Solution

1. $2x + y - 7 = 0$

$$y = -2x + 7 \qquad m = -2 \qquad (-3, 5)$$

$$y - 5 = -2(x + 3)$$
$$y - 5 = -2x - 6$$
$$2x + y + 1 = 0$$

2. $A(4, 0) \quad B(-4, 16) \quad C(18, 20)$

median BQ $\quad$ $\text{midpoint}_{AC} = \left(\frac{4+18}{2}, \frac{0+20}{2}\right) = \overset{Q}{(11, 10)}$

$$m_{BQ} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10 - 16}{11 + 4} = -\frac{6}{15} = -\frac{2}{5}$$

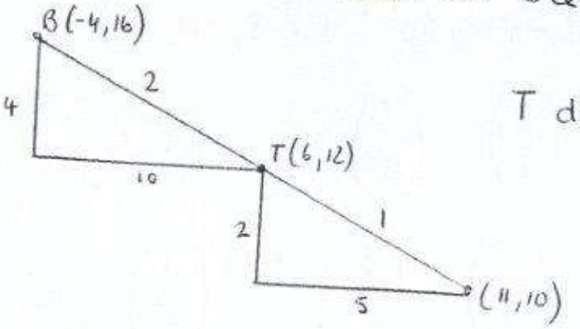
$$y - 16 = -\frac{2}{5}(x + 4)$$
$$5y - 80 = -2(x + 4)$$
$$5y - 80 = -2x - 8$$
$$2x + 5y - 72 = 0$$

(b) sub $T(6, 12)$ into $2x + 5y - 72 = 0$

$$2(6) + 5(12) - 72 = 0$$
$$LHS = RHS$$

$\therefore$ T lies on BQ

(c)



T divides BQ in the ratio 2 : 1

3. gradient$_{EG}$ $= -3$ $\quad m_{FH} = \frac{1}{3}$ since $m_{EG} \times m_{FH} = -1$

$F(6,15)$ $\quad m_{FH} = \frac{1}{3}$

$$y - 15 = \frac{1}{3}(x - 6)$$
$$3y - 45 = x - 6$$
$$x - 3y + 39 = 0$$

4. $m = \frac{1}{2}$

$$m = \tan\theta$$
$$\frac{1}{2} = \tan\theta$$
$$\theta = 26.6^\circ$$

obtuse angle $= 180 - 26.6$
$= 153.4^\circ$

5. (a) $180 - 117 = 63^\circ$

$$m = \tan 63$$
$$m = 1.96$$

(b) $180 - 38 = 142^\circ$

$$m = \tan 142$$
$$m = -0.78$$

6. $m = \tan 14^\circ$
$= 0.25$

$\therefore$ ramp is wheelchair accessible since gradient satisfies $0 < \quad \leq 0.25$.

Vectors.

1. $A(4,1,-2)$ $\quad B(5,-2,3)$ $\quad C(8,-11,18)$

$$\overrightarrow{AB} = \underline{b} - \underline{a} = \begin{pmatrix} 5 \\ -2 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \\ 5 \end{pmatrix}$$

$$\overrightarrow{BC} = \underline{c} - \underline{b} = \begin{pmatrix} 8 \\ -11 \\ 18 \end{pmatrix} - \begin{pmatrix} 5 \\ -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ -9 \\ 15 \end{pmatrix} = 3\begin{pmatrix} 1 \\ -3 \\ 5 \end{pmatrix}$$

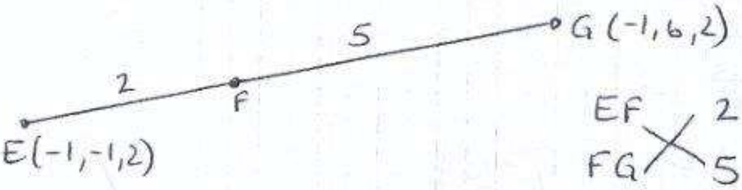
Since $3\overrightarrow{AB} = \overrightarrow{BC}$, the points are parallel and since they share point B, they are collinear.

Yes, the engineer has laid the flags correctly as they are collinear and lie in a ratio of 1 : 3.

AB : BC
1 : 3

A 1 B 3 C

2.



$$\frac{EF}{FG} \times \frac{2}{5}$$

$$5EF = 2FG$$

$$5(f - e) = 2(g - f)$$

$$5f - 5e = 2g - 2f$$

$$7f = 2g + 5e$$

$$7f = 2\begin{pmatrix}-1\\6\\2\end{pmatrix} + 5\begin{pmatrix}-1\\-1\\2\end{pmatrix} = \begin{pmatrix}-2\\12\\4\end{pmatrix} + \begin{pmatrix}-5\\-5\\10\end{pmatrix} = \begin{pmatrix}-7\\7\\14\end{pmatrix}$$

$$f = \begin{pmatrix}-1\\1\\2\end{pmatrix}$$

3. $K(6,-1,0) \quad L(4,-3,-2) \quad M(5,-1,8)$

$$\overrightarrow{LM} = m - l = \begin{pmatrix}5\\-1\\8\end{pmatrix} - \begin{pmatrix}4\\-3\\-2\end{pmatrix} = \begin{pmatrix}1\\2\\10\end{pmatrix}$$

$$\overrightarrow{LK} = k - l = \begin{pmatrix}6\\-1\\0\end{pmatrix} - \begin{pmatrix}4\\-3\\-2\end{pmatrix} = \begin{pmatrix}2\\2\\2\end{pmatrix}$$

$$\overrightarrow{LM} . \overrightarrow{LK} = 1 \times 2 + 2 \times 2 + 10 \times 2 = 2 + 4 + 20 = 26$$

$$|\overrightarrow{LM}| = \sqrt{1^2 + 2^2 + 10^2} = \sqrt{105}$$

$$\overrightarrow{LK} = \sqrt{2^2 + 2^2 + 2^2} = \sqrt{12}$$

$$\cos\theta = \frac{\overrightarrow{LM} . \overrightarrow{LK}}{|\overrightarrow{LM}||\overrightarrow{LK}|} = \frac{26}{\sqrt{105}\sqrt{12}}$$

$$\cos K\hat{L}M = 0.732$$

$$K\hat{L}M = 43^\circ$$

4. $\vec{RT} = \vec{RS} + \vec{ST}$

$= \vec{ST} - \vec{SR}$

$= \begin{pmatrix} 1 \\ 14 \\ 12 \end{pmatrix} - \begin{pmatrix} 14 \\ 5 \\ 7 \end{pmatrix} = \begin{pmatrix} -13 \\ 9 \\ 5 \end{pmatrix}$

5. $P(4,0,0) \quad Q(4,2,0) \quad U(4,2,3)$

(a) $R(0,2,0)$

$M(0,1,0)$

$N(4,2,2)$

$V(0,2,3)$

$\vec{UQ} = q - u = \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -3 \end{pmatrix}$

$\frac{1}{3}\vec{UQ} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \quad \vec{UN} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$

$\vec{UN} = \underline{n} - \underline{u} = \begin{pmatrix} 4 \\ 2 \\ 2 \end{pmatrix} - \begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$

(b) $\vec{VM} = m - v = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ -3 \end{pmatrix}$

$\vec{VN} = n - v = \begin{pmatrix} 4 \\ 2 \\ 2 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ -1 \end{pmatrix}$

$\vec{VM} \cdot \vec{VN}$

$= 0 \times 4 + (-1) \times 0 + (-3) \times (-1)$

$= 3$

$|\vec{VM}| = \sqrt{0^2 + (-1)^2 + (-3)^2}$

$= \sqrt{10}$

$|\vec{VN}| = \sqrt{4^2 + 0^2 + (-1)^2}$

$= \sqrt{17}$

(c) $\cos MVN = \dfrac{\vec{VM} \cdot \vec{VN}}{|\vec{VM}||\vec{VN}|}$

$= \dfrac{3}{\sqrt{10}\sqrt{17}}$

$\cos MVN = 0.23$

$M\hat{V}N = 76.7°$

6.(a) $B(4,4,0)$ $D(2,2,6)$ $M(2,0,0)$

(b) $\overrightarrow{DB} = b - d = \begin{pmatrix}4\\4\\0\end{pmatrix} - \begin{pmatrix}2\\2\\6\end{pmatrix} = \begin{pmatrix}2\\2\\-6\end{pmatrix}$

$\overrightarrow{DB}.\overrightarrow{DM}$
$= 2\times 0 + 2\times(-2) + (-6)\times(-6)$
$= 40$

$\overrightarrow{DM} = m - d = \begin{pmatrix}2\\0\\0\end{pmatrix} - \begin{pmatrix}2\\2\\6\end{pmatrix} = \begin{pmatrix}0\\-2\\-6\end{pmatrix}$

$|\overrightarrow{DB}| = \sqrt{2^2+2^2+(-6)^2}$
$= \sqrt{44}$

$\cos BDM = \dfrac{\overrightarrow{DB}.\overrightarrow{DM}}{|\overrightarrow{DB}||\overrightarrow{DM}|}$

$|\overrightarrow{DM}| = \sqrt{0^2+(-2)^2+(-6)^2}$
$= \sqrt{40}$

$= \dfrac{40}{\sqrt{44}\sqrt{40}}$

$\cos BDM = 0.953$

$BDM = 17.5$

Polynomials and Quadratics

1(a)
$$\begin{array}{r|rrrr} 1 & 2 & 1 & -8 & 5 \\ & & 2 & 3 & -5 \\ \hline & 2 & 3 & -5 & 0 \end{array}$$
$\therefore (x-1)$ is a factor, since $f(x) = 0$

(ii) $(x-1)(2x^2+3x-5) = 0$
$(x-1)(2x+5)(x-1) = 0$

(b) $x = 1$ $x = -5/2$

2(a)
$$\begin{array}{r|rrrr} 4 & 1 & -5 & 2 & 8 \\ & & 4 & -4 & -8 \\ \hline & 1 & -1 & -2 & 0 \end{array}$$
$\therefore (x-4)$ is a factor, since $f(x) = 0$

(ii) $(x-4)(x^2-x-2) = 0$
$(x-4)(x+1)(x-2) = 0$

(iii) $x = 4$ $x = -1$ $x = 2$

3. $$\begin{array}{r|rrrr} -1 & 2 & -5 & -1 & 6 \\ & & -2 & 7 & -6 \\ \hline & 2 & -7 & 6 & 0 \end{array}$$ $\therefore (x+1)$ is a factor, since $f(x)=0$

4. $$\begin{array}{r|rrrr} -1 & 1 & -7 & 7 & 15 \\ & & -1 & 8 & -15 \\ \hline & 1 & -8 & 15 & 0 \end{array}$$ $\therefore (x+1)$ is a factor, since $f(x)=0$

(b) $(x+1)(x^2\ 8x+15)=0$

$(x+1)(x-3)(x-5)=0$

(c) $x=-1 \quad x=3 \quad x=5$

5. $x=-6 \qquad x=4 \qquad x=1$

6. $f(x)=2x^2+4x-k \qquad b^2-4ac<0$

$a=2 \quad b=4 \quad c=-k \qquad (4)^2-4(2)(-k)<0$

$16+8k<0$

$8k<-16$

$k<-2$

7. $kx^2-x-1=0$

$a=k \quad b=-1 \quad c=-1 \qquad b^2-4ac<0$

$(-1)^2-4(k)(-1)<0$

$1+4k<0$

$4k<-1$

$k<-1/4$

8. (a) $y = x^3 + 3x^2 - 9x + 5$

$\frac{dy}{dx} = 3x^2 + 6x - 9$

S.P's occur when $\frac{dy}{dx} = 0$

$3x^2 + 6x - 9 = 0$

$3(x^2 + 2x - 3) = 0$

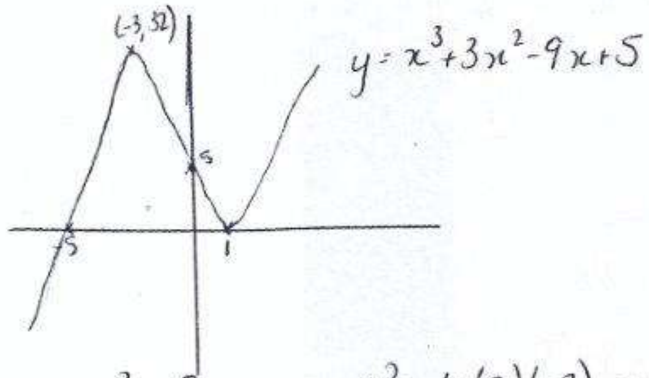
$3(x-1)(x+3) = 0$

$x = 1 \quad x = -3$

$y = 0 \quad y = 32$

x	$\xrightarrow{-4}$	-3	$\xrightarrow{-2}$	$\xrightarrow{0}$	1	$\xrightarrow{2}$
$\frac{dy}{dx}$	$+$	0	$-$	$-$	0	$+$
slope	/	—	\	\	—	/
		max T.P			min T.P	
		$(-3, 32)$			$(1, 0)$	

(b)



9. $2x^2 + px - 3 = 0$

$a = 2 \quad b = p \quad c = -3$

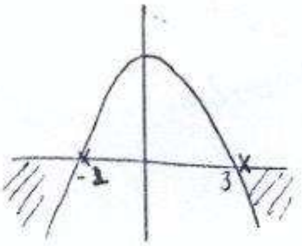
$p^2 - 4(2)(-3) = p^2 + 24$

Since $p^2 + 24 > 0$, roots are real for all values of p.

10. $6 + x - x^2 < 0$

$(3 - x)(2 + x) = 0$

$x = 3 \quad x = -2$



$x > 3$

$x < -2$

11. $1 - 6x - x^2$

$-(x + 6x) \quad + 1$

$-(x + 3)^2 + 9 + 1$

$10 - (x + 3)^2$

12.
$$
\begin{array}{r|rrrrr}
2 & 1 & a & -1 & b & -8 \\
 & & 2 & 2a+4 & 4a+6 & 8a+12+2b \\
\hline
 & 1 & a+2 & 2a+3 & 4a+6+b & 8a+2b+4=0
\end{array}
\qquad
\begin{array}{r|rrrrr}
-4 & 1 & a & -1 & b & -8 \\
 & & -4 & 16-4a & 16a-60 & -64a+24-4b \\
\hline
 & 1 & a-4 & 15-4a & 16a-60+b & -64a-4b+2
\end{array}
$$

$$8a+2b=-4 \quad \text{— ① } \times 2$$
$$64a+4b=232 \quad \text{— ② − ③}$$
$$16a+4b=-8 \quad \text{— ③}$$
$$48a=240$$
$$a=5$$

Sub $a=5$ into ①

$$40+2b=-4$$
$$2b=-44$$
$$b=-22$$

Differentiation

1. $y=3x^{-2}+2x^{3/2}$

$$\frac{dy}{dx}=-6x^{-3}+3x^{1/2}$$
$$=-\frac{6}{x^3}+3\sqrt{x}$$

2. $f(x)=5\sqrt{x}+\frac{3}{x^4}$

$$=5x^{1/2}+3x^{-4}$$
$$f'(x)=\frac{5}{2}x^{-1/2}-12x^{-5}$$
$$=\frac{5}{2\sqrt{x}}-\frac{12}{x^5}$$

3. $h=60t-2t^2$

$$\frac{dh}{dt}=60-4t \quad \text{when } t=15$$
$$v=60-4(15)$$
$$=0$$

The rocket is not moving.
- Stationary point

(b) $\frac{dh}{dt} = 60 - 4t \quad t = 20$

$= 60 - 4(20)$ The rocket is on its way down

$= -20$ - negative gradient.

4. $f(x) = 8\sin x$

$f'(x) = 8\cos x$

5. $y = \sin^3 x$

$\frac{dy}{dx} = 3\sin^2 x \cos x$

6. $y = 3x^2 - 5x - 7 \quad x = 2 \quad y = -5$

$\frac{dy}{dx} = 6x - 5$

$m = 6(2) - 5$

$= 7$

$y + 5 = 7(x - 2)$

$y + 5 = 7x - 14$

$7x - y - 19 = 0$

7. $y = \frac{1}{x^3} - \cos x$

$y = x^{-3} - \cos x$

$\frac{dy}{dx} = -3x^{-4} + \sin x$

$= \frac{-3}{x^4} + \sin x$

8. $y = \sqrt{3x^2 + 2}$

$y = (3x^2 + 2)^{1/2}$

$\frac{dy}{dx} = \frac{1}{2}(3x^2 + 2)^{-1/2} \cdot 6x$

$= \frac{3x}{\sqrt{3x^2 + 2}}$

9. $y = (8-2x^2)^{2/3}$

$\frac{dy}{dx} = \frac{2}{3}(8-2x)^{-1/3} \cdot -2$

$= -\frac{4}{3(8-2x)^{1/3}} = \frac{-4}{3\sqrt[3]{8-2x}}$

10. $y = 3\cos 5x$

$\frac{dy}{dx} = -15\sin 5x$

11. $y = 5x^3 - 12x$ $\quad (1,-7)$

$\frac{dy}{dx} = 15x^2 - 12$

$m = 15(1)^2 - 12$

$m = 3$

$y+7 = 3(x-1)$

$y+7 = 3x-3$

$3x - y - 10 = 0$

12. $y = \frac{1}{4}x^{-3}$

$\frac{dy}{dx} = -\frac{3}{4}x^{-4}$

13. $f(x) = \frac{x^2+1}{\sqrt{x}} = \frac{x^2+1}{x^{1/2}} = \frac{x^2}{x^{1/2}} + \frac{1}{x^{1/2}} = x^{3/2} + x^{-1/2}$

$f'(x) = \frac{3}{2}x^{1/2} - \frac{1}{2}x^{-3/2} = \frac{3\sqrt{x}}{2} - \frac{1}{2\sqrt{x^3}}$

$f'(4) = \frac{3\sqrt{4}}{2} - \frac{1}{2\sqrt{4^3}} = 3 - \frac{1}{16} = \frac{47}{16} = 2\frac{15}{16}$

14. $f(t) = (1-3t)^4$

$f'(t) = 4(1-3t)^3 \cdot -3$

$f'(t) = -12(1-3t)^3$

$f'(\frac{1}{6}) = -12(1-3(\frac{1}{6}))^3$

$= -12(\frac{1}{2})^3$

$= -\frac{12}{8} = -\frac{3}{2}$