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## Teacher

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## Mathematics

## Paper 1

National 5 Booster Paper A1-Solutions
Duration: 1 hour 15 minutes

Total Marks - 50

Attempt ALL questions.
You may NOT use a calculator
To earn full marks, you must show your working in your answers.
State the units for your answer where appropriate.
Write your answers clearly in the spaces provided in this booklet.
Use blue or black ink.

## Notes:

- This is a Booster Paper. Your May exam will be (a bit) harder than this.
- The Booster Papers get more challenging as you work through them.
- The final Booster Paper will be as challenging as your May exam.
- The number of marks indicated beside each question is intended as a guide and may differ slightly from SQA marking instructions.
- These original papers are produced independently of the SQA and are free of charge.
- All Booster Papers and answers can be found at www.maths180.com/BoosterPapers


## FORMULAE LIST

The roots of $a x^{2}+b x+c=0$ are $\quad x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}$

Sine Rule:
$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$

Cosine Rule:
$a^{2}=b^{2}+c^{2}-2 b c \cos A$ or $\cos A=\frac{b^{2}+c^{2}-a^{2}}{2 b c}$

Area of a triangle:
$A=\frac{1}{2} a b \sin C$

Volume of a sphere:
$V=\frac{4}{3} \pi r^{3}$

Volume of a cone:
$V=\frac{1}{3} \pi r^{2} h$

Volume of a pyramid:
$V=\frac{1}{3} A h$
Standard deviation: $\quad s=\sqrt{\frac{\sum(x-\bar{x})^{2}}{n-1}}$
or $\quad s=\sqrt{\frac{\sum x^{2}-\frac{\left(\sum x\right)^{2}}{n}}{n-1}}$, where $n$ is the sample size.

## Attempt ALL questions

MARKS

1. Multiply the brackets and simplify

$$
\begin{equation*}
(3 x-1)\left(x^{2}-2 x+3\right) \tag{3}
\end{equation*}
$$

Multiply the brackets

$$
=3 x^{3}-6 x^{2}+9 x-x^{2}+2 x-3
$$

Simplify

$$
=3 x^{3}-7 x^{2}+11 x-3
$$

2. Evaluate $17 \frac{2}{3}-8 \frac{3}{5}$ Leave your answer as a mixed number.

Subtract the whole numbers first

$$
=9 \frac{2}{3}-\frac{3}{5}
$$

Convert each fraction so they have the same denominator

$$
=9 \frac{10}{15}-\frac{9}{15}
$$

Finish the subtraction

$$
=9 \frac{1}{15}
$$

3. Decrease 840 by $13 \%$

Find $10 \%, 1 \%$ then $3 \%$ of 840

$$
\begin{gathered}
10 \% \text { of } 840=84 \\
1 \% \text { of } 840=8.4 \\
3 \% \text { of } 84=3 \times 8.4=25.2
\end{gathered}
$$

Add $10 \%$ and $3 \%$ to get $13 \%$

$$
13 \% \text { of } 84=84+25.2=109.2
$$

Subtract $13 \%$ of 840

$$
840-109.2=730.8
$$

4. Calculate the size of angle $A B C$ in the diagram below.


3
Angle ACB and the angle marked $95^{\circ}$ form a ' C -angle' so sum to $180^{\circ}$ That means that angle ACB must be $180-95=85^{\circ}$

The three corners of the triangle sum to $180^{\circ}$
So angle ABC must be $180-65-85=30^{\circ}$
5. (a) Factorise $12 x-18$

Take out the common factor

$$
=6(2 x-3)
$$

(b) Factorise $x^{2}-25$

Difference of squares

$$
=(x-5)(x+5)
$$

6. Express $x^{2}-6 x+11$ in the form $(x-a)^{2}+b$ by completing the square.

First make the squared bracket, so that it matches the $-6 x$ term

$$
\begin{aligned}
& x^{2}-6 x+11 \\
& =(x-3)^{2}+d
\end{aligned}
$$

Now multiply out the brackets to find the value of $d$

$$
=x^{2}-6 x+9+d
$$

So by comparing the +11 to the $9+d$ we get that $d=2$
State solution

$$
=(x-3)^{2}+2
$$

7. Change the subject of the formula to $k$

$$
\sqrt{\frac{k+7}{9}}=y
$$

3

Square both sides

$$
\frac{k+7}{9}=y^{2}
$$

Multiply both sides by 9

$$
k+7=9 y^{2}
$$

Subtract 7 from both sides

$$
k=9 y^{2}-7
$$

8. Fish food is on special offer.

Each jar on offer contains $30 \%$ more than the standard jar.
A jar on offer contains 390 grams of fish food.
How much does the standard jar contain?


State what we currently have

$$
130 \%=390
$$

Divide by 13 to get $10 \%$

$$
10 \%=30
$$

Multiply by 10 to get 100\%

$$
100 \%=300
$$

9. Simplify $\left(2 x^{5}\right)^{3}$

Cube both parts

$$
=2^{3}\left(x^{5}\right)^{3}
$$

Evaluate

$$
=8 \sqrt{ } x^{15}
$$

10. The diagram shows part of the graph of a quadratic function with equation $y=x^{2}+x-20$.


At $A$ and $B$ the height is 0 so set $y=0$

$$
x^{2}+x-20=0
$$

Factorise

$$
(x+5)(x-4)=0
$$

Get solutions

$$
x=-5 \text { or } x=4
$$

State co-ordinates

$$
A(-5,0) \quad B(4,0)
$$

(b) Find the coordinates of $C$, the $y$ - intercept of the graph.

At C we have that $x=0$

$$
\begin{aligned}
& y=0^{2}+0-20 \\
& y=-20
\end{aligned}
$$

State co-ordinate

$$
C(0,-20)
$$

11. Point $A(-3,7)$ and point $B(1,-3)$ are joined by a straight line
(a) Determine the gradient of this line.

Use the gradient formula

$$
\begin{aligned}
& m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& m=\frac{-3-7}{1-(-3)} \\
& m=\frac{-10}{4} \\
& m=-\frac{5}{2}
\end{aligned}
$$

(b) Determine the equation of the line.

Use $y-b=m(x-a)$ with the point $(-3,7)$ and the gradient $-\frac{5}{2}$

$$
\begin{aligned}
& y-b=m(x-a) \\
& y-7=-\frac{5}{2}(x-(-3)) \\
& y-7=-\frac{5}{2}(x+3) \\
& y-7=-\frac{5}{2} x-\frac{15}{2} \\
& y=-\frac{5}{2} x-\frac{1}{2}
\end{aligned}
$$

(c) Give the coordinates of the point where this line crosses the $y$-axis. 1

Crosses $y$-axis when $x=0$

$$
\begin{aligned}
& y=-\frac{5}{2}(0)-\frac{1}{2} \\
& y=-\frac{1}{2}
\end{aligned}
$$

State coordinates

$$
\left(0,-\frac{1}{2}\right)
$$

12. 

(a) Fully simplify $\sqrt{27}-\sqrt{12}$.

Rewrite each surd as a product that includes a square number

$$
=\sqrt{9} \sqrt{3}-\sqrt{4} \sqrt{3}
$$

Simplify

$$
=3 \sqrt{3}-2 \sqrt{3}
$$

Do the subtraction

$$
=\sqrt{3}
$$

(b) Write $\frac{15}{\sqrt{3}}$ with a rational denominator in its simplest form.

Multiply both sides by $\frac{\sqrt{3}}{\sqrt{3}}$

$$
\begin{aligned}
& \frac{15}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\
& =\frac{15 \sqrt{3}}{\sqrt{3} \sqrt{3}}
\end{aligned}
$$

Simplify $\sqrt{3} \sqrt{3}$

$$
=\frac{15 \sqrt{3}}{3}
$$

Simplify the fraction

$$
=5 \sqrt{3}
$$

13. At a florist shop, Steve buys 3 roses and 2 tulips for $£ 9.40$
(a) Write an equation to represent this information.

Define the variables (also acceptable to work in pounds not pence)
$r=$ price of a rose in pence
$t=$ price of a tulip in tulips
State equation (also acceptable to do in pounds not pence)

$$
3 r+2 t=940
$$

At the same florist shop, Natalie buys 2 roses and 4 tulips for $£ 8.40$
(b) Write an equation to represent this information.

State equation (using the same units as above, i.e. sticking with either pence or pounds)

$$
2 r+4 t=840
$$

(c) Find, algebraically, the cost of 1 rose and the cost of 1 tulip.

State the two equations together

$$
\begin{aligned}
& 3 r+2 t=940 \\
& 2 r+4 t=840
\end{aligned}
$$

Manipulate them to eliminate one variable (lots of ways to do this) for example, double the first equation

$$
6 r+4 t=1880
$$

Subtract the second equation from this

$$
4 r=1040
$$

Solve to find one variable

$$
r=260
$$

Substitute to find the other variable

$$
\begin{aligned}
& 3(260)+2 t=940 \\
& 780+2 t=940 \\
& 2 t=160 \\
& t=80
\end{aligned}
$$

State the solution
A tulip costs 80 pence, a rose costs $£ 2.60$
14. This rectangle has length given by $x+4$ and breadth given by $x-1$. All lengths are in centimetres.
(a) Show that the area can be written as $x^{2}+3 x-4$.

Area of a rectangle is base times height

$$
A=(x+4)(x-1)
$$

Expand and simplify

$$
\begin{array}{ll}
A=x^{2}-x+4 x-4 & x+4 \\
A=x^{2}+3 x-4 &
\end{array}
$$

The actual area of the rectangle measures 6 square centimetres.
(b) Find, algebraically, the value(s) of $x$.

The expression for the area from part (a) is equal to 6

$$
x^{2}+3 x-4=6
$$

Make the equation equal to zero

$$
x^{2}+3 x-10=0
$$

Factorise

$$
(x+5)(x-2)=0
$$

Find solutions

$$
x=-5 \text { or } x=2
$$

Consider which solutions are possible (as the lengths $x-1$ and $x+4$ must both be positive). The only valid solution is

$$
x=2
$$

## End of Booster Paper

