values of *p*.

10. For what values of *x* is $6 + x - x^2 < 0$?

11. Express $1 - 6x - x^2$ in the form $a - (x + b)^2$

12. Find the values of *a* and *b* if $(x - 2)$ and $(x + 4)$ are both factors of $x^4 + ax^3 - x^2 + bx - 8$

Differentiation

1. If $y = 3x^{-2} + 2x^{\frac{3}{2}}$, $x > 0$, determine $\frac{dy}{dx}$.

2. Find $f'(x)$, given that $f(x) = 5\sqrt{x} + \frac{3}{x^4}$, $x > 0$.

3. A rocket travels vertically upwards. The height (in metres) of the rocket *t* seconds after take-off can be represented by the formula $h = 60t - 2t^2$. The velocity of the rocket at time *t* is given by $v = \frac{dh}{dt}$.
   a) Find the velocity of the rocket after fifteen seconds and explain your answer in terms of the rockets movement.
   b) Find the velocity of the rocket after twenty seconds and explain your answer in terms of the rocket's movement.

4. Differentiate the function $f(x) = 8\sin x$ with respect to *x*.

5. If $y = \sin^3 x$, find $\frac{dy}{dx}$

6. A curve has equation $y = 3x^2 - 5x - 7$. Find the equation of the tangent to the curve at the point where $x = 2$.

7. If $y = \frac{1}{x^3} - \cos x$, $x \neq 0$, find $\frac{dy}{dx}$.

8. Given that $y = \sqrt{3x^2 + 2}$, find $\frac{dy}{dx}$.

9. What is the derivative of $(8 - 2x^2)^{\frac{2}{3}}$?

10. Given that $y = 3cos5x$, find $\frac{dy}{dx}$.

11. A curve has equation $y = 5x^3 - 12x$.
    What is the gradient of the tangent at the point (1, –7)?

12. What is the derivative of $\frac{1}{4x^3}$, $x \neq 0$?

13. If $f(x) = \frac{x^2+1}{\sqrt{x}}$, evaluate $f'(4)$

14. Given that $f(t) = (1 - 3t)^4$, evaluate $f'(\frac{1}{6})$