

Name SOLUTIONS

Teacher _____

Mathematics Paper 2

National 5 **Booster Paper A2**

Duration: 1 hour 50 minutes

Total Marks - 60

Attempt **ALL** questions.

You may use a calculator

To earn full marks, you must show your working in your answers.

State the units for your answer where appropriate.

Write your answers clearly in the spaces provided in this booklet.

Use **blue** or **black** ink.

Notes:

- This is a **Booster Paper**. Your May exam will be (a bit) harder than this.
- The Booster Papers get **more challenging** as you work through them.
- The final Booster Paper will be as challenging as your May exam.
- The number of marks indicated beside each question is intended as a guide and may differ slightly from SQA marking instructions.
- These original papers are produced independently of the SQA and are **free of charge**.
- All Booster Papers and answers can be found at www.maths180.com/BoosterPapers

FORMULAE LIST

The roots of $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Sine Rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine Rule: $a^2 = b^2 + c^2 - 2bc \cos A$ or $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

Area of a triangle: $A = \frac{1}{2} ab \sin C$

Volume of a sphere: $V = \frac{4}{3} \pi r^3$

Volume of a cone: $V = \frac{1}{3} \pi r^2 h$

Volume of a pyramid: $V = \frac{1}{3} Ah$

Standard deviation: $s = \sqrt{\frac{\sum (x - \bar{x})^2}{n-1}}$

or $s = \sqrt{\frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}}$, where n is the sample size.

Total marks - 60

Attempt ALL questions

1. A car was bought for £24000.
The value of the car depreciated at a rate of 8.5% every year.
Calculate the value of the car after 4 years.

3

$$\begin{aligned} \text{Multiplier} &= 100\% - 8.5\% = 91.5\% \\ &= 0.915 \end{aligned}$$

$$\begin{aligned} \text{New Value} &= 0.915^4 \times 24000 \\ &= 16822.696\dots \\ &= \underline{\underline{\pounds 16822.70}} \end{aligned}$$

2. Radio signals travel at a speed of approximately 3×10^8 metres per second.
A radio signal from Earth to a space probe takes 1.5 seconds.
Calculate the distance (in metres) from Earth to the space probe.
Give your answer in scientific notation.

3

$$\begin{aligned} D &= S \times T \\ &= 3 \times 10^8 \times 1.5 \\ &= \underline{\underline{4.5 \times 10^8}} \text{ metres} \end{aligned}$$

3. The results (in metres) of the top six athletes in the Women's long jump final at the 2004 Olympic Games in Athens are shown below.

7.07 7.05 7.05 6.96 6.85 6.83

- (a) Calculate (correct to 2 decimal places) the mean and standard deviation of these distances.

4

$$\bar{x} = \frac{7.07 + 7.05 + 7.05 + 6.96 + 6.85 + 6.83}{6}$$

$$\text{Mean} = \underline{\underline{6.97\text{m}}} \text{ (to 2 dp)}$$

x	$x - \bar{x}$	$(x - \bar{x})^2$
7.07	0.1	0.01
7.05	0.08	0.0064
7.05	0.08	0.0064
6.96	-0.01	0.0001
6.85	-0.12	0.0144
6.83	-0.14	0.0196

$$\sum (x - \bar{x})^2 = 0.0569$$

$$n = 6$$

$$s = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}}$$

$$= \sqrt{\frac{0.0569}{6 - 1}}$$

$$= \underline{\underline{0.11}} \text{ (to 2 dp)}$$

In the 2016 Olympic Games in Rio, the mean distance for the top six athletes in the Women's long jump final was 7.02 metres and the standard deviation was 0.14 metres.

- (b) Make two valid comparisons between the athletes.

2

On average, the athletes jumped further in the 2016 Games as the mean distance is greater ($7.02 > 6.97$).

The distances jumped at the 2004 Games were more consistent, as the standard deviation is lower ($0.11 < 0.14$).

4. Solve algebraically

$$6(x+7) = 5(4-x)$$

$$6x + 42 = 20 - 5x$$

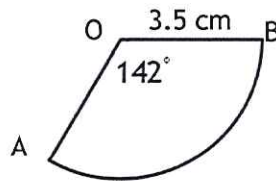
$$11x + 42 = 20$$

$$11x = -22$$

$$\underline{\underline{x = -2}}$$

3

5. The sector AOB shown here has radius of 3.5 centimetres and a sector angle of 142° .



$$d = 7\text{cm}$$

Find the length of the arc AB.

3

$$\text{Arc length} = \frac{\theta}{360} \times \pi d$$

$$= \frac{142}{360} \times \pi \times 7$$

$$= 8.674\dots$$

$$= \underline{\underline{8.67\text{cm}}} \text{ (to 3 sig fig)}$$

6. A shirt has been reduced in price by 40% to £27
Calculate the original price.

3

$$\begin{array}{rcl} 60\% & = & \text{£}27 \\ \div 3 \downarrow & & \downarrow \div 3 \\ 20\% & = & \text{£}9 \\ \times 5 \downarrow & & \downarrow \times 5 \\ 100\% & = & \text{£}45 \end{array}$$

The original price was £45

7. Simplify $\frac{3x^2 \times 4x^6}{6x^2}$

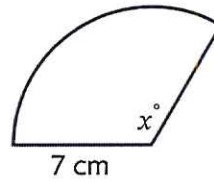
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$$\begin{aligned} &= \frac{12x^{(2+6)}}{6x^2} \\ &= \frac{12x^8}{6x^2} \\ &= 2x^{(8-2)} \\ &= \underline{\underline{2x^6}} \end{aligned}$$

8. This sector has an area of 55.6 square centimetres and radius 7 centimetres.

Calculate the size of angle x°

Give your answer to the nearest degree.



4

$$\frac{\text{Angle}}{360} = \frac{\cancel{\text{Angle}}}{\cancel{\pi r^2}} = \frac{\text{Area}}{\pi r^2}$$

$$\frac{x}{360} = \frac{55.6}{\pi \times 7^2}$$

$$\frac{x}{360} = \frac{55.6}{49\pi}$$

$$x = \frac{55.6 \times 360}{49\pi}$$

$$x = 130.02\dots$$

$$x = \underline{\underline{130^\circ}} \text{ (to nearest degree)}$$

9. Use the discriminant to determine the nature of the roots of $y = \frac{4x^2}{a} - \frac{20x}{b} + \frac{25}{c}$. 3

$$\text{Discriminant} = b^2 - 4ac$$

$$a = 4 \quad b = -20 \quad c = 25$$

$$\begin{aligned} & b^2 - 4ac \\ &= (-20)^2 - (4 \times 4 \times 25) \\ &= 400 - 400 \\ &= 0 \end{aligned}$$

As $b^2 - 4ac = 0$, there are two real, equal roots.

10. These two candles are mathematically similar.



The surface area of the small candle is 12.5 square centimetres

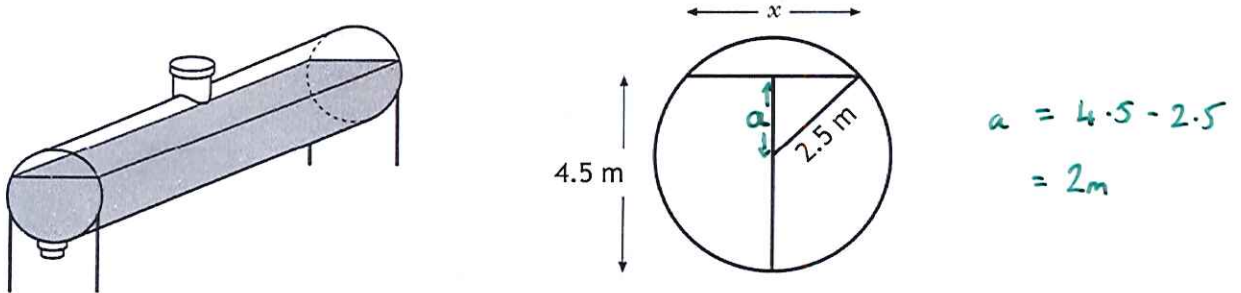
Find the surface area of the larger candle. 3

$$\text{Linear Scale Factor} = \frac{14}{7} = 2.$$

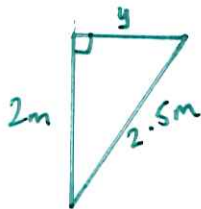
$$\begin{aligned} \text{Area Scale Factor} &= (\text{LSF})^2 \\ &= 2^2 \\ &= 4 \end{aligned}$$

$$\begin{aligned} \text{Area of Large Candle} &= \text{Small Area} \times \text{ASF} \\ &= 12.5 \times 4 \\ &= \underline{\underline{50 \text{ cm}^2}} \end{aligned}$$

10. An oil tank has a circular cross section of radius 2.5 metres.
The tank is filled to a depth of 4.5 metres.



- (a) Calculate x , the width (in metres) of the oil surface. 3

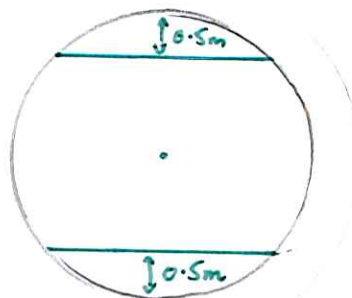


$$\begin{aligned}
 y^2 &= 2.5^2 - 2^2 \\
 &= 6.25 - 4 \\
 &= 2.25 \\
 y &= \sqrt{2.25} \\
 &= 1.5
 \end{aligned}$$

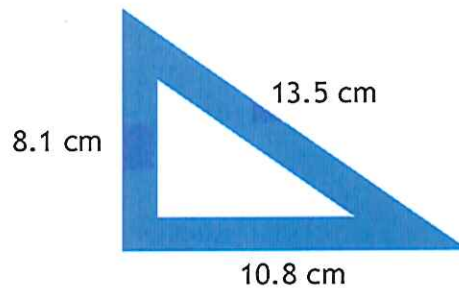
$$\begin{aligned}
 \text{Width of surface (x)} &= 1.5 \times 2 \\
 &= \underline{\underline{3\text{m}}}
 \end{aligned}$$

- (b) What other depth would give the same surface width? 1

Distance of oil from top of tank = 0.5m.
Therefore distance of oil from bottom of tank
to give the same surface width = 0.5m



12. A company is producing a new size of set square which must have a perfect right angle at one of its corners.
The company produce a prototype with sides of length 8.1 cm, 10.8 cm and 13.5 cm.



Is this set square acceptable?

You must justify your answer.

3

$$8.1^2 + 10.8^2 = 182.25$$
$$13.5^2 = 182.25$$

As $8.1^2 + 10.8^2 = 13.5^2$, by the Converse of Pythagoras, the triangle contains a right angle.
The set square is acceptable.

13. Solve the equation

$$2x^2 + 5x - 11 = 0$$

Give your answer(s) correct to two decimal places.

4

$$a = 2 \quad b = 5 \quad c = -11$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-5 \pm \sqrt{5^2 - (4 \times 2 \times -11)}}{2 \times 2}$$

$$x = \frac{-5 \pm \sqrt{113}}{4}$$

either $x = \frac{-5 + \sqrt{113}}{4}$ or $x = \frac{-5 - \sqrt{113}}{4}$

$= 1.407\dots$ $= -3.907\dots$

$= 1.41$ (2 dp) $= \underline{\underline{-3.91}}$ (2 dp)

14. Solve the equation

$$\frac{4x-5}{6} = 2x$$

Give your answer in its simplest form.

3

$$\frac{4x-5}{6} = 2x$$

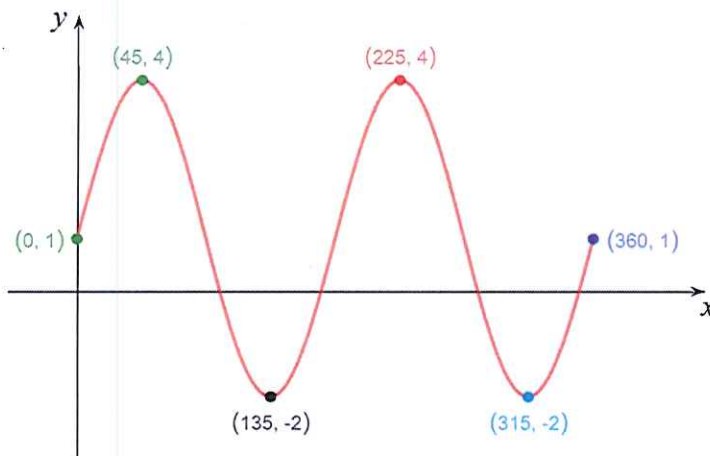
$$4x-5 = 12x$$

$$-5 = 8x$$

$$\underline{\underline{-\frac{5}{8} = x}}$$

$$(x = -\frac{5}{8})$$

15. This graph has an equation of the form $y = a \sin bx + c$



Write down the values of a , b and c .

3

$$a \text{ (amplitude)} = \underline{\underline{3}}$$

$$b \text{ (waves } 0 \leq x \leq 360) = \underline{\underline{2}}$$

$$c \text{ (vertical shift)} = \underline{\underline{1}}$$

16. (a) Write down the coordinates of the turning point of the graph of $y = (x-2)^2 + 1$.

2

x -value makes bracket 0 : $x-2=0$
 $x=2$

y -value is constant : 1

TP (2,1)

- (b) Write down equation of the axis of symmetry.

1

passing through turning point

$x=2$

- (c) Write down the coordinates of the point where the graph meets the y -axis.

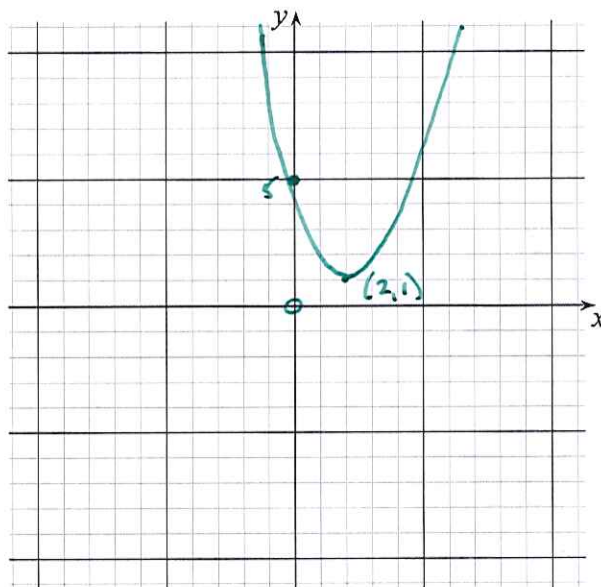
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Meets y -axis at $x=0$: $y = (0-2)^2 + 1$
 $= (-2)^2 + 1$
 $= 5$

(0,5)

- (d) Sketch and annotate fully the graph of $y = (x-2)^2 + 1$.

2

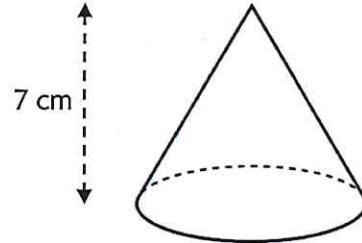


17. The cone shown has diameter 3 centimetres and vertical height 7 centimetres.

(a) Calculate the volume of this cone.

$$r = 1.5 \text{ cm}$$

$$\begin{aligned} V_{\text{cone}} &= \frac{1}{3} \pi r^2 h \\ &= \frac{1}{3} \pi \times 1.5^2 \times 7 \\ &= 16.493\dots \\ &= \underline{16.5 \text{ cm}^3} \quad (3 \text{ sig fig}) \end{aligned}$$



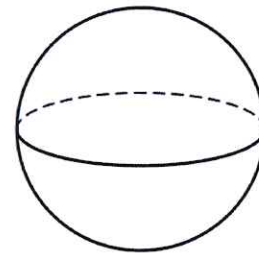
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The sphere shown here has volume 268.08 cubic centimetres.

(b) Find (to the nearest centimetre) the radius of this sphere.

$$\begin{aligned} V_{\text{sphere}} &= \frac{4}{3} \pi r^3 \\ \swarrow \quad \searrow \\ \frac{4}{3} \pi r^3 &= V \\ 4 \pi r^3 &= 3V \\ r^3 &= \frac{3V}{4\pi} \\ r &= \sqrt[3]{\frac{3V}{4\pi}} \\ r &= \sqrt[3]{\frac{3 \times 268.08}{4\pi}} \\ r &= 3.999\dots \end{aligned}$$

$$r = \underline{4 \text{ cm}} \quad (\text{to nearest cm})$$



3

End of Booster Paper A2